Lesson 1: Exponent Review

Goals

- Comprehend that repeated division by 2 is equivalent to repeated multiplication by one-half.
- Create an expression that represents repeated multiplication, and explain (orally) how the structure of the expression helps compare quantities.

Learning Targets

- I can use exponents to describe repeated multiplication.
- I understand the meaning of a term with an exponent.

Lesson Narrative

In grade 6, students worked with whole number exponents. This lesson reviews those concepts and subtly introduces the idea that repeated division by a number can be thought of as repeated multiplication by the reciprocal of that number, which plays a key role in later work on negative exponents.

Alignments

Building On

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Building Towards

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share
- Which One Doesn’t Belong?

Student Learning Goals

Let’s review exponents.

1.1 Which One Doesn’t Belong: Twos

Warm Up: 5 minutes

This warm-up prompts students to compare four expressions with exponents. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another. To allow all students to access the activity, each expression except for 2^3 has one obvious reason it does not belong. Don't let students dwell on trying to explain why 2^3 doesn't belong. During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the unit.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Before introducing the warm-up, display the expression 7^4 for all to see. Ask students if they recognize this notation and to explain what it means. Then display the expressions in the warm-up for all to see. Ask students to indicate when they have noticed one expression that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression does not belong and together, *try* to find at least one reason each expression doesn't belong.

Student Task Statement

Which expression does not belong? Be prepared to share your reasoning.

2^3	3^2
8	$2^2 \cdot 2^1$

Student Response

Answers vary. Sample responses:

- 8 does not belong because it is the only one with no exponent.
- 3^2 does not belong because it is the only one that does not equal 8.
- $2^2 \cdot 2^1$ does not belong because it is the only one that is multiplying two exponential terms.

There is not an obvious reason 2^3 doesn't belong since the other expressions have little in common.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as "base" or "exponent." Also, press students on unsubstantiated claims.

1.2 Return of the Genie

15 minutes (there is a digital version of this activity)
This activity uses the context of a genie who gives a magic coin that doubles in number each day. This context reminds students about the need for exponential notation in thinking about problems involving repeated multiplication. For the sake of simplicity, the problem was written so that the exponent is equal to the number of days.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Invite a student to read the first paragraph for the class. Make sure students understand how the magic coin works before moving on to the problem statement, perhaps by drawing a picture of a coin that doubles each day. Allow 10 minutes of work time before a whole-class discussion.

For classes using the digital version, there is an applet to help visualize the growth. If projection is available, teachers using the print version can display it from this link. https://ggbm.at/xQP9xNDm

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Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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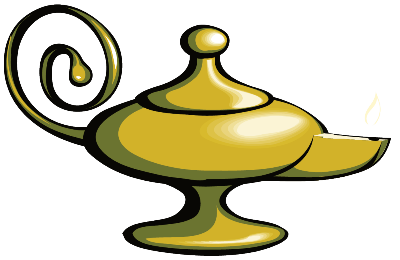
Anticipated Misconceptions

There may be some confusion about what time of day the coin doubles or how the exponent connects to the number of days. Clarify that the number of days is equal to the number of doublings.

Some students may misinterpret “How many times more coins does Mai have than Andre” as “How many more coins does Mai have than Andre.” Others may think they need to know exactly how many coins Mai and Andre have in order to answer this question. Suggest to students who are stuck that they first figure out how many times more coins Mai had on the 8th day than on the 5th day.

Student Task Statement

Mai and Andre found an old, brass bottle that contained a magical genie. They freed the genie, and it offered them each a magical $1 coin as thanks.



- The magic coin turned into 2 coins on the first day.
- The 2 coins turned into 4 coins on the second day.
- The 4 coins turned into 8 coins on the third day.

This doubling pattern continued for 28 days.

Mai was trying to calculate how many coins she would have and remembered that instead of writing $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ for the number of coins on the 6th day, she could just write 2^6.

1. The number of coins Mai had on the 28th day is very, very large. Write an expression to represent this number without computing its value.
2. Andre’s coins lost their magic on the 25th day, so Mai has a lot more coins than he does. How many times more coins does Mai have than Andre?

Student Response

1. 2^{28}, because the number of coins has doubled 28 times.
2. Mai has $2 \cdot 2 \cdot 2 = 8$ times as many coins as Andre because her coins doubled 3 times after his stopped.

Activity Synthesis

The goal is for students to understand exponential notation and use it to reason about a situation that involves repeated multiplication. Display the table for all to see. Tell students that exponents allow us to perform operations and reason about numbers that are too large to calculate by hand. Explain that the “expanded” column shows the factors being multiplied, the “exponent” column shows how to write the repeated multiplication more succinctly with exponents, and the “value” column shows the decimal value. Consider asking, “How many times larger is 2^6 than 2^4? How does expanding into factors help you see this?”

expanded	exponent	value
2	2^1	2
$2 \cdot 2 \cdot 2 \cdot 2$	2^4	16
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6	64

Access for English Language Learners

Reading, Writing: MLR3 Critique, Correct and Clarify. Display the incorrect statement: "Mai has 6 times more coins than Andre because she had 3 more doublings, and $3 \cdot 2 = 6$." Ask students to critique the response by asking, "Do you agree with the author's reasoning? Why or why not?" Give students 2–3 minutes of quiet think time to write feedback to the author that identifies how to improve the solution and expand on his/her work. Invite students to share written feedback with a partner before selecting 2–3 students to share with the whole class. Listen for students who refer to repeated multiplication and use the language of exponents in their feedback to the author. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Maximize meta-awareness, Optimize output (for explanation)

1.3 Broken Coin

15 minutes (there is a digital version of this activity)
The broken coin prompts students to think about repeated division, laying the foundation for later work on negative exponents. Understanding repeated division by 2 as being equivalent to repeated multiplication by $\frac{1}{2}$ will later allow students to make sense of negative exponents.

Look for students who express their answers to question 2 as $\left(\frac{1}{2}\right)^{28}$ and those who write $\frac{1}{2^{28}}$. Ask them to share their responses later.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Allow 10 minutes of work time, 2 minutes for partner discussion, and follow with a brief whole-class discussion. It is expected that some students will multiply 6 times for question 1. For the second question, give students time to realize that they need to use a more efficient method to describe the number.

For students using the digital activity, there is an applet to help visualize the coin halving each day.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of the actions that occur in the situation to support connections between new situations and prior understandings. Remind students that repeated multiplication by $\frac{1}{2}$ is equivalent to repeated division by 2.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Anticipated Misconceptions

If students spend more than several minutes on trying to multiply $\left(\frac{1}{2}\right)^{28}$, remind them of the more efficient exponential notation.

If students are wondering how to represent dividing repeatedly, ask if they can think of division by 2 as multiplication, perhaps by another value.

Student Task Statement

After a while, Jada picks up a coin that seems different than the others. She notices that the next day, only half of the coin is left!

- On the second day, only $\frac{1}{4}$ of the coin is left.
- On the third day, $\frac{1}{8}$ of the coin remains.

1. What fraction of the coin remains after 6 days?
2. What fraction of the coin remains after 28 days? Write an expression to describe this without computing its value.
3. Does the coin disappear completely? If so, after how many days?

Student Response

1. $\frac{1}{64}$, ($\left(\frac{1}{2}\right)^6 = \frac{1}{64}$)
2. $\frac{1}{2^{28}}$, or $(\frac{1}{2})^{28}$ ($\frac{1}{2}$ is multiplied by itself 28 times).

3. The coin never completely disappears because each time that half of the coin disappears, there is still half left.

Are You Ready for More?

Every animal has two parents. Each of its parents also has two parents.

1. Draw a family tree showing an animal, its parents, its grandparents, and its great-grandparents.
2. We say that the animal's eight great-grandparents are "three generations back" from the animal. At which generation back would an animal have 262,144 ancestors?

Student Response

1. A diagram with a tree structure that shows the animal on one level (0 generations back), its two parents on the next, its four grandparents on the next, and its eight great-grandparents last (3 generations back).
2. 18 generations, because $2^{18} = 262{,}144$.

Activity Synthesis

The goal is for students to understand that dividing by 2 repeatedly corresponds to multiplying by $\frac{1}{2}$ repeatedly. Ask students to discuss their responses with their partner. Select previously identified students to share their responses to the second question, highlighting the difference between $\left(\frac{1}{2}\right)^{28}$ and $\frac{1}{2^{28}}$. Ask students whether they agree or disagree with either of those responses. Students should come away with the idea that repeatedly dividing by 2 is the same as repeatedly multiplying by $\frac{1}{2}$. Here are some questions to consider:

- "Why are exponents useful when thinking about the coin after many days?" (It is shorter to write than a lot of 2s.)
- "What does your partner think about the last question? Do you agree? Why or why not?"

Access for English Language Learners

Speaking, Listening: *MLR7 Compare and Connect.* As students share different approaches for representing the fraction of the coin remaining after 6 days (or 28 days), ask students to identify how the approaches are alike, and how they are different. Invite students to connect the approaches by asking, "Where is the number of days represented in each approach?" Ask students to describe what worked well with their approach. These exchanges strengthen students' mathematical language use and reasoning based on exponents.
Design Principle(s): Maximize meta-awareness, Support sense-making

Lesson Synthesis

The goal of the discussion is to check whether students understand that exponents indicate repeated multiplication. Consider recording and displaying student responses for all to see during the discussion.

Here are some questions to consider for discussion:

- "What does it mean when we write 2^{42}?" (2^{42} means that 2 has been repeatedly multiplied 42 times. To expand this into factors would show 42 factors that are 2.)
- "How many times larger is 2^{45} than 2^{42}?" (2^{45} is 8 times larger than 2^{42} because it has 3 more factors that are 2, so it has been multiplied by 2 an extra 3 times.)
- "What does it mean when I write $\left(\frac{1}{2}\right)^{42}$?" ($\left(\frac{1}{2}\right)^{42}$ means that $\frac{1}{2}$ has been repeatedly multiplied 42 times. To expand this into factors would show 42 factors that are $\frac{1}{2}$.)
- "Which is greater, $\left(\frac{1}{2}\right)^{42}$ or $\left(\frac{1}{2}\right)^{45}$? Why?" ($\left(\frac{1}{2}\right)^{42}$ is greater since multiplying by $\frac{1}{2}$ results in a value closer to 0 and $\left(\frac{1}{2}\right)^{45}$ has been multiplied by $\frac{1}{2}$ three extra times.)

1.4 Exponent Check

Cool Down: 5 minutes

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Student Task Statement

1. What is the value of 3^4?
2. How many times bigger is 3^{15} compared to 3^{12}?

Student Response

1. 81, because $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$.
2. 3^{15} is 27 times larger than 3^{12}, because 3^{15} has 3 more factors that are 3 and $3^3 = 27$.

Student Lesson Summary

Exponents make it easy to show repeated multiplication. For example,

$$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2.$$

One advantage to writing 2^6 is that we can see right away that this is 2 to the *sixth* power. When this is written out using multiplication, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, we need to count the number of factors. Imagine writing out 2^{100} using multiplication!

Let’s say you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25}, we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: in this case, we can see right away that it is on the 25th day that the number of grains of rice has doubled! That’s a lot of rice (more than a cubic meter)!

Glossary

- exponent

Lesson 1 Practice Problems

Problem 1

Statement

Write each expression using an exponent:

a. $1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

b. $1 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

c. $1 \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$

d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

Solution

a. 7^5

b. $(\frac{4}{5})^5$

c. $(9.3)^8$

d. 2^8

Problem 2

Statement

Evaluate each expression:

a. 2^5

b. 3^3

c. 4^3

a. 6^2

b. $\left(\frac{1}{2}\right)^4$

c. $\left(\frac{1}{3}\right)^2$

Solution

a. 32

b. 27

c. 64

d. 36

e. $\frac{1}{16}$

f. $\frac{1}{9}$

Problem 3

Statement

Clare made $160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

a. How much money will Clare have in her account after 1 year? After 2 years?

b. How much money will Clare have in her account after 5 years? Explain your reasoning.

c. Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account.

Solution

a. $164.80, $169.74

b. $185.48. Reasoning varies. Sample reasoning: $160 \cdot 1.03^5$ (or multiply 160 by 1.03 five times)

c. $160 \cdot 1.03^{30}$

Problem 4

Statement

The equation $y = 5{,}280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

Solution

There are 5,280 feet in every mile. For example, each additional mile that someone travels is equivalent to traveling an additional 5,280 feet.

(From Unit 3, Lesson 1.)

Problem 5

Statement

The points $(2, 4)$ and $(6, 7)$ lie on a line. What is the slope of the line?

A. 2

B. 1

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Solution

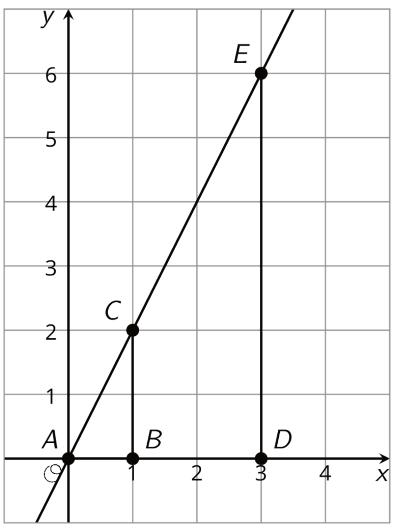
D

(From Unit 3, Lesson 5.)

Problem 6

Statement

The diagram shows a pair of similar figures, one contained in the other. Name a point and a scale factor for a dilation that moves the larger figure to the smaller one.



Solution

Center: A, scale factor: $\frac{1}{3}$

(From Unit 2, Lesson 6.)