## Lesson 4: Positive Rational Exponents

* Let’s use roots to write exponents that are fractions.

### 4.1: Math Talk: Regrouping Fractions

Find the value of each expression mentally.

$\frac{1}{2}⋅5⋅4$

$\frac{5}{2}⋅4$

$\frac{2}{3}⋅7⋅\frac{3}{2}$

$7⋅\frac{5}{3}⋅\frac{3}{7}$

### 4.2: You Can Use Any Fraction As an Exponent

1. Use exponent rules to explain why these expressions are equal to each other:

|  |  |
| --- | --- |
| * $\left(5^{\frac{1}{3}}\right)^{2}$
 | * $\left(5^{2}\right)^{\frac{1}{3}}$
 |

*
1. Write $5^{\frac{2}{3}}$ using radicals.
2. Write $5^{\frac{4}{3}}$ using radicals. Show your reasoning using exponent rules.

### 4.3: Fractional Powers Are Just Numbers

1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| * $x$
 | * 0
 | * $\frac{1}{3}$
 | * $\frac{2}{3}$
 | * 1
 | * $\frac{4}{3}$
 | * $\frac{5}{3}$
 | * 2
 |
| * $2^{x}$ (using exponents)
 | * $2^{0}$
 | * $2^{\frac{1}{3}}$
 | * $2^{\frac{2}{3}}$
 | * $2^{1}$
 | * $2^{\frac{4}{3}}$
 | * $2^{\frac{5}{3}}$
 | * $2^{2}$
 |
| * $2^{x}$ (decimal approximation)
 | *
 | *
 | *
 | *
 | *
 | *
 | *
 |

* 1. Plot the points that you filled in.
	+ 
	1. Connect the points as smoothly as you can.
	2. Use this graph of $y=2^{x}$ to estimate the value of the other powers in the table, and write your estimates in the table.
1. Let’s investigate $2^{\frac{1}{3}}$:
	1. Write $2^{\frac{1}{3}}$ using radical notation.
	2. What is $\left(2^{\frac{1}{3}}\right)^{3}$?
	3. Raise your estimate from the table of $2^{\frac{1}{3}}$ to the third power. What should it be? How close did you get?
2. Let’s investigate $2^{\frac{2}{3}}$:
	1. Write $2^{\frac{2}{3}}$ using radical notation.
	2. What is the value of $\left(2^{\frac{2}{3}}\right)^{3}$?
	3. Raise your estimate from the table of $2^{\frac{2}{3}}$ to the third power. What should it be? How close did you get?

#### Are you ready for more?

Answer these questions using the fact that $(1.26)^{3}=2.000376$.

1. Explain why $\sqrt[3]{2}$ is very close to $1.26$. Is it larger or smaller than $1.26$?
2. Is it possible to write $\sqrt[3]{2}$ exactly with a finite decimal expansion? Explain how you know.

### Lesson 4 Summary

Using exponent rules, we know $3^{\frac{1}{4}}$ is the same as $\sqrt[4]{3}$ because $\left(3^{\frac{1}{4}}\right)^{4}=3$. But what about $3^{\frac{5}{4}}$?

Using exponent rules,

$3^{\frac{5}{4}}=\left(3^{5}\right)^{\frac{1}{4}}$

which means that

$3^{\frac{5}{4}}=\sqrt[4]{3^{5}}$

Since $3^{5}=243$, we could just write $3^{\frac{5}{4}}=\sqrt[4]{243}$.

Alternatively, we could express the fraction $\frac{5}{4}$ as $\frac{1}{4}⋅5$ instead. Using exponent rules, we get

$3^{\frac{5}{4}}=\left(3^{\frac{1}{4}}\right)^{5}=\left(\sqrt[4]{3}\right)^{5}$

Here are more examples of exponents that are fractions and their equivalents:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ | 0 | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 | $\frac{4}{3}$ | $\frac{5}{3}$ | 2 |
| $5^{x}$ (using exponents) | $5^{0}$ | $5^{\frac{1}{3}}$ | $5^{\frac{2}{3}}$ | $5^{1}$ | $5^{\frac{4}{3}}$ | $5^{\frac{5}{3}}$ | $5^{2}$ |
| $5^{x}$ (equivalent expression) | 1 | $\sqrt[3]{5}$ | $\sqrt[3]{5^{2}}$ or $\sqrt[3]{25}$ | 5 | $\sqrt[3]{5^{4}}$ or $\sqrt[3]{625}$ | $\sqrt[3]{5^{5}}$ or $\sqrt[3]{3125}$ | 25 |



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