Lesson 12: What is Surface Area?

Goals

- Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method.
- Comprehend that the term “surface area” (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.

Learning Targets

- I know what the surface area of a three-dimensional object means.

Lesson Narrative

This lesson introduces students to the concept of **surface area**. They use what they learned about area of rectangles to find the surface area of prisms with rectangular **faces.**

Students begin exploring surface area in concrete terms, by estimating and then calculating the number of square sticky notes it would take to cover a filing cabinet. Because students are not given specific techniques ahead of time, they need to make sense of the problem and persevere in solving it (MP1). The first activity is meant to be open and exploratory. In the second activity, they then learn that the surface area (in square units) is the number of unit squares it takes to cover all the surfaces of a three-dimensional figure without gaps or overlaps (MP6).

Later in the lesson, students use cubes to build rectangular prisms and then determine their surface areas.

Alignments

Addressing

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

- Poll the Class
- Think Pair Share

Required Materials

Snap cubes

Required Preparation

- Prepare 12 cubes per student and extra copies of isometric dot paper for Building with Snap Cubes activity.
- Build several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes for the cool-down.

Student Learning Goals

Let's cover the surfaces of some three-dimensional objects.

12.1 Covering the Cabinet (Part 1)

Warm Up: 5 minutes
This activity prepares students to think about surface area, which they explore in this lesson and upcoming lessons. Students watch a video of a cabinet being gradually tiled with non-overlapping sticky notes. The cabinet was left only partially tiled, which raises the question of the number of sticky notes it takes to cover the entire rectangular prism. Students estimate the answer to this question.

This activity was inspired by Andrew Stadel. Media used with permission. http://www.estimation180.com/filecabinet.

Building Towards

- 6.G.A.4

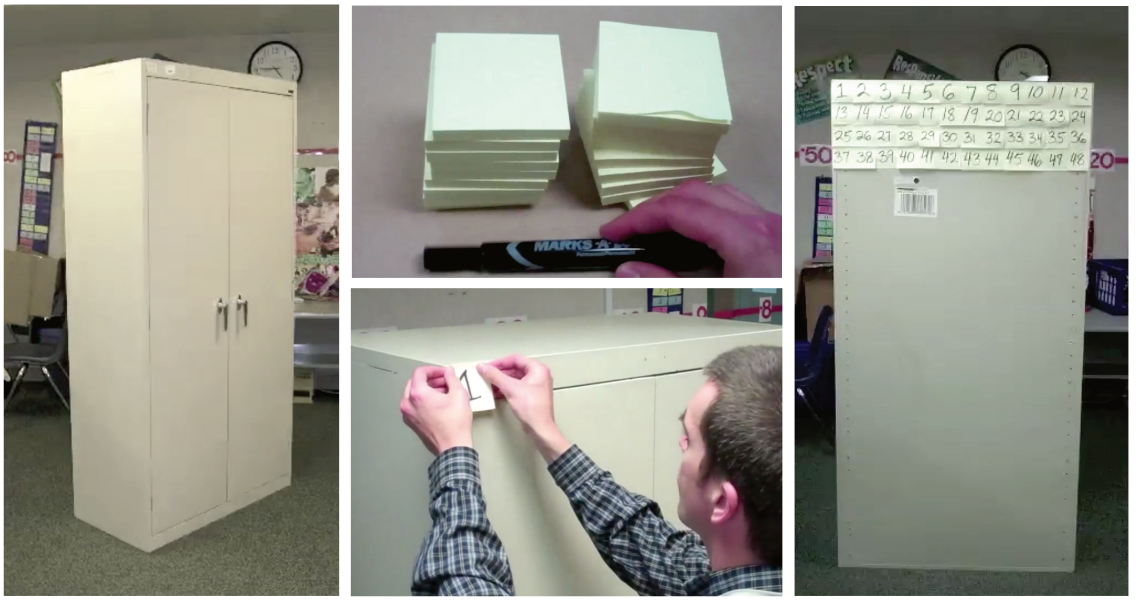
Instructional Routines

- Notice and Wonder
- Poll the Class

Launch

Arrange students in groups of 2. Show the video of a teacher beginning to cover a large cabinet with sticky notes or display the following still images for all to see. Before starting the video or displaying the image, ask students be prepared to share one thing they notice and one thing they wonder.

Video 'File Cabinet - Act 1' available here: https://player.vimeo.com/video/304136534.



Give students a minute to share their observation and question with a partner. Invite a few students to share their questions with the class. If the question “How many sticky notes would it take to cover the entire cabinet?” is not mentioned, ask if anyone wondered how many sticky notes it would take to cover the entire cabinet.

Give students a minute to make an estimate.

> **Student Task Statement**
>
> Your teacher will show you a video about a cabinet or some pictures of it.
>
> Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

Student Response

Estimates vary. The actual number of sticky notes is 935. Good estimates are in the 800–1,200 range.

Activity Synthesis

Poll the class for students' estimates, and record them for all to see. Invite a couple of students to share how they made their estimate. Explain to students that they will now think about how to answer this question.

12.2 Covering the Cabinet (Part 2)

20 minutes

After making an estimate of the number of sticky notes on a cabinet in the warm-up, students now brainstorm ways to find that number more accurately and then go about calculating an

answer. The activity prompts students to transfer their understandings of the area of polygons to find the **surface area** of a three-dimensional object.

Students learn that the surface area of a three-dimensional figure is the total area of all its faces. Since the area of a region is the number of square units it takes to cover the region without gaps and overlaps, surface area can be thought of as the number of square units that needed to cover all sides of an object without gaps and overlaps. The square sticky notes illustrate this idea in a concrete way.

As students work, notice the varying approaches taken to determine the number of sticky notes needed to tile the faces of the cabinet (excluding the bottom). Identify students with different strategies to share later.

Addressing

- 6.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Give students 1 minute of quiet time to think about the first question and another minute to share their responses with their group. Ask students to pause afterwards.

Select some students to share how they might go about finding out the number of sticky notes and what information they would need. Students may ask for some measurements:

- The measurements of the cabinet in terms of sticky notes: Tell students that the cabinet is 24 by 12 by 6.
- The measurements of the cabinet in inches or centimeters: Tell students that you don't have that information and prompt them to think of another piece of information they could use.
- The measurements of each sticky note: Share that it is 3 inches by 3 inches.

If no students mention needing the edge measurements of the cabinet in terms of sticky notes, let them begin working on the second question and provide the information when they realize that it is needed. Give students 8–10 minutes for the second question.

Anticipated Misconceptions

Students may treat all sides as if they were congruent rectangles. That is, they find the area of the front of the cabinet and then just multiply by 5, or act as if the top is the only side that is not congruent to the others. If there is a real cabinet (or any other large object in the shape of a rectangular prism) in the classroom, consider showing students that only the sides opposite each other can be presumed to be identical.

Students may neglect the fact that the bottom of the cabinet will not be covered. Point out that the bottom is inaccessible because of the floor.

Student Task Statement

Earlier, you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

2. Use the information you have to find the number of sticky notes to cover the cabinet. Show your reasoning.

Student Response

1. Find the area of each side of the cabinet, excluding the bottom, and add them together. Needed information: measurements of the cabinet edge lengths in sticky notes.

2. Answers vary. Strategies may be a combination of the following two strategies:
   - Multiply the number of sticky notes along each edge of each side. Add all of the products.
   - Multiply the edge lengths of each side of the cabinet to find the area of each side. Add all of the areas.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

Student Response

Two cabinets: 1,582 sticky notes. Three cabinets: 2,229 sticky notes. Twenty cabinets: 13,228 sticky notes.

Activity Synthesis

Invite previously identified students or groups to share their answer and strategy. On a visual display, record each answer and each distinct process for determining the surface area (i.e. multiplying the side lengths of each rectangular face and adding up the products). After each presentation, poll the class on whether others had the same answer or process.

Play the video that reveals the actual number of sticky notes needed to cover the cabinet. If students' answers vary from that shown on the video, discuss possible reasons for the differences. (For example, students may not have accounted for the cabinet's door handles. Some may have made a calculation error.)

Tell students that the question they have been trying to answer is one about the surface area of the cabinet. Explain that the **surface area** of a three-dimensional figure is the total area of all its surfaces. We call the flat surfaces on a three-dimensional figure its **faces**.

The surface area of a rectangular prism would then be the combined area of all six of its faces. In the context of this problem, we excluded the bottom face, since it is sitting on the ground and will not be tiled with sticky notes. Discuss:

- "What unit of measurement are we using to represent the surface area of the cabinet?" (Square sticky notes)
- "Would the surface area change if we used larger or smaller sticky notes? How?" (Yes, if we use larger sticky notes, we would need fewer. If we use smaller ones, we would need more.)

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students share their strategies for determining the number of sticky notes that cover the cabinet, ask students to make connections between the various strategies. Some students will calculate the number of sticky notes that will cover each of the five faces of the cabinet and add them together. Other students may realize that opposite faces of the cabinet are congruent so it is only necessary to calculate the area of three faces of the cabinet. Encourage students to explain why both methods result in the same answer. This will promote students' use of mathematical language as they make sense of the various methods for finding the surface area of a rectangular prism. *Design Principles(s): Cultivate conversation; Maximize meta-awareness*

12.3 Building with Snap Cubes

20 minutes (there is a digital version of this activity)
This activity encourages students to apply strategies for finding the area of polygons to finding the *surface area* of rectangular prisms. Students use 12 cubes to build a prism, think about its surface area, and use isometric dot paper to draw their prism.

As students build their prisms, notice those with different designs and those with the same design but different approaches to finding surface area (e.g. by counting individual square, by multiplying the edge lengths of rectangular faces, etc.).

Addressing

- 6.G.A.4

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Read the first two lines of the task statement together. Remind students that we refer to the flat surfaces of a three-dimensional figure as **faces**.

Give students a minute to think about how we know the surface area of the shown prism is 32 square units. Ask 1–2 students to explain their reasoning to the class. Use students' explanations to highlight the meaning of surface area, i.e., that the area of all the faces need to be accounted for, including those we cannot see when looking at a two-dimensional drawing.

Tell students they will use 12 cubes to build a different prism, draw it, and find its surface area. Consider doing a quick demonstration on how to draw a simple prism on isometric dot paper. (Start with one cube and then add a cube in each dimension.) Tell students that in this activity, we call each face of a single cube, "1 square unit."

Give each student 12 cubes to build a prism and 6–8 minutes of quiet work time. If students are using snap cubes, say that we will pretend all of the faces are completely smooth, so they do not need to worry about the "innies and outies" of the snap cubes.

As students work, consider arranging two students with contrasting designs or strategies as partners. Ask partners to share their answers, explanations, and drawings. Stress that each partner should focus their explanation on how they went about finding surface area. The listener should think about whether the explanation makes sense or if anything is amiss in the reasoning.

For students in digital classrooms, an applet can be used to build and draw prisms. Physical cubes are still recommended and preferred for the building of the figures, however.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Support multiple forms of communication. Some students may benefit from an explicit demonstration and additional practice to learn how to draw cubes using isometric dot paper. Invite students who are unable to represent their figures using isometric dot paper to explain their reasoning orally, using virtual or concrete manipulatives.
Supports accessibility for: Visual-spatial processing; Conceptual processing; Fine-motor skills

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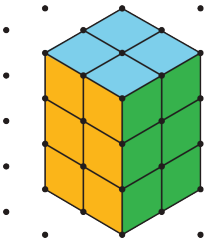
Anticipated Misconceptions

Students may count the faces of the individual snap cubes rather than faces of the completed prism. Help them understand that the faces are the visible ones on the outside of the figure.

Student Task Statement

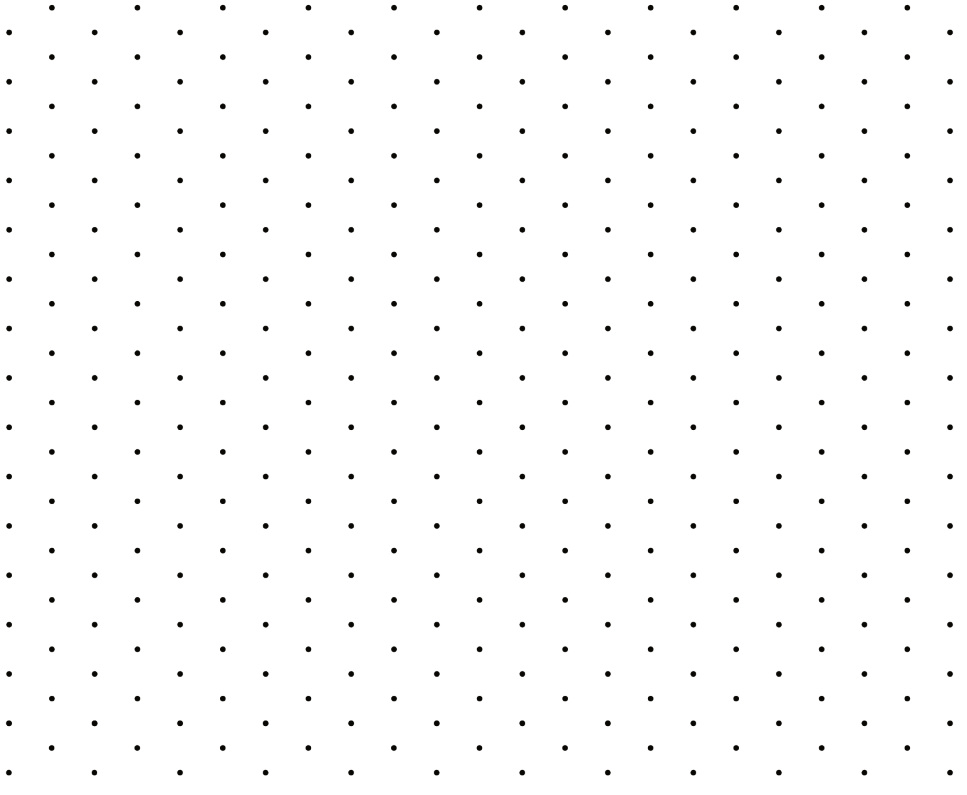
Here is a sketch of a rectangular prism built from 12 cubes:



It has six **faces**, but you can only see three of them in the sketch. It has a **surface area** of 32 square units.

Your teacher will give you 12 snap cubes. Use all of your snap cubes to build a different rectangular prism (with different edge lengths than the prism shown here).

1. How many faces does your figure have?
2. What is the surface area of your figure in square units?
3. Draw your figure on isometric dot paper. Color each face a different color.



Student Response

1. There are 6 faces—front, back, left, right, top, and bottom.
2. Answers vary based on design. Sample responses:
   - For a prism that is 12 units by 1 unit by 1 unit, the surface area is 50 square units. $(4 \cdot 12) + (2 \cdot 1) = 50$
   - For a prism that is 6 units by 2 units by 1 unit, the surface area is 40 square units. $(2 \cdot 12) + (2 \cdot 6) + (2 \cdot 2) = 40$
   - For a prism that is 4 units by 3 units by 1 unit, the surface area is 38 square units. $(2 \cdot 12) + (2 \cdot 4) + (2 \cdot 3) = 38$

3. Drawings vary, but all prisms should have one edge length that is 1 unit.

Activity Synthesis

After partner discussions, select a student to highlight for the class the strategy (or strategies) for finding surface area methodically. Point out that in this activity each face of their prism is a rectangle, and that we can find the area of each rectangle by multiplying its side lengths and then add the areas of all the faces.

Explain that later, when we encounter non-rectangular prisms, we can likewise reason about the area of each face the way we reasoned about the area of a polygon.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language (e.g., cubes, faces, surface area, square units, etc.), to restate and/or revoice the strategy (or strategies) presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. This will provide more students with an opportunity to produce language that describes strategies for finding surface area.
Design Principle(s): Support sense-making; Maximize meta-awareness

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Lesson Synthesis

In this lesson, we found the surface areas of a cabinet and of rectangular prisms built out of cubes.

- "What does it mean to find the **surface area** of a three-dimensional figure?" (It means finding the number of unit squares that cover the entire surface of the object without gaps or overlaps.)
- "How can we find the number of unit squares that cover the entire surface of an object?" (We can count them, or we can find the area of each **face** of the object and add the areas of all faces.)
- "How are finding surface area and finding area alike? How are they different?" (They both involve finding the number of unit squares that cover a region entirely without gaps and overlaps. Both have to do with two-dimensional regions. Finding area involves a single polygon. Finding surface area means finding the sum of the areas of multiple polygons (faces) of which a three-dimensional figure is composed.)

12.4 A Snap Cube Prism

Cool Down: 5 minutes
Addressing

- 6.G.A.4

Launch

Prepare several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes. Display one for all to see and pass the rest around for students to examine, if needed.

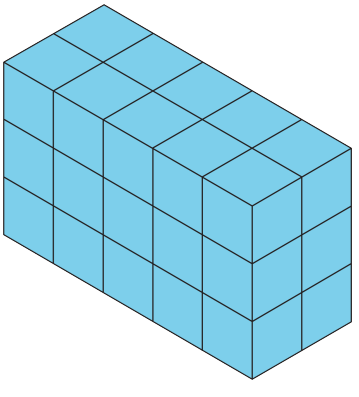
Anticipated Misconceptions

Students may not include the bottom face as it is not visible when the prism is sitting on a table.

Students may find the number of cubes instead of the surface area due to their previous volume work with rectangular prisms in grade 5.

Student Task Statement

A rectangular prism made is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units? Explain or show your reasoning.



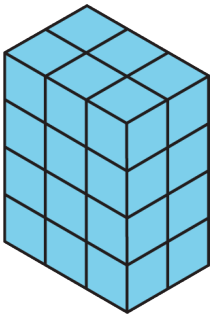
Student Response

The surface area is $2 \cdot [(3 \cdot 5) + (2 \cdot 5) + (2 \cdot 3)] = 62$ (or 62 square units).

Student Lesson Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



So the surface area of a rectangular prism that has edge-lengths 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Glossary

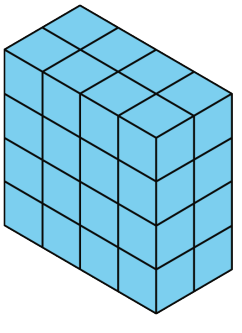
- face
- surface area

Lesson 12 Practice Problems

Problem 1

Statement

What is the surface area of this rectangular prism?



A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units

Solution

D

Problem 2

Statement

Which description can represent the surface area of this trunk?

A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

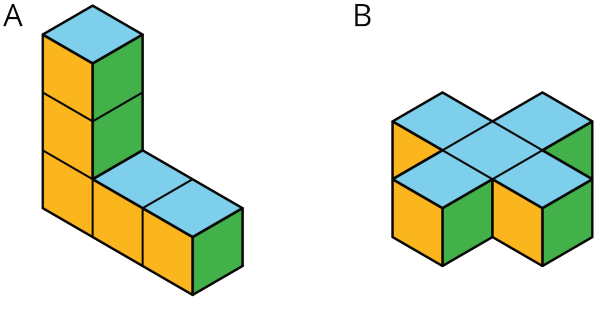
Solution

B

Problem 3

Statement

Which figure has a greater surface area?



Solution

Figure A and Figure B have the same surface area of 22 square units.

Problem 4

Statement

A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

Solution

88 square units. Two faces are 4 units by 2 units, amounting to 16 square units. Two faces are 4 units by 6 units, amounting to 48 square units. Two faces are 2 units by 6 units, amounting to 24 square units. $16 + 48 + 24 = 88$.

Problem 5

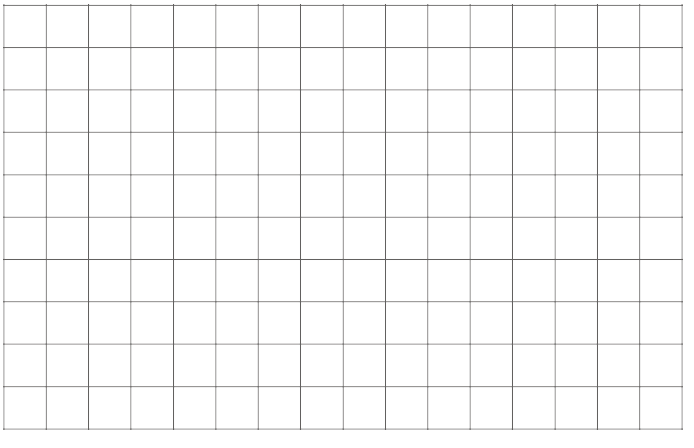
Statement

Draw an example of each of these triangles on the grid.

a. A right triangle with an area of 6 square units.

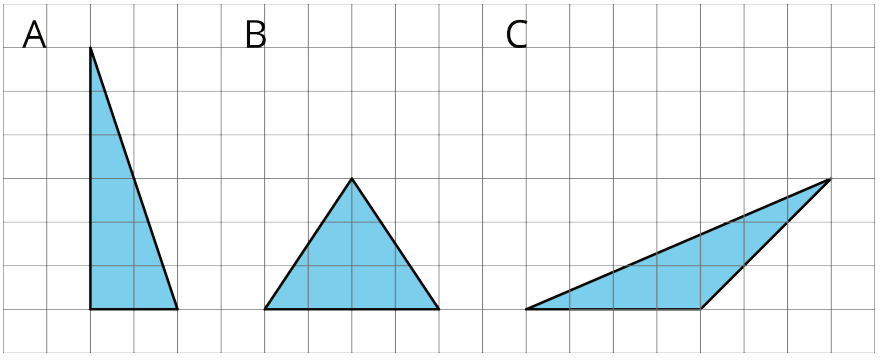
b. An acute triangle with an area of 6 square units.

c. An obtuse triangle with an area of 6 square units.



Solution

Answers vary. Sample response:

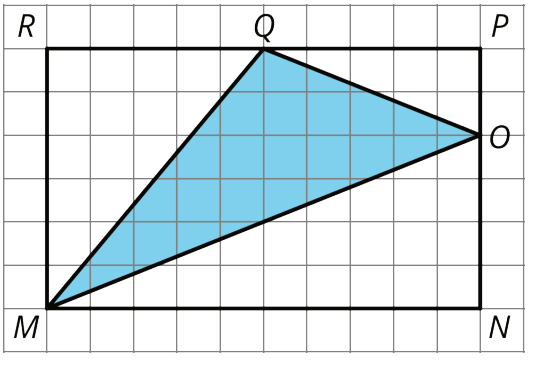


(From Unit 1, Lesson 9.)

Problem 6

Statement

Find the area of triangle MOQ in square units. Show your reasoning.



Solution

20 square units. Reasoning varies. Sample reasoning: The area of triangle MOQ can be found by subtracting the areas of the three right triangles from the area of rectangle $MNPR$.

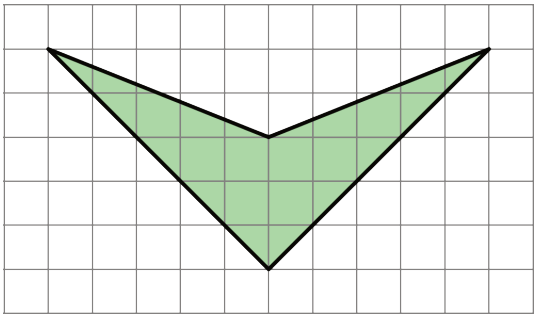
- The area of rectangle $MNPR$ is $10 \cdot 6$ or 60 square units.
- The area of triangle QRM is $\frac{1}{2} \cdot 6 \cdot 5$ or 15 square units.
- The area of triangle MNO is $\frac{1}{2} \cdot 10 \cdot 4$ or 20 square units.
- The area of triangle OPQ is $\frac{1}{2} \cdot 2 \cdot 5$ or 5 square units. $60 - (15 + 20 + 5) = 20$.

(From Unit 1, Lesson 10.)

Problem 7

Statement

Find the area of this shape. Show your reasoning.



Solution

15 square units. Reasoning varies. Sample reasoning:

- The shape can be decomposed into two identical triangles with a vertical cut down the middle. Each triangle has base 3 units and height 5 units, so its area is $\frac{1}{2} \cdot 3 \cdot 5$ or 7.5 square units. $2 \cdot (7.5) = 15$.
- The shape can be decomposed into two identical triangles and rearranged into a parallelogram with base 3 units and height 5 units. $3 \cdot 5 = 15$.

(From Unit 1, Lesson 3.)