Lesson 6: Even More Graphs of Functions

Goals

- Compare and contrast (orally) peers' graphs that represent the same context.
- Comprehend that graphs representing the same context can appear different, depending on the variables chosen.
- Draw the graph of a function that represents a context, and explain (orally) which quantity is a function of which.

Learning Targets

- I can draw the graph of a function that represents a real-world situation.

Lesson Narrative

This lesson focuses on qualitative aspects of graphs, so there are no units or scale on the axes. In the warm up, students analyze two different graphs that represent the same situation (based on a series of photos). Depending on which quantities are chosen as the dependent and independent variable, both graphs describe different aspects of the same story. The two functions represented have the same independent variable (time), but different dependent variables (distance from edge of lawn vs. distance from the camera).

In the following activity, students identify independent and dependent variables from contexts and select an appropriate graph to match their choices. Different choices are possible, so students must be precise about which choice they are making and explain how the choice relates to the graph (MP6). In the final activity, students create a graph from a story. In doing so, students have to make many choices about the aspects of a situation they want to represent with a mathematical object—this is an important part of modeling with mathematics (MP4). Depending on the variables chosen, graphs of the same situation can appear to be different but still tell the same story.

Alignments

Addressing

- 8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Students are asked to make displays of their work in groups of 2–3. Prepare materials for creating a visual display in this way such as markers, chart paper, board space, etc.

Student Learning Goals

Let's draw a graph from a story.

6.1 Dog Run

Warm Up: 5 minutes
The purpose of this warm-up is for students to realize there are different dependent variables that can be used when making a model of a context and the choice of which we use affects how a graph of a function looks. In the warm-up, students compare two graphs and then determine what the creator of each graph chose as their dependent variable. During the partner discussion, students should listen to their partner's argument and decide if they agree or disagree with what they are saying (MP3).

Addressing

- 8.F.B.5

Launch

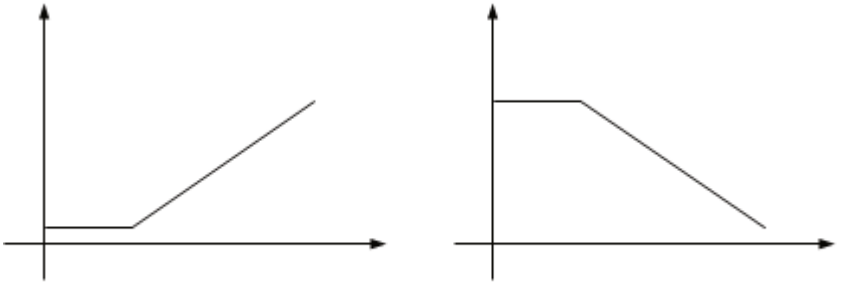
Display the 5 pictures of the dog from the print statement for all to see. Ask "What is happening in this succession of pictures?" After a brief quiet think time, invite students to share their answers. (As time elapses, the dog is moving.) Tell students they are going to consider two graphs describing the dog's movement.

Arrange students in groups of 2. Give 1–2 minutes of quiet work time and then have them share their responses with their partner to see if they agree or disagree with what variables Diego and Lin graphed. If partners do not agree, encourage students to make sense of their partner's thinking and reach a consensus. Follow with a whole-class discussion.

Student Task Statement

Here are five pictures of a dog taken at equal intervals of time.

Diego and Lin drew different graphs to represent this situation:



They both used time as the independent variable. What do you think each one used for the dependent variable? Explain your reasoning.

Student Response

Both graphs are good depictions of the scenario, using different variables as the dependent variables. For the dependent variables, Diego used the distance from the edge of the grass and Lin used the distance from the camera.

Activity Synthesis

The goal of this discussion is for students to understand that the same situation can be represented in different ways depending on what variables you choose to represent.

Select students to share the different variables that they think Lin and Diego used in their graphs. If any partners disagreed at first, ask those groups to share how they decided on their final response for what variables Diego and Lin were using.

6.2 Which Graph is It?

10 minutes

The purpose of this activity is for students to sketch a graph showing the qualitative features of the function described in the problem. Building on the warm-up, in the first problem students decide what independent and dependent variables were used to create a given graph. In the second problem, students choose the variables and make their own sketch of the context. The problems are designed to have multiple correct solutions based on which quantities students identify in the descriptions. For example, Jada's information could be viewed from the perspective that the time it

takes her to swim a lap depends on how much she practices. Alternatively, we could also say that the number of seconds she takes off her time depends on how much she practices.

Watch for groups choosing different graphs for the first problem and groups sketching different graphs for the last problem to share during discussion.

Addressing

- 8.F.B.5

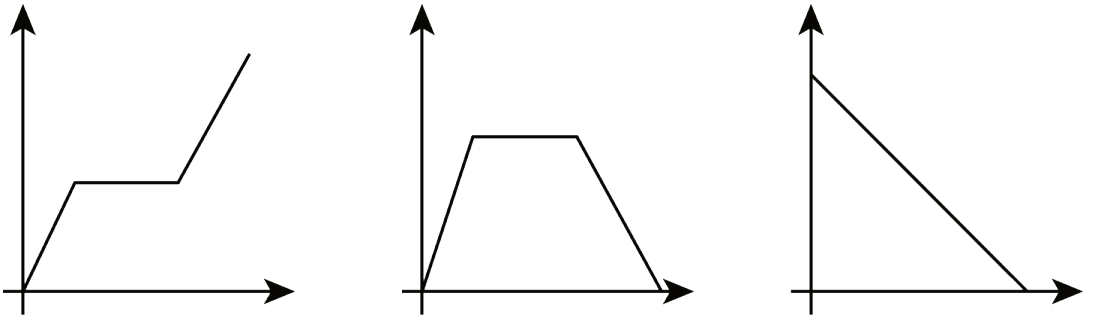
Instructional Routines

- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 2. Display the statement and graphs for all to see:

Elena filled up the tub and gave her small dog a bath. Then she let the water out of the tub.



Ask groups to decide which graph best fits the provided context and what independent and dependent variables were used to create it. Give 1–2 minutes for partners to discuss and then select 2–3 groups to share their graph pick and choice for variables. (The second graph shows the amount of water increasing, then staying steady, then decreasing over time. The amount of water in the bath tub is a function of time.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information as the graph through different modalities by using tables. If students are unsure where to begin, suggest they create a table to represent the independent and dependent variables.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Access for English Language Learners

Speaking: MLR8 Discussion Supports. As students explain which graph they think matches the water in Elena's bathtub, revoice student ideas to demonstrate mathematical language use. Press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. This will help students to produce and make sense of the language needed to communicate their own ideas.
Design Principle(s): Support sense-making; Optimize output (for explanation)

Anticipated Misconceptions

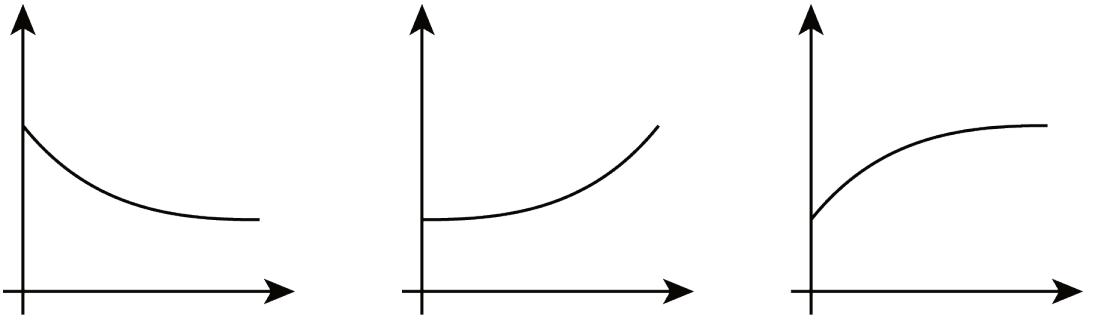
Students may have a difficult time representing the "jump" when money is added to the jar. Remind them of some of the function graphs they have seen in the past, such as with only discrete points plotted, to remind them that graphs do not have to be a single, connected line.

Student Task Statement

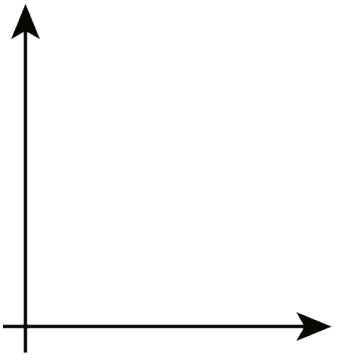
For each situation,

- name the independent and dependent variables
- pick the graph that best fits the situation, or sketch the graph if one isn't provided
- label the axes
- answer the question: which quantity is a function of which? Be prepared to explain your reasoning.

1. Jada is training for a swimming race. The more she practices, the less time it takes for her to swim one lap.

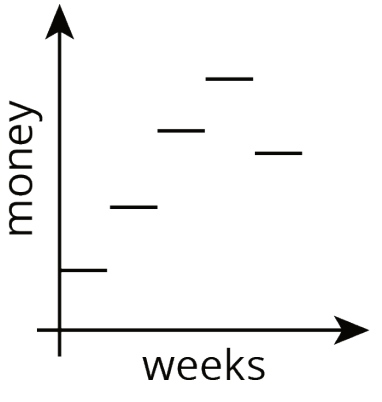


2. Andre adds some money to a jar in his room each week for 3 weeks and then takes some out in week 4.



Student Response

Answers vary. Sample responses:

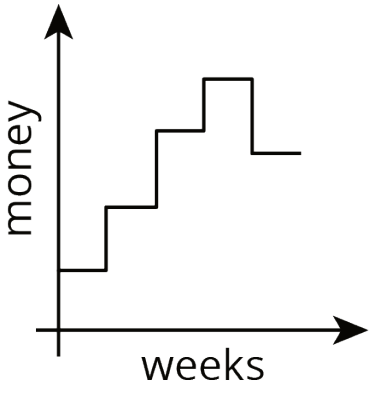
1. If practice time is the independent variable and time to swim one lap is the dependent variable, then the first graph is the best choice because an increase in practice time corresponds to a decrease in lap completion time. (If the quantities are assigned in the other order, the first graph will also be the best choice.) If practice time is the independent variable and pace is the dependent variable, then the second graph could be a good choice. If practice time is the independent variable and time she takes off her total lap time is the dependent variable, then the third graph is the best choice because increase practice time leads to an increase in how much time she has dropped from her lap time.
2. 

   The graph shows money in the jar increasing once per week three times, followed by a decrease in week 4. The amount of money in the jar is a function of the number of weeks.

Activity Synthesis

Select previously identified groups to share their responses and record and display their graphs for all to see.

If time allows, begin the discussion of the last problem by displaying this graph and asking students what they think of it as a possible representation of the amount of money in Andre's savings jar:



Give students 1 minute to consider the graph. Invite students to explain their thoughts about why it is or is not a good representation. (For example, the vertical lines would mean that at the same time the jar has two different amounts of money in it, which isn't possible, so this is not a good representation.) Remind students that functions only have one input for each output, so relationships whose graphs have vertical lines cannot be functions.

One way to represent a function that steps in this way uses open and closed circles to show that the function has only one value at each particular time. Discuss with students how to redraw this graph with open and closed circles so that the graph represents a function.

6.3 Sketching a Story about a Boy and a Bike

20 minutes (there is a digital version of this activity)
The purpose of this task is for students to sketch a graph from a story. In order to make the sketch, students must select two quantities from the story to graph, decide which is the independent variable and which is the dependent variable, and create and label their axes based on their decisions (MP4).

Monitor for displays that are correct but different from each other in important ways for students to focus on during the discussion. For example, they may differ by what variables were graphed such as:

- distance from home as a function of time
- distance from park as a function of time
- total distance traveled as a function of time

Addressing

- 8.F.B.5

Instructional Routines

- Group Presentations
- MLR2: Collect and Display

Launch

Arrange students in groups of 2–3. Distribute tools for creating a visual display. Before students begin, it may be necessary to demonstrate how to "create a set of axes" so that a first-quadrant graph can be sketched and is large enough to be seen from a distance. Group work time followed by a whole-class discussion.

If using the digital activity, the Activity Narrative is still valid, and student graphs will be used to drive the class discussion. What varies from the print activity is students will have access to an applet to create their graphs. Students will be able to share their graphs by projecting their screen. They may need to save their graph by either exporting the image or "printscreen" to a word document.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as the applet or graphing software. Some students may benefit from a checklist or list of steps to be able to use the applet or software.
Supports accessibility for: Organization; Conceptual processing; Attention

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. As students work in groups, capture the vocabulary and phrases students use to describe the relationship between the variables they selected over time. Listen for students who refer to the story to justify their decisions as they create the shape of the graph. Record their language on a visual display that can be referenced in future discussions. This will help students to produce and make sense of the language needed to communicate about the relationship between quantities represented by functions graphically and in story contexts.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Students may try and start graphing before they have clearly articulated and labeled their axes with their chosen variables. Encourage groups to make sure all are in agreement on what variables they are graphing before they create their display.

Student Task Statement

Your teacher will give you tools for creating a visual display. With your group, create a display that shows your response to each question.

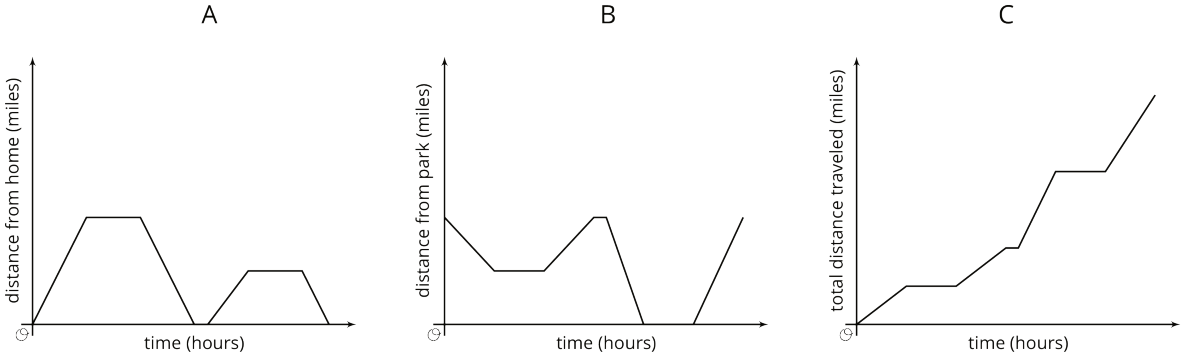
Here is a story: "Noah was at home. He got on his bike and rode to his friend's house and stayed there for awhile. Then he rode home again. Then he rode to the park. Then he rode home again."

1. Create a set of axes and sketch a graph of this story.
2. What are the two quantities? Label the axes with their names and units of measure. (For example, if this were a story about pouring water into a pitcher, one of your labels might say “volume (liters).”)
3. Which quantity is a function of which? Explain your reasoning.
4. Based on your graph, is his friend’s house or the park closer to Noah's home? Explain how you know.
5. Read the story and all your responses again. Does everything make sense? If not, make changes to your work.

Student Response

Answers vary. Sample response:

1. There are several ways of choosing which variables to use to tell the story, and once the variables are chosen, several ways to draw a graph to represent the story. These solutions here will reference the graphs A, B, and C, which are all possible graphical interpretations of the story.



2. In A, the quantities are the time elapsed (in hours), and the distance Noah is from home (in miles).
   In B, the quantities are the time elapsed (in hours), and the distance Noah is from the park (in miles).
   In C, the quantities are the time elapsed (in hours), and the total distance Noah has traveled (in miles).

3. In A, the distance Noah is from home is a function of time.
   In B, the distance Noah is from the park is a function of time.
   In C, the distance Noah has traveled is a function of time.

4. In A, his friend's house is farther than the park, since the graph indicates that while at his friend's house, Noah is further from home than when he is at the park.
   In B, it is impossible to tell from the graph which is farther from Noah's house. The graph indicates that the friend's house is closer to the park than it is to Noah's house, but not enough information about the distance between Noah's house and his friend's house to answer the question.
   In C, the park is closer than his friend's house, since the graph of Noah's total distance traveled changes more when he travels to the park than it does when he travels to his friend's house.

Are You Ready for More?

It is the year 3000. Noah's descendants are still racing around the park, but thanks to incredible technological advances, now with much more powerful gadgets at their disposal. How might their newfound access to teleportation and time-travel devices alter the graph of stories of their daily adventures? Could they affect whether or not the distance from home is a function of the time elapsed?

Student Response

Answers vary.

Activity Synthesis

Select previously identified groups to share their visual displays for all to see throughout the discussion. Give students 2–3 minutes of quiet think time to review the 3 displays and identify differences and similarities. Invite students to share what they identified and record and display the responses for all to see. If not brought up by students, ask:

- "What is the same and different about the two quantities that each group chose to use in their visual display?"
- "Did each group use the same units of measure? Why does it make sense to use the unit of measure seen in any of these examples?"
- "Why does it make sense to have time be the independent variable in this situation?"
- "Do all of these examples make sense in relation to the situation?"

Conclude the discussion by refocusing students on the input-output pairs described by the different graphs. For example, on a graph where distance from home is a function of time, there should be three inputs where the output is zero since he starts at home, returns home from his friend's house, and ends at home.

Alternatively, have the class do a "gallery walk" in which students leave written feedback on sticky-notes for the other groups. Here is guidance for the kind of feedback students should aim to give each other:

- “What is one thing that group did that would have made your project better if you had done it?”
- “What is one thing your group did that would have improved their project if they did it too?”
- “Does their answer make sense in the situation?”
- “Is their answer about which house is closer to the park clear and correct?”
- “If there was a mistake, what could they be more careful about in similar problems?”

Lesson Synthesis

Keep students in the same groups. Remind them of the multiple representations of the situation in the last activity. Tell students to imagine a situation that could be modeled in at least two different ways, depending upon which variables are chosen for the axes. Give time for students to write a clear explanation of the situation, the variables chosen, and how the choices would affect the appearance of the graph. If time allows, have students sketch an example of each of their graphs and share.

6.4 Walking Home From School

Cool Down: 5 minutes

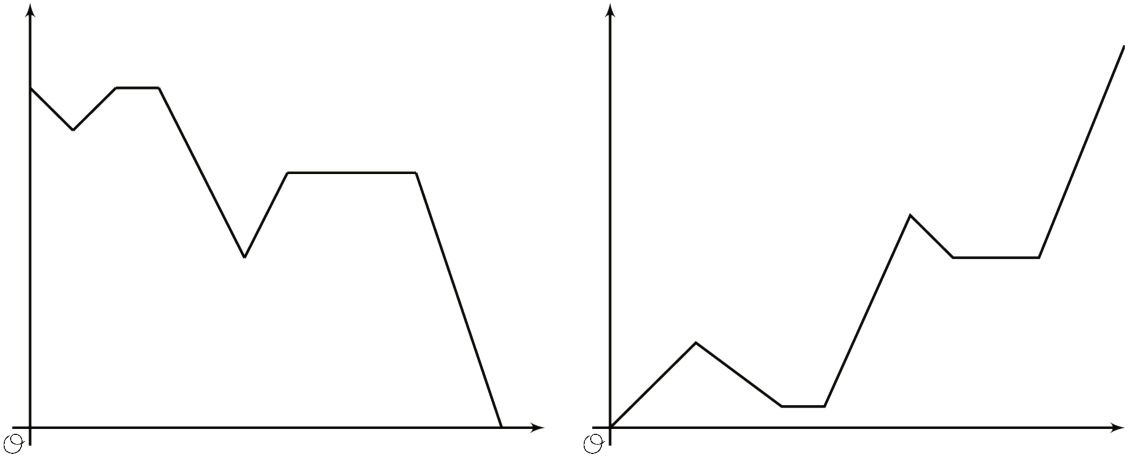
Addressing

- 8.F.B.5

Student Task Statement

Elena starts to walk home from school, but has to turn around and go back because she left something in her locker. On her way back home (the second time), she runs into her friend who invites her to the library to do homework with her. She stays at the library and then heads home to do her chores. Determine:

- Which graph fits Elena's story.
- What the two quantities are.
- Which quantity is a function of which.

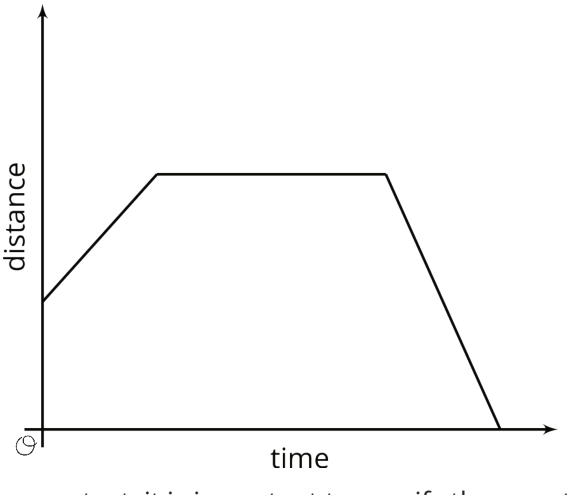


Student Response

The first graph most directly reflects Elena's story if the vertical axis represents Elena's distance from home and the horizontal axis represents the time since she started to walk home from school the first time. The graph then demonstrates that the distance from home is a function of the time elapsed.

Student Lesson Summary

Here is a graph showing Andre's distance as a function of time.



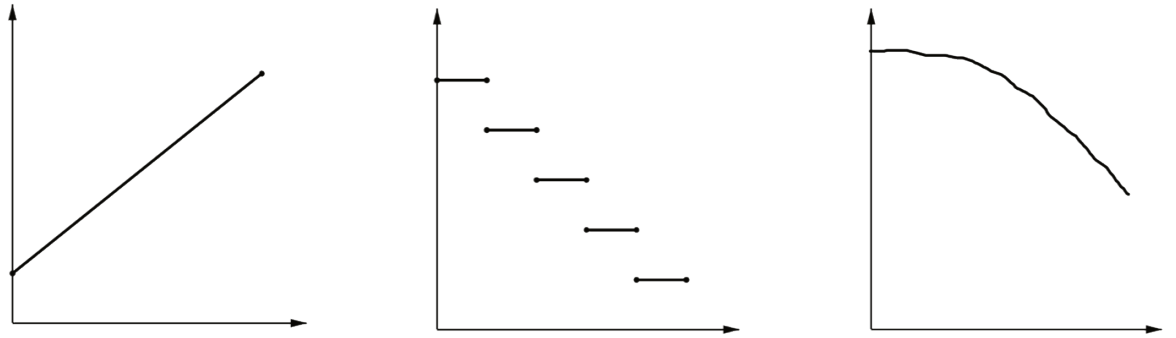
For a graph representing a context, it is important to specify the quantities represented on each axis. For example, if this is showing distance from home, then Andre starts at some distance from home (maybe at his friend's house), moves further away (maybe to a park), then returns home. If instead the graph is showing distance from school, the story may be Andre starts out at home, moves further away (maybe to a friend's house), then goes to school. What could the story be if the graph is showing distance from a park?

Lesson 6 Practice Problems

Problem 1

Statement

Match the graph to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.



A B C

a. Tyler pours the same amount of milk from a bottle every morning.

b. A plant grows the same amount every week.

c. The day started very warm but then it got colder.

d. A carnival has an entry fee of $5 and tickets for rides cost $1 each.

Solution

a. B. Independent variable and horizontal axis label: time (hours). Dependent variable and vertical axis label: amount of milk in bottle (ounces).

b. A. Independent variable and horizontal axis label: time (weeks). Dependent variable and vertical axis label: height of plant (inches).

c. C. Independent variable and horizontal axis label: time (hours). Dependent variable and vertical axis label: temperature (degrees Celsius).

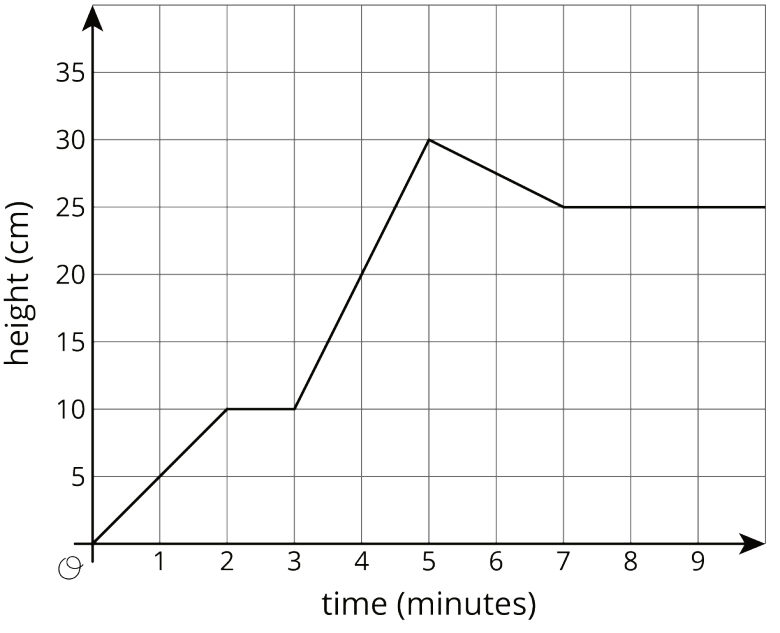
d. A. Independent variable and horizontal axis label: number of rides. Dependent variable and vertical axis label: cost to attend carnival in dollars.

Problem 2

Statement

Jada fills her aquarium with water.

The graph shows the height of the water, in cm, in the aquarium as a function of time in minutes. Invent a story of how Jada fills the aquarium that fits the graph.



Solution

Answers vary. One possible story: Jada turns on the water faucet, and the water in the aquarium is increasing at a constant rate for the first two minutes to a height of 10 cm. Then Jada's mom calls her to take out the trash, so she turns off the faucet for the minute it takes her to take out the trash. After she comes back, she turns on the water higher than before, and the water increases to a height of 30 cm in the next two minutes. This is high enough, and Jada turns off the water. Unfortunately, there is a slow leak, and the water height decreases to 25 cm. After two minutes, Jada notices the leak. She stops it, and the water stays constant after that.

Problem 3

Statement

Recall the formula for area of a circle.

a. Write an equation relating a circle's radius, r, and area, A.

b. Is area a function of the radius? Is radius a function of the area?

c. Fill in the missing parts of the table.

r	3		$\frac{1}{2}$	
A		16π		100π

Solution

a. $A = \pi r^2$

b. Yes for both. Any radius results in one and only one area. Any area results in one and only one radius, assuming that radii have to be positive.

c.

r	3	4	$\frac{1}{2}$	10
A	9π	16π	$\frac{1}{4}\pi$	100π

(From Unit 5, Lesson 4.)

Problem 4

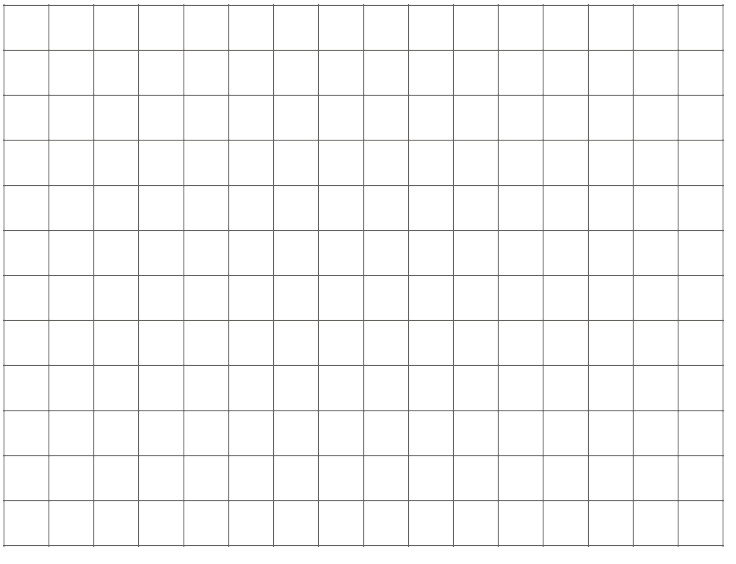
Statement

The points with coordinates $(4, 8)$, $(2, 10)$, and $(5, 7)$ all lie on the line $2x + 2y = 24$.

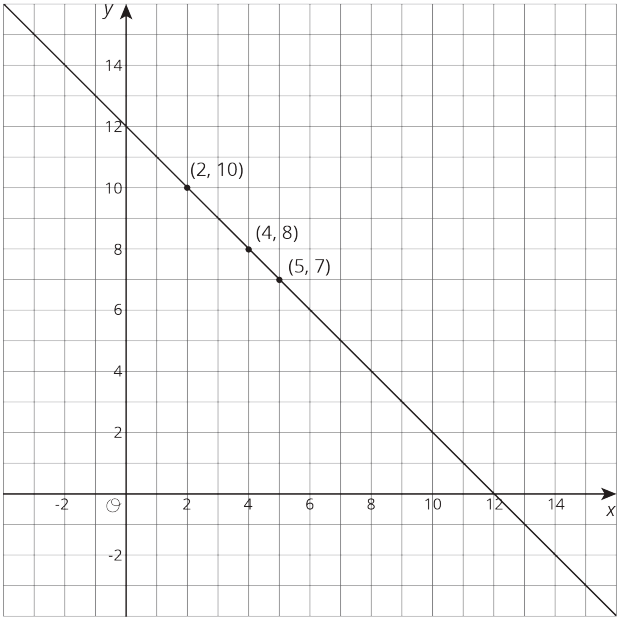
a. Create a graph, plot the points, and sketch the line.

b. What is the slope of the line you graphed?

c. What does this slope tell you about the relationship between lengths and widths of rectangles with perimeter 24?



Solution

a. 

b. -1

c. A slope of -1 means that for rectangles of perimeter 24, every extra unit of length put into the width is one less unit of length that can be put into the length.

(From Unit 3, Lesson 11.)