Lesson 4: How Many Groups? (Part 1)

Goals

- Coordinate multiplication equations and pattern block diagrams in which the yellow hexagon represents one whole.
- Create a diagram to represent and solve a problem asking “How many groups?” in which the divisor is a unit fraction, and explain (orally) the solution method.

Learning Targets

- I can find how many groups there are when the amount in each group is not a whole number.
- I can use diagrams and multiplication and division equations to represent “how many groups?” questions.

Lesson Narrative

This lesson and the next one extend the “how many groups?” interpretation of division to situations where the “group” can be fractional. This builds on the work in earlier grades on dividing whole numbers by unit fractions.

Students use pattern blocks to answer questions about how many times a fraction goes into another number (e.g., how many $\frac{2}{3}$s are in 2?), and to represent multiplication and division equations involving fractions. In this lesson, they focus on situations where the quotient (the number of groups) is a whole number.

This lesson is the first in a group of six lessons that trace out a gradual progression of learning—from reasoning with specific quantities, to using a symbolic formula for division of fractions (MP8).

Alignments

Building On

- 5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- 5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Addressing

- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Instructional Routines

- MLR3: Clarify, Critique, Correct

Required Materials

Pattern blocks

Required Preparation

Prepare enough pattern blocks so that each group of 3–4 students has at least 2 hexagons and 6 of each of the other shapes (triangle, rhombus, and trapezoid).

> ### Student Learning Goals
> Let’s play with blocks and diagrams to think about division with fractions.

4.1 Equal-sized Groups

Warm Up: 5 minutes
This warm-up reviews the idea of multiplication as representing equal-sized groups and the relationship between multiplication and division.

There are multiple equations students can write for each of the problems; the equations that connect multiplication and division to equal-sized groups are the important ones to highlight. As students work, identify students whose equations reflect these ideas.

Building On

- 5.NF.B.4
- 5.NF.B.7

Launch

Give students 2 minutes of quiet think time, followed by a whole-class discussion.

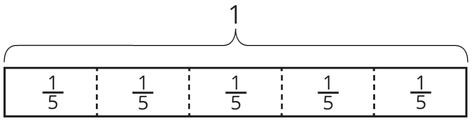
Anticipated Misconceptions

Some students may struggle to frame repeated addition as multiplication. To help them see the connection, refer to one of their addition statements and ask questions such as, “How many same-sized groups are being added?” or “What is in each group?”.

> ### Student Task Statement
> Write a multiplication equation and a division equation for each sentence or diagram.

1. Eight \$5 bills are worth \$40.

2. There are 9 thirds in 3 ones.

3. 

Student Response

1. $8 \cdot 5 = 40$ (or $5 \cdot 8 = 40$) and $40 \div 5 = 8$ (or $40 \div 8 = 5$)
2. $9 \cdot \frac{1}{3} = 3$ (or $\frac{1}{3} \cdot 9 = 3$) and $3 \div 9 = \frac{1}{3}$ (or $3 \div \frac{1}{3} = 9$)
3. $5 \cdot \frac{1}{5} = 1$ (or $\frac{1}{5} \cdot 5 = 1$) and $1 \div 5 = \frac{1}{5}$ (or $1 \div \frac{1}{5} = 5$)

Activity Synthesis

Select 1–2 students to share their responses. Record the responses for all to see. Ask students to indicate whether they agree or disagree with each one.

As students present the equations for each problem, connect the pieces in each equation to the idea of equal-sized groups. Ask questions such as:

- “Which number in the multiplication equation refers to the number of groups?”
- “Which number in the multiplication equation refers to how much is in each group?”
- “In this case, what does the division $3 \div 9 = \frac{1}{3}$ mean?”
- “In this case, what does the division $1 \div \frac{1}{5} = 5$ mean?”

4.2 Reasoning with Pattern Blocks

25 minutes (there is a digital version of this activity)
In this activity, students use the relationships between the areas of geometric shapes to reason about division situations that involve fractions. The focus is on the “how many groups?” interpretation of division.

Students start by using pattern blocks to represent multiplication of a whole number and a fraction. For example, if a hexagon represents 1 and six triangles make a hexagon, then each triangle represents $\frac{1}{6}$. They can then use six triangles to represent $6 \cdot \frac{1}{6} = 1$.

Later, students use the blocks to reason in the opposite direction, answering questions such as, “How many $\frac{1}{2}$s are in 4?” These kinds of questions serve as a stepping stone to more abstract questions such as, “What is 4 divided by $\frac{1}{2}$?”

Addressing

- 6.NS.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 3–4. Provide access to pattern blocks. Give students 10–12 minutes to collaborate on the first three questions and 3–4 minutes of quiet think time for the last question.

Remind students of the following:

- We can think of a fraction such as $\frac{1}{2}$ or $\frac{1}{3}$ in relation to 1 whole. In this task, the hexagon is 1 whole.
- We worked with the same shapes earlier in the course. We saw that two triangles make a rhombus, because if we place two triangles (joined along one side with no gap) on top of a rhombus, the triangles would match the rhombus exactly. This means that a triangle is half of a rhombus.

Classrooms with no access to pattern blocks or those using the digital materials can use the provided applet. Physical pattern blocks are still preferred, however.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. Once students have determined what fraction of a hexagon each of the shapes represents, pause the class. Invite students to demonstrate and share their strategies for finding each fraction using pattern blocks to justify their reasoning. Create a display that includes an image of each shape labeled with the name and the fraction it represents of a hexagon. Keep this display visible as students move on to the next problems.
Supports accessibility for: Conceptual processing; Memory

Access for English Language Learners

Representing, Writing: MLR3 Clarify, Critique, Correct. At the appropriate time, pause the class for a brief discussion of the first question. Display the following incorrect response that reflects a possible common misunderstanding: "The area of the rhombus is 3 because 3 fit inside the hexagon." Ask students, "Do you agree with the statement? Why or why not?" Invite students to identify the error, correct the statement, and draw a diagram to represent the situation. Improved statements should include fractional language and direct connections to the diagram. This will help students evaluate and improve on the written mathematical arguments of others.
Design Principle(s): Maximize meta-awareness; Optimize output (for justification)

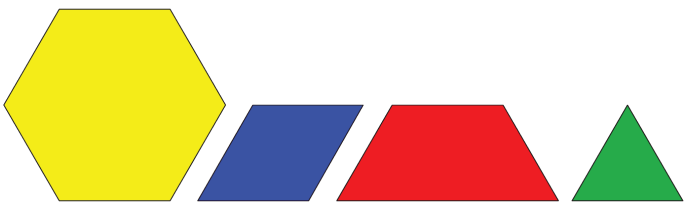
Anticipated Misconceptions

Some students may not remember the names of the shapes for these blocks. Consider reviewing the names of these shapes before beginning the activity and having students write them next to the pictures for reference.

Some students may simply look at the blocks and incorrectly guess the size of each block relative to the hexagon. Encourage them to place the blocks on top of the hexagon, to use non-hexagons to compose a hexagon, or to otherwise manipulate the blocks in order to make comparisons.

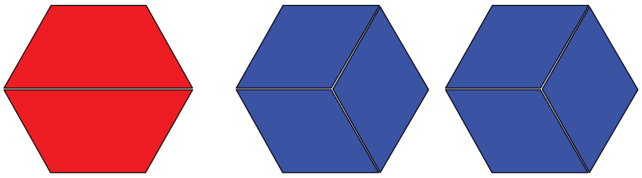
Student Task Statement

Your teacher will give you pattern blocks as shown here. Use them to answer the questions.



1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.
    - 1 triangle
    - 1 rhombus
    - 1 trapezoid
    - 4 triangles
    - 3 rhombuses
    - 2 hexagons
    - 1 hexagon and 1 trapezoid

2. Here are Elena's diagrams for $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Do you think these diagrams represent the equations? Explain or show your reasoning.



$2 \cdot \frac{1}{2} = 1$ $\qquad$ $6 \cdot \frac{1}{3} = 2$

3. Use pattern blocks to represent each multiplication equation. Remember that a hexagon represents 1 whole.

   a. $3 \cdot \frac{1}{6} = \frac{1}{2}$

   b. $2 \cdot \frac{3}{2} = 3$

4. Answer the questions. If you get stuck, consider using pattern blocks.

   a. How many $\frac{1}{2}$s are in 4?

   b. How many $\frac{2}{3}$s are in 2?

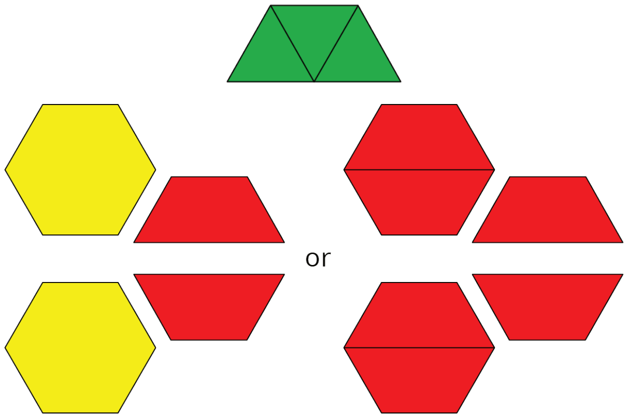
   c. How many $\frac{1}{6}$s are in $1\frac{1}{2}$?

Student Response

1.
   - $\frac{1}{6}$
   - $\frac{2}{6}$ (or $\frac{1}{3}$)
   - $\frac{1}{2}$
   - $\frac{4}{6}$ (or $\frac{2}{3}$)
   - 1
   - 2
   - $1\frac{1}{2}$

2. Agree. Sample reasoning: In the first representation, each trapezoid is $\frac{1}{2}$ of a hexagon, so 2 of them make 1 whole or
   1 hexagon. In the second representation, each rhombus is $\frac{1}{3}$ of a hexagon, so 3 rhombuses make 1 hexagon or 1 whole and 6 rhombuses make 2 wholes.

a.



3. b.

4. a. 8

b. 3

c. 9

Activity Synthesis

Select a few students to show their pattern-block arrangements or drawings for $3 \cdot \frac{1}{6} = \frac{1}{2}$ and $2 \cdot \frac{3}{2} = 3$. After each person shares, poll the class to see if others did it the same way or had alternative solutions.

Select other students to share their responses and reasoning for the last set of questions. If no one reasoned about the questions by using pattern blocks, show how the blocks could be used to answer the questions. For instance:

- For "how many $\frac{1}{2}$s are in 4?", we could use 8 trapezoids (each representing $\frac{1}{2}$) to make 4 hexagons.
- For "how many $\frac{2}{3}$s are in 2?", we could use 2 rhombuses (each representing $\frac{2}{3}$) to make 2 hexagons.
- For "how many $\frac{1}{6}$s are in $1\frac{1}{2}$?", we could use 9 triangles (each representing $\frac{1}{6}$) to make $1\frac{1}{2}$ hexagons.

Highlight that, in each case, we know the size of each group (or each block) and are trying to find out how many groups (or how many blocks) are needed to equal a particular area.

You may choose to use the applet at https://ggbm.at/VmEqZvke in the discussion.

Lesson Synthesis

In this lesson, we learned that we can reason about division with fractions as we have done in division with whole numbers—by thinking in terms of equal-sized groups. We can use pattern blocks, diagrams, and equations to think about questions such as "how many $\frac{3}{4}$s are in 6?"

- "How do we know which number represents the size of a group, and which represents a total?" (We can often tell by the context of the problem, or by interpreting the question carefully. For the question "how many $\frac{3}{4}$s are in 6?," we are interested in groups of $\frac{3}{4}$s and we have a total amount of 6.)
- "How do diagrams or pattern blocks help us find the answers to these questions?" (Diagrams often allow us to count or see the number of groups.)
- "What equations can we write to represent the question 'how many $\frac{3}{4}$s are in 6'?" (We can start with multiplication: "there are ? groups of $\frac{3}{4}$ in 6" can be written as $? \cdot \frac{3}{4} = 6$. The division equation $6 \div \frac{3}{4} = ?$ represents the same question.)

4.3 Halves, Thirds, and Sixths

Cool Down: 5 minutes

Addressing

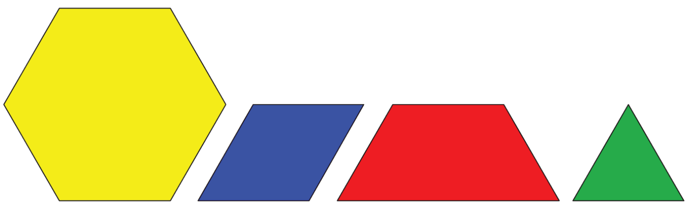
- 6.NS.A.1

Launch

Give students continued access to pattern blocks, if needed.

Student Task Statement

1. The hexagon represents 1 whole.



   Draw a pattern-block diagram that represents the equation $4 \cdot \frac{1}{3} = 1\frac{1}{3}$.

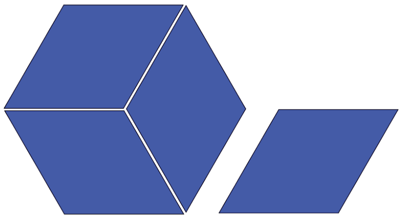
2. Answer the following questions. If you get stuck, consider using pattern blocks.

   a. How many $\frac{1}{2}$s are in $3\frac{1}{2}$?

   b. How many $\frac{1}{3}$s are in $2\frac{2}{3}$?

   c. How many $\frac{1}{6}$s are in $\frac{2}{3}$?

Student Response

1. 

2. a. There are seven $\frac{1}{2}$s in $3\frac{1}{2}$.

   b. There are eight $\frac{1}{3}$s in $2\frac{2}{3}$.

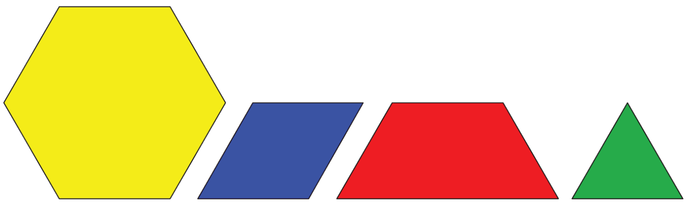
   c. There are four $\frac{1}{6}$s in $\frac{2}{3}$.

Student Lesson Summary

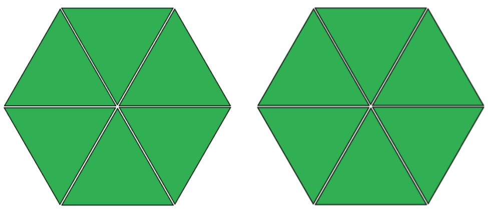
Some problems that involve equal-sized groups also involve fractions. Here is an example: "How many $\frac{1}{6}$ are in 2?" We can express this question with multiplication and division equations.

$$? \cdot \frac{1}{6} = 2$$
$$2 \div \frac{1}{6} = ?$$

Pattern-block diagrams can help us make sense of such problems. Here is a set of pattern blocks.



If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$, because 6 triangles make 1 hexagon. We can use the triangle to represent the $\frac{1}{6}$ in the problem.



Twelve triangles make 2 hexagons, which means there are 12 groups of $\frac{1}{6}$ in 2.

If we write the 12 in the place of the "?" in the original equations, we have:

$$12 \cdot \frac{1}{6} = 2$$

$$2 \div \frac{1}{6} = 12$$

Lesson 4 Practice Problems

Problem 1

Statement

Consider the problem: A shopper buys cat food in bags of 3 lbs. Her cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?

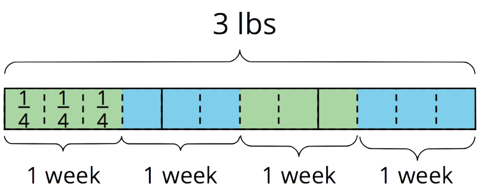
a. Draw a diagram to represent the situation and label your diagram so it can be followed by others. Answer the question.

b. Write a multiplication or division equation to represent the situation.

c. Multiply your answer in the first question (the number of weeks) by $\frac{3}{4}$. Did you get 3 as a result? If not, revise your previous work.

Solution

a.



There are 4 servings of $\frac{3}{4}$ lbs in the 3 lbs bag. The bag lasts 4 weeks.

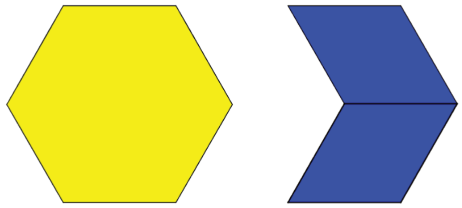
b. $? \cdot \frac{3}{4} = 3$ or $3 \div \frac{3}{4} = ?$

c. The answer is correct because $4 \cdot \frac{3}{4} = 3$.

Problem 2

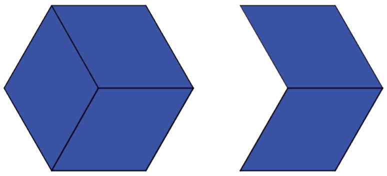
Statement

Use the diagram to answer the question: How many $\frac{1}{3}$s are in $1\frac{2}{3}$? The hexagon represents 1 whole. Explain or show your reasoning.



Solution

If the hexagon represents 1, then the rhombus represents $\frac{1}{3}$ because the hexagon is composed of three rhombuses. The diagram of one hexagon and two rhombuses matches up exactly with five rhombuses. So there are five $\frac{1}{3}$s in $1\frac{2}{3}$.



Problem 3

Statement

Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?

A. How many 3s are in $\frac{1}{8}$?

B. What is 3 groups of $\frac{1}{8}$?

C. How many $\frac{1}{8}$s are in 3?

D. What is $\frac{1}{8}$ of 3?

Solution

C

Problem 4

Statement

Write two division equations for each multiplication equation.

a. $15 \cdot \frac{2}{5} = 6$

b. $6 \cdot \frac{4}{3} = 8$

c. $16 \cdot \frac{7}{8} = 14$

Solution

a. $6 \div \frac{2}{5} = 15$ and $6 \div 15 = \frac{2}{5}$

b. $8 \div 6 = \frac{4}{3}$ and $8 \div \frac{4}{3} = 6$

c. $14 \div 16 = \frac{7}{8}$ and $14 \div \frac{7}{8} = 16$

Problem 5

Statement

Noah and his friends are going to an amusement park. The total cost of admission for 8 students is $100, and all students share the cost equally. Noah brought $13 for his ticket. Did he bring enough money to get into the park? Explain your reasoning.

Solution

Responses vary. Sample response: Yes, he did bring enough money, since $100 \div 8 = 12.5$. So if the friends share the cost equally, each pays $12.50. (Also $8 \cdot 13$ is bigger than 100, so if everybody brought $13, they would have more money than they need.)

(From Unit 4, Lesson 2.)

Problem 6

Statement

Write a division expression with a quotient that is:

a. greater than $8 \div 0.001$

b. less than $8 \div 0.001$

c. between $8 \div 0.001$ and $8 \div \frac{1}{10}$

Solution

Answers vary. Sample responses:

a. $9 \div 0.001$ or $8 \div 0.0001$

b. $7 \div 0.01$ or $8 \div 0.01$

c. $8 \div 0.01$ or $6 \div 0.001$

(From Unit 4, Lesson 1.)

Problem 7

Statement

Find each unknown number.

a. 12 is 150% of what number?

b. 5 is 50% of what number?

c. 10% of what number is 300?

d. 5% of what number is 72?

e. 20 is 80% of what number?

Solution

a. 8

b. 10

c. 3,000

d. 1,440

e. 25

Sample reasoning likely to include reasoning about benchmark percentages, or about percentages as rates per 100. For example, for "5% of what is 72."

- To reason about benchmark percentages, reason that if 5% is 72, then 10% is 144. 100% is ten times as much, so 100% is 1,440.
- To reason about rates per 100, create a double number line or a table of equivalent ratios, as shown. Since 5 is multiplied by 20 to reach 100, multiply 72 by 20 as well.

amount	percentage
72	5
1,440	100

(From Unit 3, Lesson 14.)