Lesson 7: Connecting Representations of Functions

Goals

- Compare and contrast (orally) representations of functions, and describe (orally) the strengths and weaknesses of each type of representation.
- Interpret multiple representations of functions, including graphs, tables, and equations, and explain (orally) how to find information in each type of representation.

Learning Targets

- I can compare inputs and outputs of functions that are represented in different ways.

Lesson Narrative

In this lesson, students compare two functions represented in different ways (graph and table, graph and equation, and table and verbal description). In each case, students use the different representations to find outputs for different inputs. Even though they use different representations, students are looking for the same information about the contexts and need to interpret each representation appropriately.

In a graph, students identify the input on the horizontal axis, then find the corresponding coordinate point on the graph, which lets them read the associated output. In a table, they find the input value in the first row (or column) and read the output value in the second. For functions given by equations, students substitute the input value into the expression on the right side of the equation and compute the corresponding output value on the left. Students also look for inputs corresponding to a given output by trying to reverse these procedures.

Each representation gives us the ability to find input-output pairs. However, each representation has strengths and weaknesses. Graphs require estimation but easily let us identify important features such as highest point or steepest section. Tables immediately let us find output values but only for limited input values. Equations let us precisely compute outputs for all inputs, but only one at a time. Comparing the different strengths of these representations helps students make decisions about how to use these tools strategically in the future.

Note that this lesson specifically avoids comparisons of linear functions to other linear functions, in order to avoid students associating “function” with only linear relationships. In a later lesson, students revisit some of these ideas and compare linear functions.

Alignments

Addressing

- 8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

- 8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1, 1)$, $(2, 4)$ and $(3, 9)$, which are not on a straight line.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- Think Pair Share

Student Learning Goals

Let’s connect tables, equations, graphs, and stories of functions.

7.1 Which are the Same? Which are Different?

Warm Up: 5 minutes

The purpose of this warm-up is for students to identify connections between three different representations of functions: equation, graph, and table. Two of the functions displayed are the same but with different variable names. It is important for students to focus on comparing input-output pairs when deciding how two functions are the same or different.

Addressing

- 8.F.A.2

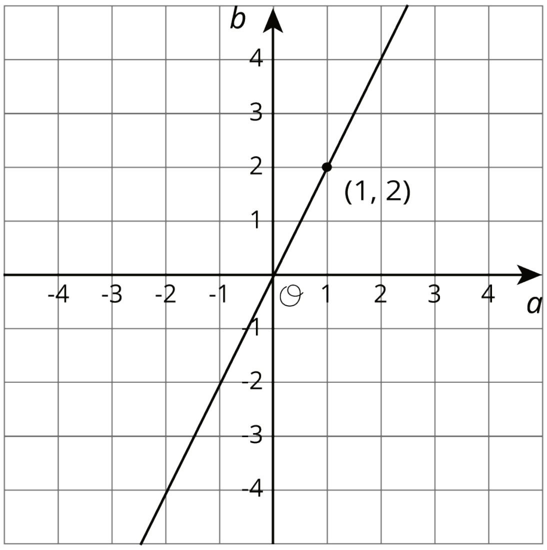
Launch

Give students 1–2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are three different ways of representing functions. How are they alike? How are they different?

$$y = 2x$$



p	-2	-1	0	1	2	3
q	4	2	0	-2	-4	-6

Student Response

Answers vary. Possible responses:

- The equations for the first two would have the same form but different variables.
- The graphs for the first two are identical except for the labels on the axes.
- A table of values for both the equation and graph would have the same ordered pairs but the variables names would be different.
- The third one has opposite outputs for the same input as the first two. The graph would be a line reflected across the y-axis as compared with the first two.

Activity Synthesis

Ask students to share ways the representations are alike and different. Record and display the responses for all to see. To help students clarify their thinking, ask students to reference the equation, graph, or table when appropriate. If the relationship between the inputs and outputs in each representation does not arise, ask students what they notice about that relationship in each representation.

7.2 Comparing Temperatures

10 minutes

This is the first of three activities where students make connections between different functions represented in different ways. In this activity, students are given a graph and a table of temperatures from two different cities and are asked to make sense of the representations in order to answer questions about the context.

Addressing

- 8.F.A.2
- 8.F.A.3

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time and then time to share responses with their partner. Follow with a whole-class discussion.

Access for Students with Disabilities

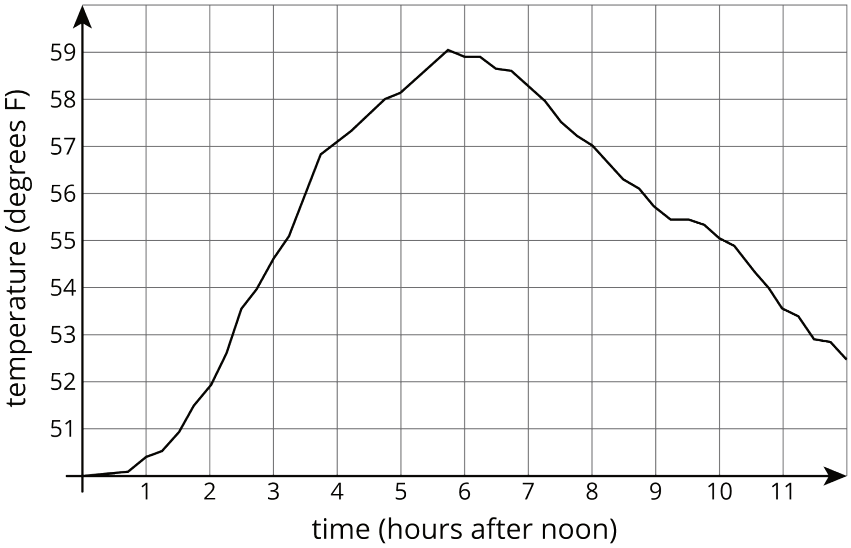
Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, ask students to use the same color to highlight the temperatures for each city that occurred at the same time such as blue for 4:00 pm.
Supports accessibility for: Visual-spatial processing

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display the graph and table to students without revealing the questions that follow. Invite students to work with their partner to write possible questions that could be answered by the two different representations for the two cities. Select 2–3 groups to share their questions with the class. Highlight questions that invite comparisons between the two cities. This helps students produce the language of mathematical questions and talk about the relationships between the graph and table.
Design Principle(s): Maximize meta-awareness; Support sense-making

Student Task Statement

The graph shows the temperature between noon and midnight in City A on a certain day.



The table shows the temperature, T, in degrees Fahrenheit, for h hours after noon, in City B.

h	1	2	3	4	5	6
T	82	78	75	62	58	59

1. Which city was warmer at 4:00 p.m.?
2. Which city had a bigger change in temperature between 1:00 p.m. and 5:00 p.m.?
3. How much greater was the highest recorded temperature in City B than the highest recorded temperature in City A during this time?
4. Compare the outputs of the functions when the input is 3.

Student Response

1. City B. From the graph, the temperature in City A is 57°F at 4:00 p.m., and from the table, the temperature in City B is 62°F
2. City B. From the graph, the temperature in City A increased about 7.5 degrees, from just under 50.5°F to just over 58°F. From the table, the temperature in B decreased 24 degrees, from 82°F degrees down to 58°F
3. About 23 degrees. From the graph, the highest recorded temperature in City A is about 59°F. From the table, the highest recorded temperature in City B is 82°F. $82 - 59 = 23$.
4. The first function gives the temperature in City A at 3:00 p.m., which is about 54.5°F. The second function gives the temperature in City B at 3:00 p.m., which is 75°F. City B is hotter than City A at that time by about 20.5 degrees, since $75 - 54.5 = 20.5$.

Activity Synthesis

Display the graph and table for all to see. Select groups to share how they used the two different representations to get their answers for each question. To further student thinking about the advantages and disadvantages of each representation, ask:

- “Which representation do you think is better for identifying the highest recorded temperature in a city?” (The graph, since I just have to find the highest part. In the table I have to read all the values in order to find the highest temperature.)
- “Which representation do you think is quicker for figuring out the change in temperature between 1:00 p.m. and 5:00 p.m.?” (The table was quicker since the numbers are given and I only have to subtract. In the graph I had to figure out the temperature values for both times before I could subtract.)

7.3 Comparing Volumes

10 minutes (there is a digital version of this activity)
This is the second of three activities where students make connections between different functions represented in different ways. In this activity, students are given an equation and a graph of the volumes of two different objects. Students then compare inputs and outputs of both functions and what those values mean in the context of the shapes.

Addressing

- 8.F.A.2
- 8.F.A.3

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time and then time to share their responses with their partner. Follow with a whole-class discussion.

If using the digital activity, students will have an interactive version of the graph that the print statement uses. Using this version, students can click on the graph to determine coordinates, which might be helpful. The focus of the discussion should remain on how and why students used the graph and equation.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example show or provide students with a cube and sphere. Discuss the relationship between the side length or radius and the volume of the object.
Supports accessibility for: Conceptual processing

Anticipated Misconceptions

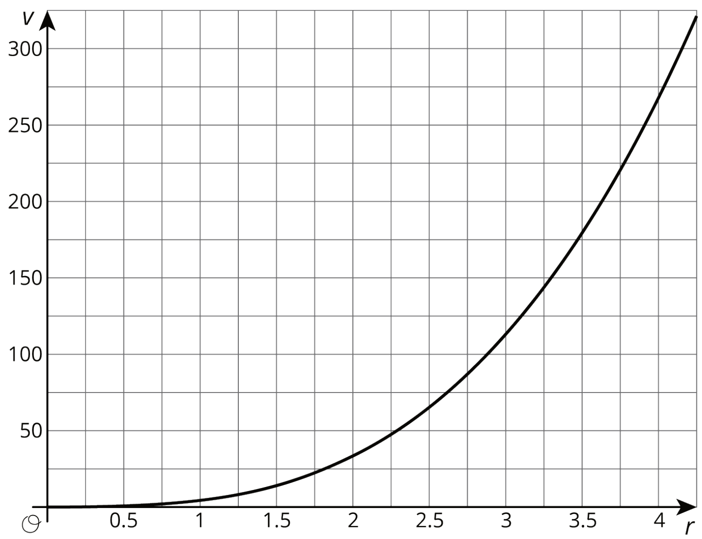
Some students may struggle with the many parts of the second question. These two questions can help scaffold the question for students who need it:

- "What information is given to you and what can you do with it?"
- "What information is the focus of the question and what would you like to know to be able to answer that question."

Student Task Statement

The **volume**, V, of a cube with edge length s cm is given by the equation $V = s^3$.

The volume of a sphere is a function of its radius (in centimeters), and the graph of this relationship is shown here.



1. Is the volume of a cube with edge length $s = 3$ greater or less than the volume of a sphere with radius 3?
2. If a sphere has the same volume as a cube with edge length 5, estimate the radius of the sphere.

3. Compare the outputs of the two volume functions when the inputs are 2.

Student Response

1. Less. The volume of a cube with edge length is 27 cm^3, since $27 = 3^3$. From the graph, the volume of a sphere of radius 3 cm^3 is over 100 cm^3.
2. About 3.1 cm. The volume of a cube with edge length 5 cm is 125 cm^3, since $125 = 5^3$. From the graph, the volume of a sphere with radius 3.1 cm is about 125 cm^3.
3. The output of the cube volume function is 8 cm^3 when the input is 2 cm, since $8 = 2^3$. From the graph, the output of the sphere volume function when the input is 2 cm is about 35 cm^3.

Are You Ready for More?

Estimate the edge length of a cube that has the same volume as a sphere with radius 2.5.

Student Response

About 4. From the graph, the volume of a sphere of radius 2.5 is about 65, whereas a cube of side length 4 has volume $4^3 = 64$.

Activity Synthesis

The purpose of this discussion is for students to think about how they used the information from the different representations to answer questions around the context.

Display the equation and graph for all to see. Invite groups to share how they used the representations to answer the questions. Consider asking the following questions to have students expand on their answers:

- "How did you use the given representations to find an answer? How did you use the equation? The graph?"
- "For which problems was it nicer to use the equation? The graph? Explain your reasoning."

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. As students work, capture the vocabulary and phrases students use to describe the connections across the two different representations (e.g., equation and graph). Listen for students who justify their ideas by explaining how or why they know something is true based on the equation or graph. Scribe students' language on a visual display that can be referenced in future discussions. This will help students to produce and make sense of the language needed to communicate about the relationships between quantities represented by functions graphically and in equations.
Design Principle(s): Support sense-making; Maximize meta-awareness

7.4 It's Not a Race

Optional: 10 minutes

In this activity, students continue their work comparing properties of functions represented in different ways. Students are given a verbal description and a table to compare and decide whose family traveled farther over the same time intervals. The purpose of this activity is for students to continue building their skill interpreting and comparing functions.

Addressing

- 8.F.A.2
- 8.F.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Give students 3–5 minutes of quiet work time, followed with whole-class discussion.

Anticipated Misconceptions

Students may miss that Elena's family's speed has different units than what is needed to compare with Andre's family. Point out to students that the units are important and possibly ask them to find out how many miles Elena's family travels in 1 minute rather than 1 hour.

Student Task Statement

Elena's family is driving on the freeway at 55 miles per hour.

Andre's family is driving on the same freeway, but not at a constant speed. The table shows how far Andre's family has traveled, d, in miles, every minute for 10 minutes.

t	1	2	3	4	5	6	7	8	9	10
d	0.9	1.9	3.0	4.1	5.1	6.2	6.8	7.4	8	9.1

1. How many miles per minute is 55 miles per hour?
2. Who had traveled farther after 5 minutes? After 10 minutes?
3. How long did it take Elena's family to travel as far as Andre's family had traveled after 8 minutes?
4. For both families, the distance in miles is a function of time in minutes. Compare the outputs of these functions when the input is 3.

Student Response

1. 0.92 miles per minute.
2.
    a. Andre had traveled farther after 5 minutes. Elena traveled 4.6 miles, because $5 \cdot 0.92 = 4.6$. From the table we see that Andre traveled 5.1 miles in that same time.

    b. Elena had traveled farther after 10 minutes. Elena traveled 9.2 miles, because $10 \cdot 0.92 = 9.2$. Andre traveled 9.1 miles in that same time.
3. About 8.04 minutes. After 8 minutes, Andre has traveled 7.4 miles. To find the number of minutes, t, it takes Elena to travel 7.4 miles at 0.92 miles per minute, we solve the equation $0.92t = 7.4$ to find that t is approximately 8.04 minutes.
4. The function for Andre's family gives an output of 3.0 miles on an input of 3 minutes. The function for Elena's family gives an output of 2.76 on an input of 3 minutes, since $3 \cdot 0.92 = 2.76$. Therefore, Andre's family has traveled a greater distance after 3 minutes than Elena's family did.

Activity Synthesis

The purpose of this discussion is for students to think about how they use a verbal description and table to answer questions related to the context. Ask students to share their solutions and how they used the equation and graph. Consider asking some of the following questions:

- "How did you use the table to get information? How did you use the verbal description?"
- "What did you prefer about using the description to solve the problem? What did you prefer about using the table to solve the problem?"

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Language; Social-emotional skills; Attention

---

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to reflect on their problem-solving strategies. Ask students to write an initial response to the prompt, “What did you prefer about using the description to solve the problem? What did you prefer about using the table to solve the problem?” Give students time to meet with 2–3 partners, to share and get feedback on their responses. Provide prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “What do you mean?”, “Can you give an example?”, and “Can you say that another way?”, etc.). Give students 1–2 minutes to revise their writing based on the feedback they received. This will help students produce a written generalization about interpreting and comparing functions.
Design Principle(s): Optimize output (for generalization)

Lesson Synthesis

We looked at several different ways functions are represented today: graphs, tables, equations, and verbal descriptions. We can use each representation to find outputs for different inputs. In each case we may perform different actions, but we are looking for the same information. Consider discussing these questions to emphasize the strengths and weaknesses of the representations with students:

- “Think about each of the representations of a function that we have used today. What is easy or hard about using each representation? What type of question do you prefer to answer with each?”
- “If we had only used tables in the volume activity, how could that have made it easier? Harder?” (It would have been easier to find the volumes when the inputs were the same value, but it would have been harder if the tables didn't have exactly the same inputs.)
- “If we had only used graphs to represent the functions in the car activity, how would that have been easier? How would it have been harder?” (It is easier to compare which is farther at a specific time, but it is harder to have an accurate answer, since graphs require estimation.)

7.5 Comparing Different Areas

Cool Down: 5 minutes

Addressing

- 8.F.A.2

Student Task Statement

The table shows the area of a square for specific side lengths.

side length (inches)	0.5	1	2	3
area (square inches)	0.25	1	4	9

The area A of a circle with radius r is given by the equation $A = \pi \cdot r^2$.

Is the area of a square with side length 2 inches greater than or less than the area of a circle with radius 1.2 inches?

Student Response

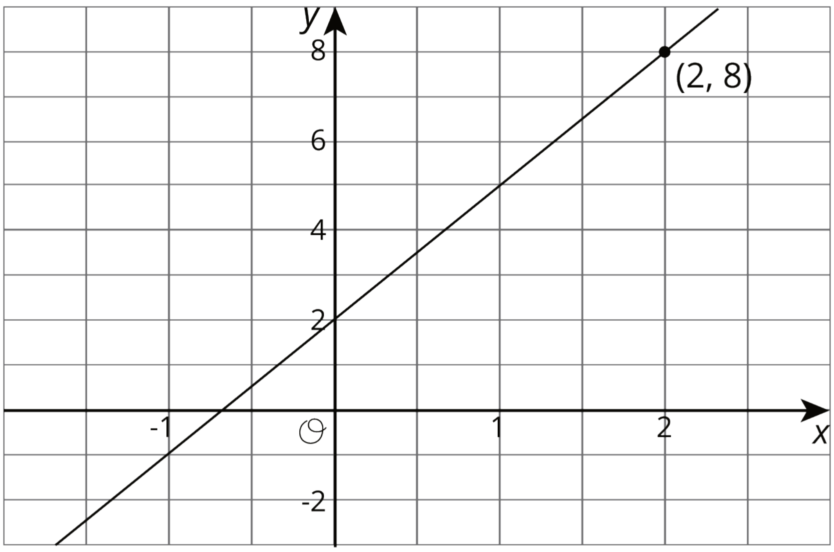
Less than. From the table the area of a square of side length 2 inches is 4 square inches,whereas from the equation the area of a circle with radius 1.2 inches is about 4.52 square inches.

Student Lesson Summary

Functions are all about getting outputs from inputs. For each way of representing a function—equation, graph, table, or verbal description—we can determine the output for a given input.

Let's say we have a function represented by the equation $y = 3x + 2$ where y is the dependent variable and x is the independent variable. If we wanted to find the output that goes with 2, we can input 2 into the equation for x and finding the corresponding value of y. In this case, when x is 2, y is 8 since $3 \cdot 2 + 2 = 8$.

If we had a graph of this function instead, then the coordinates of points on the graph are the input-output pairs. So we would read the y-coordinate of the point on the graph that corresponds to a value of 2 for x. Looking at the graph of this function here, we can see the point $(2, 8)$ on it, so the output is 8 when the input is 2.



A table representing this function shows the input-output pairs directly (although only for select inputs).

x	-1	0	1	2	3
y	-1	2	5	8	11

Again, the table shows that if the input is 2, the output is 8.

Glossary

- volume

Lesson 7 Practice Problems

Problem 1

Statement

The equation and the tables represent two different functions. Use the equation $b = 4a - 5$ and the table to answer the questions. This table represents c as a function of a.

a	-3	0	2	5	10	12
c	-20	7	3	21	19	45

a. When a is -3, is b or c greater?

b. When c is 21, what is the value of a? What is the value of b that goes with this value of a?

c. When a is 6, is b or c greater?

d. For what values of a do we know that c is greater than b?

Solution

a. b

b. $a = 5, b = 15$

c. There is not enough information to answer this question, since 6 is not in the table for a.

d. 0, 5, and 12

Problem 2

Statement

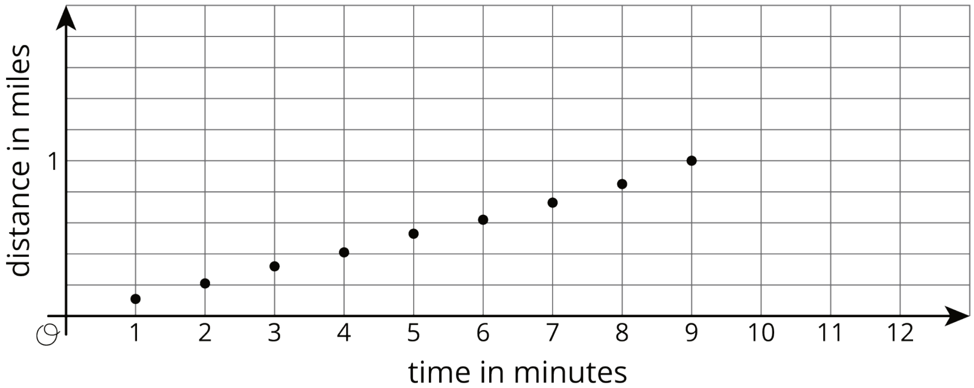
Elena and Lin are training for a race. Elena runs her mile at a constant speed of 7.5 miles per hour.

Lin’s total distances are recorded every minute:

time (minutes)	1	2	3	4	5	6	7	8	9
distance (miles)	0.11	0.21	0.32	0.41	0.53	0.62	0.73	0.85	1

a. Who finished their mile first?

b. This is a graph of Lin’s progress. Draw a graph to represent Elena’s mile on the same axes.

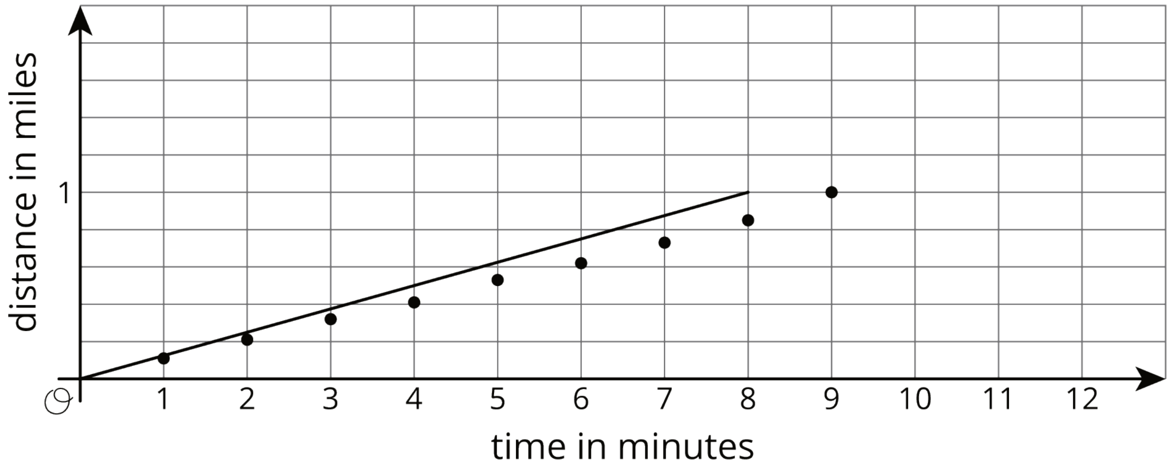


c. For these models, is distance a function of time? Is time a function of distance? Explain how you know.

Solution

a. Elena finished her mile first. It took her 8 minutes to complete her mile, but took Lin 9 minutes.

b.



c. In both models, distance is a function of time, and time is also a function of distance. Given a time for either runner the distance can be found, and vice versa.

Problem 3

Statement

Match each function rule with the value that could not be a possible input for that function.

A. 3 divided by the input

B. Add 4 to the input, then divide this value into 3

C. Subtract 3 from the input, then divide this value into 1

1. 3
2. 4
3. -4
4. 0
5. 1

Solution

- A: 4
- B: 3
- C: 1

(From Unit 5, Lesson 2.)

Problem 4

Statement

Find a value of x that makes the equation true. Explain your reasoning, and check that your answer is correct.

$$-(-2x + 1) = 9 - 14x$$

Solution

$x = \frac{5}{8}$. This is the same as $2x - 1 = 9 - 14x$. If 1 is added to each side, that results in $2x = 10 - 14x$. If $14x$ is added to each side, then $16x = 10$. Both sides are then multiplied by $\frac{1}{16}$ to find $x = \frac{10}{16}$ or $\frac{5}{8}$. This is correct because $-(-2(\frac{5}{8}) + 1) = \frac{5}{4} - 1 = \frac{1}{4}$ and $9 - 14(\frac{5}{8}) = \frac{36}{4} - \frac{35}{4} = \frac{1}{4}$.

(From Unit 4, Lesson 4.)