## Lesson 14: Proving the Pythagorean Theorem

* Let’s prove the Pythagorean Theorem.

### 14.1: Notice and Wonder: Variable Version



What do you notice? What do you wonder?

### 14.2: Prove Pythagoras Right



Elena is playing with the equivalent ratios she wrote in the warm-up. She rewrites $\frac{a}{x}=\frac{c}{a} as a^{2}=xc$. Diego notices and comments, “I got $b^{2}=yc$. The $a^{2}$ and $b^{2}$ remind me of the Pythagorean Theorem.” Elena says, “The Pythagorean Theorem says that $a^{2}+b^{2}=c^{2}$. I bet we could figure out how to show that.”

1. How did Elena get from $\frac{a}{x}=\frac{c}{a} to a^{2}=xc$?
2. What equivalent ratios of side lengths did Diego use to get $b^{2}=yc$?
3. Prove $a^{2}+b^{2}=c^{2}$ in a right triangle with legs length $a$ and $b$ and hypotenuse length $c$.

### 14.3: An Alternate Approach



When Pythagoras proved his theorem he used the 2 images shown here. Can you figure out how he used these diagrams to prove $a^{2}+b^{2}=c^{2}$ in a right triangle with hypotenuse length $c$?

#### Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

* Cut out 2 congruent right triangles
* Label the long sides $b$, the short sides $a$ and the hypotenuses $c$.
* Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

How does this diagram prove the Pythagorean Theorem?



### Lesson 14 Summary

In any right triangle with legs $a$ and $b$ and hypotenuse $c$, we know that $a^{2}+b^{2}=c^{2}$. We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle, $\frac{c}{a}=\frac{a}{d}$. Because the largest triangle is similar to the middle triangle, $\frac{c}{b}=\frac{b}{e}$. We can rewrite these equations as $a^{2}=cd$ and $b^{2}=ce$.

We can add the 2 equations to get that $a^{2}+b^{2}=cd+ce$ or $a^{2}+b^{2}=c(d+e)$. From the original diagram we can see that $d+e=c$, so $a^{2}+b^{2}=c(c)$ or $a^{2}+b^{2}=c^{2}$.

Using the Pythagorean Theorem we can describe a triangle's angles without ever drawing it. For example, a triangle with side lengths 8, 15, and 17 is right because $17^{2}=8^{2}+15^{2}$. A triangle with side lengths 8, 15, and 18 is obtuse because $18^{2}>8^{2}+15^{2}$. A triangle with side lengths 8, 15, and 16 is acute because $16^{2}<8^{2}+15^{2}$.



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