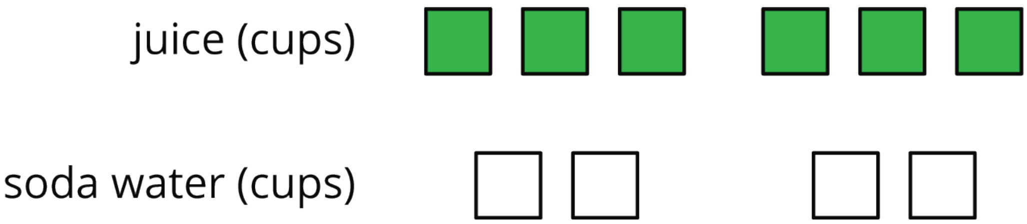
Family Support Materials

Ratios, Rates, and Percentages

What are Ratios?

Family Support Materials 1

A **ratio** is an association between two or more quantities. For example, say we have a drink recipe made with cups of juice and cups of soda water. Ratios can be represented with diagrams like those below.



Here are some correct ways to describe this diagram:

- The ratio of cups of juice to cups of soda water is $6 : 4$.
- The ratio of cups of soda water to cups of juice is 4 to 6.
- There are 3 cups of juice for every 2 cups of soda water.

The ratios $6 : 4$, $3 : 2$, and $12 : 8$ are **equivalent** because each ratio of juice to soda water would make a drink that tastes the same.

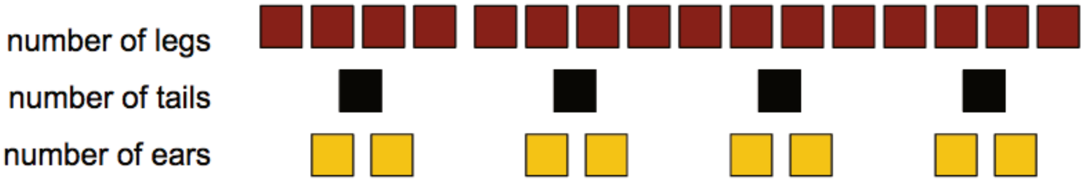
Here is a task to try with your student:

There are 4 horses in a stall. Each horse has 4 legs, 1 tail, and 2 ears.

1. Draw a diagram that shows the ratio of legs, tails, and ears in the stall.
2. Complete each statement.
    - The ratio of _______ to _______ to _______ is _______ : _______ : _______.
    - There are _______ ears for every tail. There are _______ legs for every ear.

Solution:

1. Answers vary. Sample response:



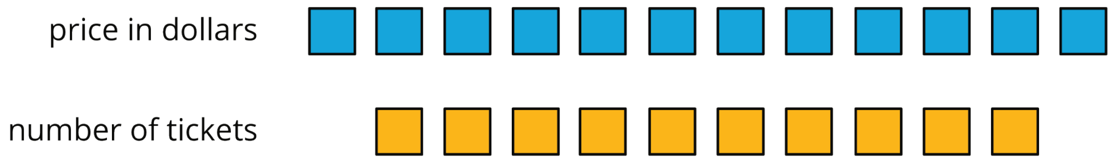
2. Answers vary. Sample response: The ratio of legs to tails to ears is $16 : 4 : 8$. There are 2 ears for every tail. There are 2 legs for every ear.

Representing Equivalent Ratios

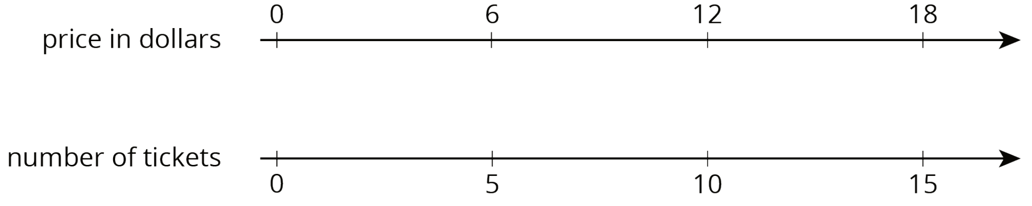
Family Support Materials 2

There are different ways to represent ratios.

Let’s say the 6th grade class is selling raffle tickets at a price of \$6 for 5 tickets. Some students may use diagrams with shapes to represent the situation. For example, here is a diagram representing 10 tickets for \$12.



Drawing so many shapes becomes impractical. Double number line diagrams are easier to work with. The one below represents the price in dollars for different numbers of raffle tickets all sold *at the same rate* of \$12 for 10 tickets.



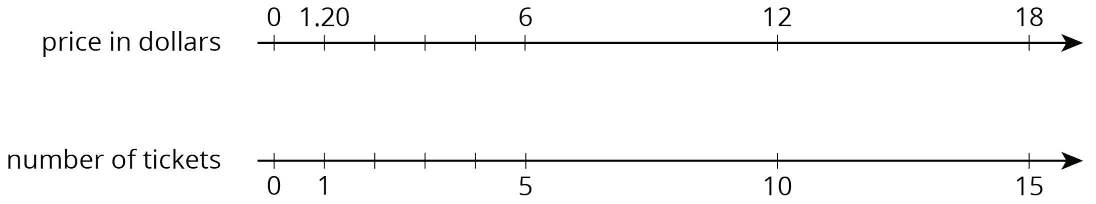
Here is a task to try with your student:

Raffle tickets cost \$6 for 5 tickets.

1. How many tickets can you get for \$90?
2. What is the price of 1 ticket?

Solution:

1. 75 tickets. Possible strategies: Extend the double number line shown and observe that \$90 is lined up with 75 tickets. Or, since 90 is 6 times 15, compute 5 times 15.
2. \$1.20. Possible strategies: Divide the number line into 5 equal intervals, as shown. Reason that the price in dollars of 1 ticket must be $6 \div 5$.


price in dollars
0
1.20
6
12
18
number of tickets
0
1
5
10
15

Rates

Family Support Materials 3

Who biked faster: Andre, who biked 25 miles in 2 hours, or Lin, who biked 30 miles in 3 hours? One strategy would be to calculate a **unit rate** for each person. A unit rate is an equivalent ratio expressed as something "per 1." For example, Andre's rate could be written as "$12\frac{1}{2}$ miles in 1 hour" or "$12\frac{1}{2}$ miles *per 1 hour*." Lin's rate could be written "10 miles per 1 hour." By finding the unit rates, we can compare the distance each person went in 1 hour to see that Andre biked faster.

Every ratio has *two* unit rates. In this example, we could also compute *hours per mile*: how many hours it took each person to cover 1 mile. Although not every rate has a special name, rates in "miles per hour" are commonly called **speed** and rates in "hours per mile" are commonly called **pace**.

Andre:

distance (miles)	time (hours)
25	2
1	0.08
12.5	1

Lin:

distance (miles)	time (hours)
30	3
10	1
1	0.1

Here is a task to try with your student:

Dry dog food is sold in bulk: 4 pounds for $16.00.

1. At this rate, what is the cost *per pound* of dog food?
2. At this rate, what is the amount of dog food you can buy *per dollar*?

Solution:

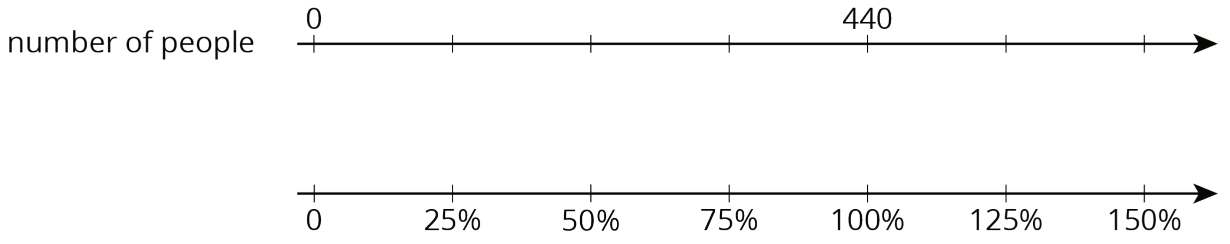
1. \$4.00 per pound because $16 \div 4 = 4$.
2. You get $\frac{1}{4}$ or 0.25 of a pound per dollar because $4 \div 16 = 0.25$.

dog food (pounds)	cost (dollars)
4	16
1	4
0.25	1

Percentages

Family Support Materials 4

Let's say 440 people attended a school fundraiser last year. If 330 people were adults, what percentage of people were adults? If it's expected that the attendance this year will be 125% of last year, how many attendees are expected this year? A double number line can be used to reason about these questions.



Students use their understanding of "rates per 1" to find **percentages**, which we can think of as "rates per 100." Double number lines and tables continue to support their thinking. The example about attendees of a fundraiser could also be organized in a table:

number of people	percentage
440	100%
110	25%
330	75%
550	125%

Toward the end of the unit, students develop more sophisticated strategies for finding percentages. For example, you can find 125% of 440 attendees by computing $\frac{125}{100} \cdot 440$. With practice, students will use these more efficient strategies and understand why they work.

Here is a task to try with your student:

For each question, explain your reasoning. If you get stuck, try creating a table or double number line for the situation.

1. A bottle of juice contains 16 ounces, and you drink 25% of the bottle. How many ounces did you drink?
2. You get 9 questions right in a trivia game, which is 75% of the questions. How many questions are in the game?
3. You planned to walk 8 miles, but you ended up walking 12 miles. What percentage of your planned distance did you walk?

Solution:

Any correct reasoning that a student understands and can explain is acceptable. Sample reasoning:

1. 4. 25% of the bottle is $\frac{1}{4}$ of the bottle, and $\frac{1}{4}$ of 16 is 4.
2. 12. If 9 questions is 75%, we can divide each by 3 to know that 3 questions is 25%. Multiplying each by 4 shows that 12 questions is 100%.
3. 150%. If 8 miles is 100%, then 4 miles is 50%, and 12 miles is 150%.