Lesson 14: Four Representations

Goals

- Calculate the constant of proportionality for a proportional relationship in an unfamiliar context, and express it (in writing) using the correct units.
- Critique (orally and in writing) presentations of proportional and nonproportional relationships.
- Invent and describe (in writing and using other representations) a proportional relationship and a nonproportional relationship.

Learning Targets

- I can make connections between the graphs, tables, and equations of a proportional relationship.
- I can use units to help me understand information about proportional relationships.

Lesson Narrative

In this lesson, students examine tables, equations, and graphs of proportional relationships, and use them to reason about relationships that are proportional as well as relationships that are not proportional.

This lesson requires students to use everything they have learned since the beginning of the unit. This unit is meant to help students understand:

- What is a proportional relationship?
- What kinds of situations can be represented by proportional relationships?
- What form does an equation of a proportional relationship have?
- What do graphs of proportional relationships look like?

The next lesson in this unit is intended as a bridge to Unit 7.4 which is about applications of proportional relationships. It gives an example of how proportional relationships, and the derived units they give rise to, help to solve problems about the real world.

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR7: Compare and Connect

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Students are asked to make displays of their work in groups of 2. Prepare materials for creating a visual display in this way such as markers, chart paper, board space, etc.

Student Learning Goals

Let's contrast relationships that are and are not proportional in four different ways.

14.1 Which is the Bluest?

Warm Up: 5 minutes
In this warm-up, students are asked to reason which group of blocks is the bluest and explain how they arrived at that decision. The goal is to prompt students to visualize and articulate different ways they can use ratios, equivalent ratios and proportions to support their reasoning.

Building On

- 6.RP.A.3

Addressing

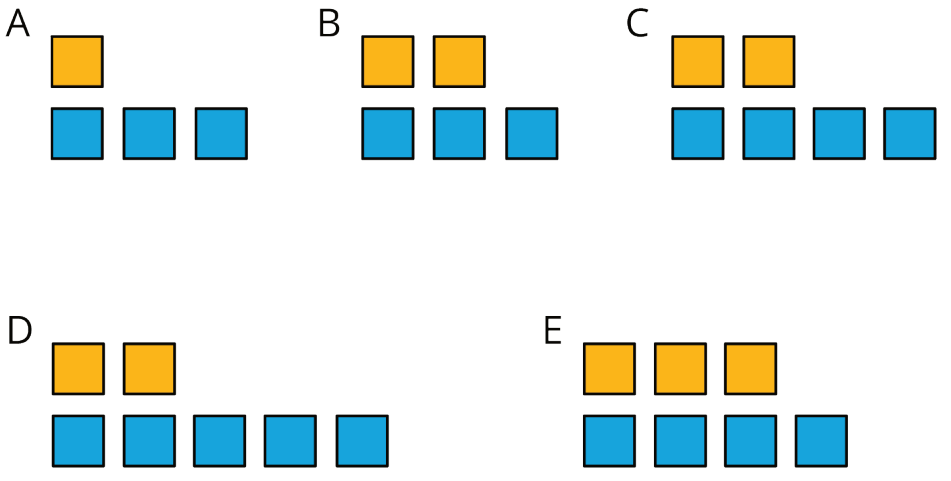
- 7.RP.A.2

Launch

Students in groups of 2. Tell students you will show them five groups of blocks. Their job is to determine which group of blocks is the bluest. Display the image for all to see. Give students 2 minutes of quiet think time. Encourage students who have one way of supporting their decision to think about another way while they wait.

Student Task Statement

1. Which group of blocks is the bluest?



2. Order the groups of blocks from least blue to bluest.

Student Response

1. A or D. A if looking at the amount of blue per yellow. D if looking at the total amount of blue or difference between blue and yellow.
2. Answers vary. Sample response: E, B, C, D, A when ordering by the amount of blue per total blocks.

Activity Synthesis

Ask students to share which group of blocks is the bluest and their reasoning. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- Did anyone choose the same group of blocks but would explain it differently?
- Does anyone want to add an observation to the way ____ saw the blocks?
- Do you agree or disagree? Why? Ask students to order the groups of blocks from less blue to bluest after deciding on the bluest group of blocks.

14.2 One Scenario, Four Representations

20 minutes

In this activity, students choose from different lists of things to define their own proportional and nonproportional relationships. Some of the things on the list will be familiar and others will be unfamiliar. This is a significant change from previous activities where students were always given two quantities and they had to decide if they were proportional or not. This new step gives students

the opportunity to think about what quantities are related to some of the items on the lists, which is an important step of modeling with mathematics (MP4).

This activity and the next go together. Students use the work from this activity to make a visual display of their work in the next activity.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2.

The names of things in the task may be unfamiliar to both English Language Learners and fluent English speakers. Before students start, take some time to ensure they know the meaning of their chosen things or ask them to do some research on their meanings. You might ask each pair of students to choose a different unfamiliar word, spend five minutes to research it, and prepare a drawing or explanation to share with the class.

It is best to approve of students' choices before they work. For example, if students choose "legs" and "earthworms," that will not make for a very interesting relationship.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a graphing calculator or graphing software. Some students may benefit from a checklist or list of steps to be able to use the calculator or software.
Supports accessibility for: Organization; Conceptual processing; Attention

Access for English Language Learners

Conversing, Writing: MLR2 Collect and Display. While pairs are working, circulate and listen to students talk about the relationships between quantities and justify whether the relationships are proportional. Write down common or important phrases you hear students say about the relationships. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language in their written work and in paired and whole group discussions.
Design Principle(s): Optimize output (for justification); Support sense-making

Anticipated Misconceptions

As students work, pay attention to the numbers they use in their tables. Students can be haphazard when choosing values, and their numbers may end up being unfriendly. You can ask them questions that encourage them to reason about what numbers would be friendlier. Also, depending on the things chosen, they may need to consider scales for their axes. Watch out for scales like {1, 2, 3, . . . } for the number of legs on a centipede!

Creating a relationship that is not proportional may present too significant a challenge for struggling learners. An accommodation would be to change their task to creating only a proportional relationship, or even assigning two quantities that are straightforward like "starfish legs vs. number of starfish."

Student Task Statement

1. Select two things from different lists. Make up a situation where there is a *proportional relationship* between quantities that involve these things.

creatures	length	time	volume
○ starfish	○ centimeters	○ nanoseconds	○ milliliters
○ centipedes	○ cubits	○ minutes	○ gallons
○ earthworms	○ kilometers	○ years	○ bushels
○ dinosaurs	○ parsecs	○ millennia	○ cubic miles

body parts	area	weight	substance
○ legs	○ square microns	○ nanograms	○ helium
○ eyes	○ acres	○ ounces	○ oobleck
○ neurons	○ hides	○ deben	○ pitch
○ digits	○ square light-years	○ metric tonnes	○ glue

2. Select two other things from the lists, and make up a situation where there is a relationship between quantities that involve these things, but the relationship is *not* proportional.
3. Your teacher will give you two copies of the "One Scenario, Four Representations" sheet. For each of your situations, describe the relationships in detail. If you get stuck, consider asking your teacher for a copy of the sample response.
    a. Write one or more sentences describing the relationship between the things you chose.

b. Make a table with titles in each column and at least 6 pairs of numbers relating the two things.

c. Graph the situation and label the axes.

d. Write an equation showing the relationship and explain in your own words what each number and letter in your equation means.

e. Explain how you know whether each relationship is proportional or not proportional. Give as many reasons as you can.

Student Response

Answers vary. Sample response:

One Scenario, Four Representations

The two quantities are: d yards or distance traveled during in the race in yards; t minutes or time in minutes that has elapsed in the race.

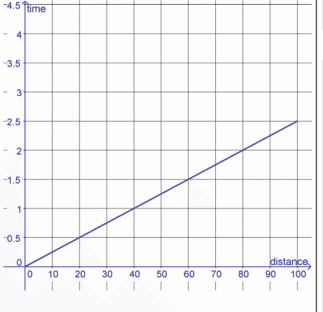
Verbal description: One or more complete sentences describing the relationship.

Adan and Mike are teammates in a 100-yd three-legged race. Their friend Ceril is timing them. Ceril notices that they pass the 20-yd marker at 1/2 minute, the 40-yd marker at 1 minute, and the 60-yd marker at 1.5 minutes.

Table of Values

d	t
20	1/2
40	1
60	1.5
80	2
100	2.5
1	1/40

Graph
Label each axis!



Equation: $t = \frac{1}{40}d$

Explain in words what each letter and number in your equation means:

t represents the time in minutes that has elapsed in the race, d represents the distance in yards they have traveled, and 1/40 is the constant of proportionality. It takes them 1/40 of a minute to travel 1 yard.

Explain how you know the relationship is proportional. Find as many reasons as you can. This relationship is proportional because: each value of d in the table can be multiplied by 1/40 to get the corresponding value of t. The graph is part of a line that goes through the origin and Quadrant I. The equation can be written in the form d = kt.

Activity Synthesis

Ask groups to trade their work with another group to give feedback about their analysis.

14.3 Make a Poster

Optional: 15 minutes

In this activity, students make a visual display of their scenarios from the previous activity (after sharing their rough draft with another group and getting some feedback).

When the posters are complete and displayed around the room, students view each others' work and use sentence starters to give feedback and to critique the reasoning of others (MP3).

Addressing

- 7.RP.A.2

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Keep students in the same groups. Tell them that they should incorporate the feedback they received when making their posters.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a task checklist which makes all the required components of the poster explicit.
Supports accessibility for: Attention; Social-emotional skills

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Student Task Statement

Create a visual display of your two situations that includes all the information from the previous activity.

Student Response

Answers vary.

Activity Synthesis

When the posters are complete, hang them around the room. Provide students with these sentence starters, and give them an opportunity to view their classmates' work and write their responses. This provides a structured way for students to critique the reasoning of others (MP3).

The most surprising combination of things was _____ because _____.

The group _____ should check their work where they _____.

I really liked when the group _____ did this _____ because _____.

Access for English Language Learners

Representing, Reading, Writing: MLR7 Compare and Connect. Use this routine when students share their visual displays. Direct attention to the different ways pairs explained quantities and their relationships and justified whether the relationship was proportional (e.g., distance traveled in yards and time elapsed in minutes, for every one minute they traveled 5 yards, the relationship is proportional because the graph of the line passes through the origin). Emphasize the language used to describe the proportional relationships and justify whether the relationship was proportional. These exchanges strengthen students' mathematical language use and reasoning of proportional relationships.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Reflect on the following questions.

- "Describe any part of your work today that you would do differently, if you could start over."
- "Tell me about something new you learned in this class recently."
- "Tell me about any questions you still have, or anything that is confusing you."

14.4 Explain Their Work

Cool Down: 5 minutes

Addressing

- 7.RP.A

Student Task Statement

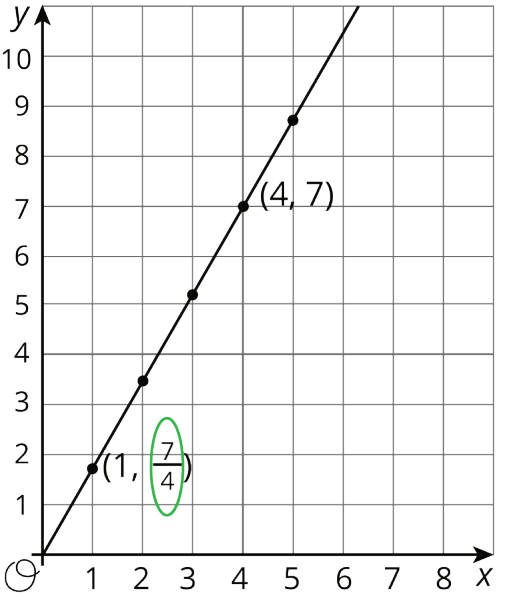
Choose a relationship that another group found and explain why it is a proportional relationship. Make sure to include the quantities they used and any important constants of proportionality.

Student Response

Answers vary. Sample response: In a 100 yard, Three Legged Race, distance in yards and time in minutes are proportional since each value of distance could be multiplied by $\frac{1}{40}$ to get the time. The constant of proportionality they used was $\frac{1}{40}$.

Student Lesson Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that you can multiply by any number in the first column to get the associated number in the second column, then the relationship between the quantities is not a proportional relationship.

Lesson 14 Practice Problems

Problem 1

Statement

The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.

a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.

b. Create a graph of the relationship.

c. What does 2.95 represent in this situation?

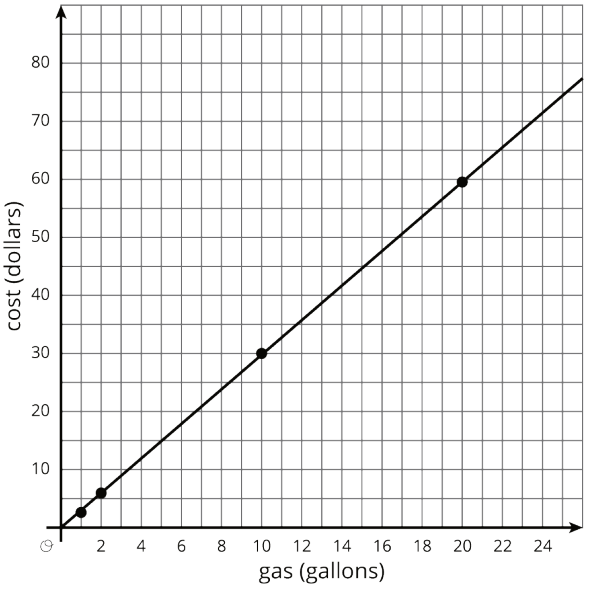
d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? How did she come up with that?

Solution

a. Answers vary. Sample response:

gallons of gas (g)	cost in dollars (c)
1	2.95
2	5.90
10	29.50
20	59.00

b. Answers vary. Sample response:



c. One gallon of gas costs $2.95. Or, gas costs $2.95 per gallon. Or, 2.95 dollars per gallon is the constant of proportionality.

d. Since 2.95 is close to 3, Jada's mom reasoned that if it cost about 3 dollars per gallon, the reciprocal rate must be $\frac{1}{3}$ gallon per dollar. gallons of gas (g) cost (c).

Problem 2

Statement

There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

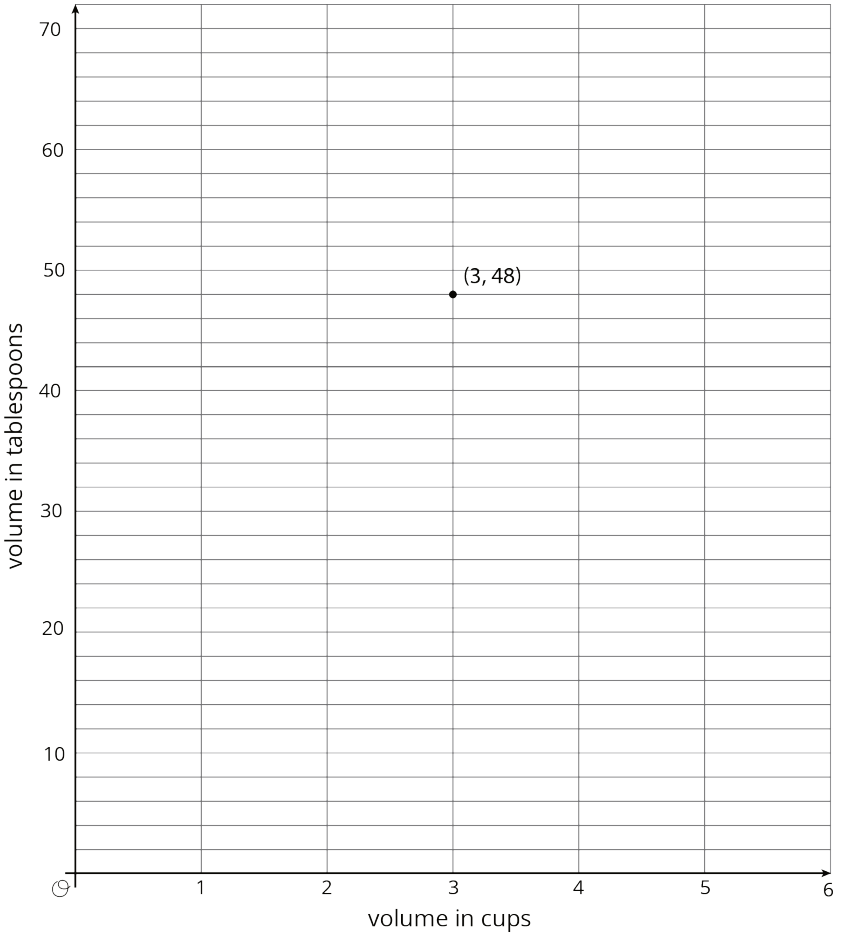
a. Plot and label at least two more points that represent the relationship.

b. Use a straightedge to draw a line that represents this proportional relationship.

c. For which value y is $(1, y)$ on the line you just drew?

d. What is the constant of proportionality for this relationship?

e. Write an equation representing this relationship. Use c for cups and t for tablespoons.



Solution

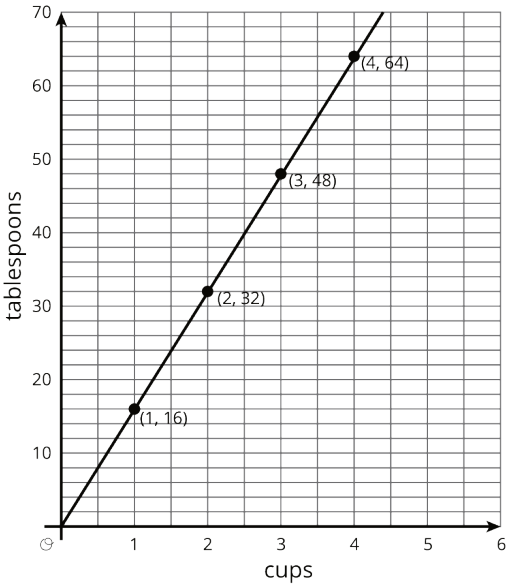
a. See below

b. See below

c. 16

d. 16 tablespoons per cup

e. $t = 16c$


70
60
50
40
30
20
10
tablespoons
(4, 64)
(3, 48)
(2, 32)
(1, 16)
1
2
3
4
5
6
cups