Lesson 2: Meanings of Division

Goals

- Identify or generate a multiplication equation that represents the same relationship as a division expression, and explain (orally) the reasoning.
- Interpret and create tape diagrams that represent situations involving equal-sized groups.
- Recognize there are two different ways to interpret a division expression, i.e., asking “how many groups?” or “how many in each group?”

Learning Targets

- I can explain how multiplication and division are related.
- I can explain two ways of interpreting a division expression such as $27 \div 3$.
- When given a division equation, I can write a multiplication equation that represents the same situation.

Lesson Narrative

In this lesson, students revisit the relationship between multiplication and division that they learned in prior grades. Specifically, students recall that we can think of multiplication as expressing the number of equal-size groups, and that we can find a product if we know thenumber of groups and thesize of each group. They interpret division as a way of finding a missing factor, which can either be the number of groups, or the size of one group. They do so in the context of concrete situations and by using diagrams and equations to support their reasoning.

As they represent division situations with diagrams and equations and interpret division equations in context, students reason quantitatively and abstractly (MP2).

Alignments

Building On

- 3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

Building Towards

- 6.NS.A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

> **Student Learning Goals**
>
> Let’s explore ways to think about division.

2.1 A Division Expression

Warm Up: 5 minutes

The purpose of this warm-up is to review students' prior understanding of division and elicit the ways in which they interpret a division expression. This review prepares them to explore the meanings of division in the lesson.

Some students may simply write the value of the expression because they struggle to put into words how they think about the problem. Encourage them to think of a story with a question, in which the expression could be used to answer the question.

Building On

- 3.OA.A.2

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Ask students to write a list of all of the ways they think about $20 \div 4$. Explain that they can write what the expression means to them, how they think about it when evaluating the expression, or a situation that matches the expression.

Give students 1 minute of quiet think time, followed by 1 minute of partner discussion. During discussion, ask students to share their responses and notice what they have in common.

> **Student Task Statement**
>
> Here is an expression: $20 \div 4$.
>
> What are some ways to think about this expression? Describe at least two meanings you think it could have.

Student Response

Answers vary. Possible responses:

- How many groups of 4 are in 20?
- How many are in each group if we split 20 into 4 groups?
- How many 4’s are in 20?
- What times 4 equals 20?

- What is the other side length of a rectangle with a side length of 4 and an area of 20?
- What is $\frac{1}{4}$ of 20?
- What is $\frac{20}{4}$?

Activity Synthesis

Invite partners to share the interpretations of $20 \div 4$ that they had in common. Record and display these responses for all to see. Ask students to notice any themes or trends in the range of responses.

Highlight the two ways students will be thinking about division in this unit:

- Division means partitioning a number or a quantity into equal groups and finding out *how many groups can be made*.
- Division means partitioning a number or a quantity into equal groups, and finding out *how much is in each group*.

2.2 Bags of Almonds

25 minutes
This activity prompts students to explore two ways of thinking about division by connecting it to multiplication, thinking about what it means in the context of a situation, and drawing visual representations.

Building On

- 3.OA.A.2

Building Towards

- 6.NS.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Ask students to keep their materials closed. Display the following question for all to see:

A baker has 12 pounds of almonds. She puts them in bags, so that each bag has the same weight. In terms of pounds and bags of almonds, what could $12 \div 6$ mean?

Give students a minute of quiet think time and 1–2 minutes to explain their thinking to their partner. Ask a few students who interpreted the expression differently to share their interpretations. If students do not bring up one of the two ways to interpret the 6, ask them about it: Could the 6 represent the number of bags (or the amount in each bag)?

Once students see that the divisor could be interpreted in two ways, ask students to open the materials and give students 4–5 minutes to complete the first question.

Reconvene as a class afterwards. Select a couple of students to explain Clare and Tyler's diagrams and equations. Highlight that, in this context, $12 \div 6$ could mean 12 pounds of almonds being divided equally into 6 bags, or 12 pounds of almonds being divided so that each bag has 6 pounds.

Give students quiet time to complete the rest of the activity.

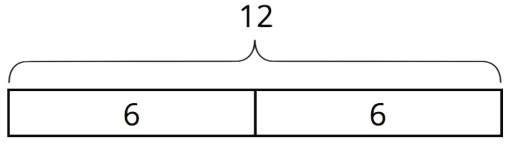
Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Provide students with access to blank tape diagrams. Encourage students to annotate diagrams with details to show how each value is represented—for example number of pounds of almonds in total, number of pounds in one bag, or number of bags of almonds.
Supports accessibility for: Visual-spatial processing; Organization

Student Task Statement

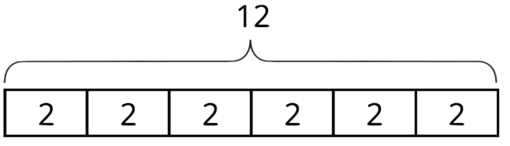
A baker has 12 pounds of almonds. She puts them in bags, so that each bag has the same weight.

Clare and Tyler drew diagrams and wrote equations to show how they were thinking about $12 \div 6$.



___ • 6 = 12

Clare’s diagram and equation



6 • ___ = 12

Tyler’s diagram and equation

1. How do you think Clare and Tyler thought about $12 \div 6$? Explain what each diagram and the parts of each equation could mean about the situation with the bags of almonds. Make sure to include the meaning of the missing number.

   Pause here for a class discussion.

2. Explain what each division expression could mean about the situation with the bags of almonds. Then draw a diagram and write a multiplication equation to show how you are thinking about the expression.

   a. $12 \div 4$

b. $12 \div 2$

c. $12 \div \frac{1}{2}$

Student Response

1. Clare might have seen 6 as the number of pounds in a bag. The missing factor is then how many bags the baker would have. Tyler might have seen 6 as the number of bags. The missing factor would then mean how many pounds of almonds are in each bag.

2. Diagrams and interpretations vary.
   a. $12 \div 4$ could mean, "How many pounds in 1 bag if there were 12 pounds in 4 bags?" ($4 \cdot \_\_\_ = 12$) or "How many bags can we get if we put 4 pounds in each bag?" ($\_\_\_ \cdot 4 = 12$).

   b. $12 \div 2$ could mean, "How many pounds in 1 bag if there were 12 pounds in 2 bags?" ($2 \cdot \_\_\_ = 12$) or "How many bags can we get if we put 2 pounds in each bag?" ($\_\_\_ \cdot 2 = 12$).

   c. $12 \div \frac{1}{2}$ could mean, "How many pounds in 1 bag if there were 12 pounds in $\frac{1}{2}$ bag?" ($\frac{1}{2} \cdot \_\_\_ = 12$) or "How many bags can we get if we put $\frac{1}{2}$ pound in each bag?" ($\_\_\_ \cdot \frac{1}{2} = 12$).

Are You Ready for More?

A loaf of bread is cut into slices.

1. If each slice is $\frac{1}{2}$ of a loaf, how many slices are there?
2. If each slice is $\frac{1}{5}$ of a loaf, how many slices are there?
3. What happens to the number of slices as each slice gets smaller?
4. What would dividing by 0 mean in this situation about slicing bread?

Student Response

1. 2
2. 5
3. The number of slices grows as the slices get smaller. This process is limited by how small the bread can get physically.
4. Dividing the bread into slices of length 0, which doesn’t make sense.

Activity Synthesis

Select a few students to share their diagrams and equations for the problems in the last question. After each explanation, highlight the connections between the expression, the diagram, and the context. Make sure students understand that the division expression $12 \div 6$ can be interpreted as the answer to the question “6 times what number equals 12?” or the question, “What number times 6 equals 12?" (or "How many 6s are in 12?"). More generally, division can be interpreted as a way to find two values:

- The size of each group when we know the number of groups and a total amount
- How many groups are in a total amount given the size of one group

Note that students may write either $\_\_\_ \cdot 6 = 12$ or $6 \cdot \_\_\_ = 12$ for each interpretation as long as they understand what each factor represents. Because we tend to say “___ groups of ___” in these materials, we follow that order in writing the multiplication:

$$(\text{number of groups}) \cdot (\text{size of each group}) = \text{total amount}$$

When discussing $12 \div 2$, make explicit how its multiplication equations and diagram connect to those of $12 \div 6$ in the first question. Students may see that the diagrams for $2 \cdot \_\_\_ = 12$ and $\_\_\_ \cdot 6 = 12$ are partitioned the same way. Point out that:

- In $2 \cdot \_\_\_ = 12$, the size of each group (each bag) was unknown, but because there are 2 equal groups in 12, we concluded that there were 6 pounds in each group.
- In $\_\_\_ \cdot 6 = 12$, we know each group (each bag) has 6 pounds of almonds, so there must be 2 groups of 6 in 12 pounds.

This discussion will be helpful in upcoming work, as students use their understanding of representations of division to divide fractions.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each diagram and expression that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we explored the relationship between multiplication and division in order to understand the meanings of division. We know that multiplication can represent the number of equal-size groups. For instance, $3 \cdot 5 = 15$ can mean 3 groups of 5 make 15. Let's review how we can use the same idea of equal-size groups to think about division.

- "How can we interpret $20 \div 8$?" (We can think of it as "how many groups of 8 are in 20?" or "how much is in each group if there are 20 in 8 groups?)
- "Suppose we interpret it as 'how many groups of 8 are in 20?'. How might we draw a diagram to show this?" (A bar that represents 20 divided into equal parts of 8.) "What multiplication equation can we write?" ($? \cdot 8 = 20$ or $8 \cdot ? = 20$, as long as we are clear what each factor represents.)
- "If we think of it as 'how much is in each group if there are 20 in 8 groups?', how would the diagram be different?" (A bar that represents 20 divided into 8 equal parts.) "What multiplication equation can we write?" ($8 \cdot ? = 20$ or $? \cdot 8 = 20$, as long as we know what each factor represents.)

2.3 Groups on A Field Trip

Cool Down: 5 minutes

Building Towards

- 6.NS.A

Student Task Statement

1. During a field trip, 60 students are put into equal-sized groups.

   a. Describe two ways to interpret $60 \div 5$ in this context.

   b. Find the quotient.

   c. Explain what the quotient would mean in each of the two interpretations you described.

2. Consider the division expression $7\frac{1}{2} \div 2$. Select **all** multiplication equations that correspond to this division expression.

   a. $2 \cdot ? = 7\frac{1}{2}$

   b. $7\frac{1}{2} \cdot ? = 2$

   c. $2 \cdot 7\frac{1}{2} = ?$

   d. $? \cdot 7\frac{1}{2} = 2$

   e. $? \cdot 2 = 7\frac{1}{2}$

Student Response

1. a. $60 \div 5$ could represent, "How many students are in each group if there are 5 groups?" (or "How many groups can be formed if there are 5 students per group?")

   b. $60 \div 5 = 12$

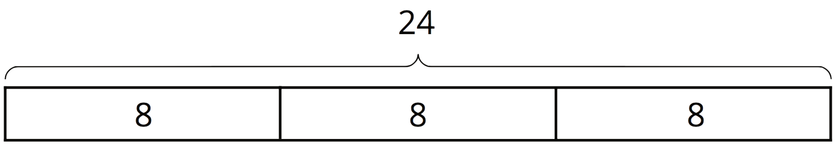
c. The quotient could mean there are 12 students in each of the 5 groups (or there are 12 groups with 5 students in each group).

2. A ($2 \cdot ? = 7\frac{1}{2}$) and E ($? \cdot 2 = 7\frac{1}{2}$)

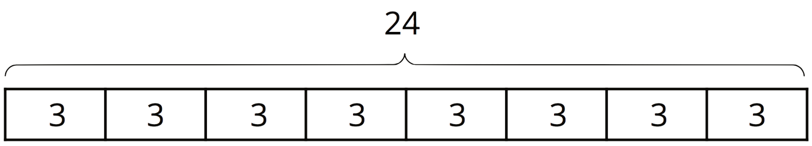
Student Lesson Summary

Suppose 24 bagels are being distributed into boxes. The expression $24 \div 3$ could be understood in two ways:

- 24 bagels are distributed equally into 3 boxes, as represented by this diagram:



- 24 bagels are distributed into boxes, 3 bagels in each box, as represented by this diagram:



In both interpretations, the quotient is the same ($24 \div 3 = 8$), but it has different meanings in each case. In the first case, the 8 represents the number of bagels in each of the 3 boxes. In the second, it represents the number of boxes that were formed with 3 bagels in each box.

These two ways of seeing division are related to how 3, 8, and 24 are related in a multiplication. Both $3 \cdot 8$ and $8 \cdot 3$ equal 24.

- $3 \cdot 8 = 24$ can be read as “3 groups of 8 make 24.”
- $8 \cdot 3 = 24$ can be read as “8 groups of 3 make 24.”

If 3 and 24 are the only numbers given, the multiplication equations would be:

$$3 \cdot ? = 24$$
$$? \cdot 3 = 24$$

In both cases, the division $24 \div 3$ can be used to find the value of the “?” But now we see that it can be interpreted in more than one way, because the “?” can refer to *the size of a group* (as in “3 groups of what number make 24?”), or to *the number of groups* (as in “How many groups of 3 make 24?”).

Lesson 2 Practice Problems

Problem 1

Statement

Twenty pounds of strawberries are being shared equally by a group of friends. The equation $20 \div 5 = 4$ represents the division of strawberries.

a. If the 5 represents the number of people, what does the 4 represent?

b. If the 5 represents the pounds of strawberries per person, what does the 4 represent?

Solution

a. The number of pounds of strawberry each person got.

b. The number of friends who were sharing the strawberries.

Problem 2

Statement

A sixth-grade science club needs $180 to pay for the tickets to a science museum. All tickets cost the same amount.

What could $180 \div 15$ mean in this situation? Describe two different possible meanings of this expression. Then, find the quotient and explain what it means in each case.

Solution

$180 \div 15$ could mean: “How many tickets could the club buy with $180 if each ticket costs $15?” or “How much does each ticket cost if $180 buys 15 tickets?” The answer is $180 \div 15 = 12$. In the first case, it means the club could buy 12 tickets. In the second, it means each ticket costs $12.

Problem 3

Statement

Write a multiplication equation that corresponds to each division equation.

a. $10 \div 5 = ?$

b. $4.5 \div 3 = ?$

c. $\frac{1}{2} \div 4 = ?$

Solution

Answers vary. Sample responses:

a. $? \cdot 5 = 10$ or $5 \cdot ? = 10$

b. $? \cdot 3 = 4.5$ or $3 \cdot ? = 4.5$

c. $? \cdot 4 = \frac{1}{2}$ or $4 \cdot ? = \frac{1}{2}$

Problem 4

Statement

Write a division or multiplication equation that represents each situation. Use a "?" for the unknown quantity.

a. 2.5 gallons of water are poured into 5 equally sized bottles. How much water is in each bottle?

b. A large bucket of 200 golf balls is divided into 4 smaller buckets. How many golf balls are in each small bucket?

c. Sixteen socks are put into pairs. How many pairs are there?

Solution

a. $2.5 \div 5 = ?$ or $5 \cdot ? = 2.5$

b. $200 \div 4 = ?$ or $4 \cdot ? = 200$

c. $16 \div 2 = ?$ or $? \cdot 2 = 16$

Problem 5

Statement

Find a value for a that makes each statement true.

a. $a \div 6$ is greater than 1

b. $a \div 6$ is equal to 1

c. $a \div 6$ is less than 1

d. $a \div 6$ is equal to a whole number

Solution

a. Answers vary. (Any number $a > 6$, for example $a = 7$)

b. $a = 6$

c. Answers vary. (Any positive number $a < 6$, for example $a = 3$)

d. If a is a multiple of 6, then $a \div 6$ is a whole number.

(From Unit 4, Lesson 1.)

Problem 6

Statement

Complete the table. Write each percentage as a percent of 1.

fraction	decimal	percentage
$\frac{1}{4}$	0.25	25% of 1
	0.1	
		75% of 1
$\frac{1}{5}$		
	1.5	
		140% of 1

Solution

fraction	decimal	percentage
$\frac{1}{4}$	0.25	25% of 1
$\frac{1}{10}$	0.1	10% of 1
$\frac{3}{4}$	0.75	75% of 1
$\frac{1}{5}$	0.2	20% of 1
$\frac{3}{2}$	1.5	150% of 1
$\frac{7}{5}$	1.4	140% of 1

(From Unit 3, Lesson 14.)

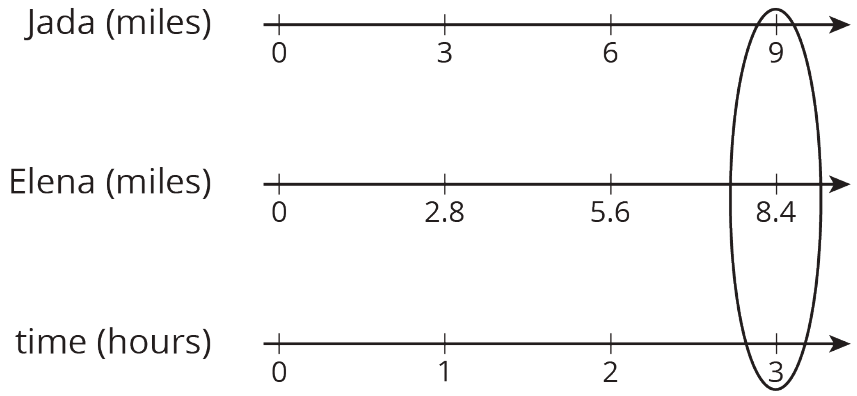
Problem 7

Statement

Jada walks at a speed of 3 miles per hour. Elena walks at a speed of 2.8 miles per hour. If they both begin walking along a walking trail at the same time, how much farther will Jada walk after 3 hours? Explain your reasoning.

Solution

Jada will have walked 0.6 miles farther. Sample reasoning:



After 3 hours Jada will have walked 9 miles, and Elena will have walked 8.4 miles. $9 - 8.4 = 0.6$.

(From Unit 3, Lesson 8.)