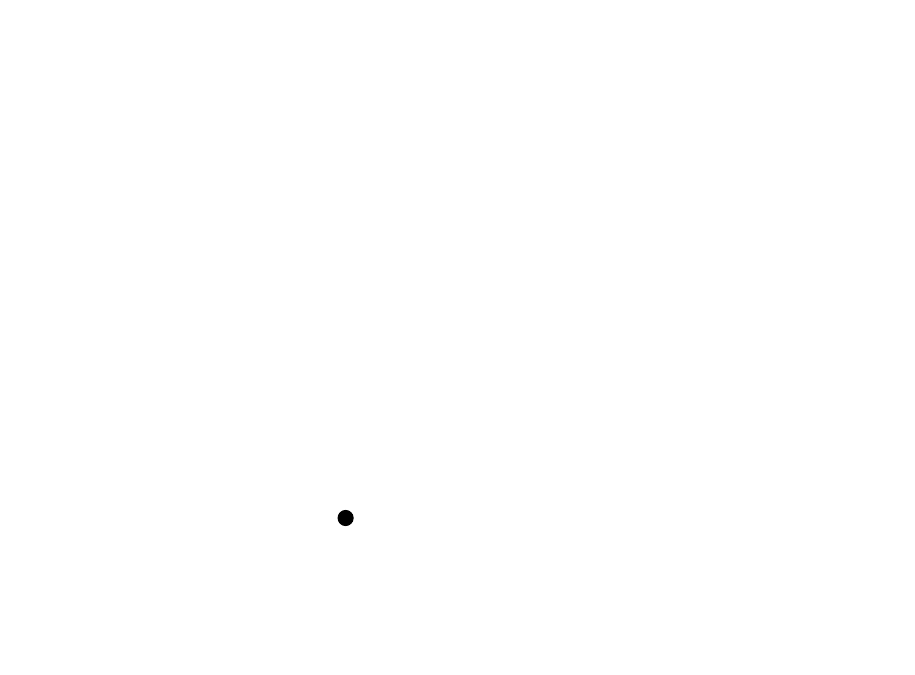
## Lesson 14: Proving the Pythagorean Theorem

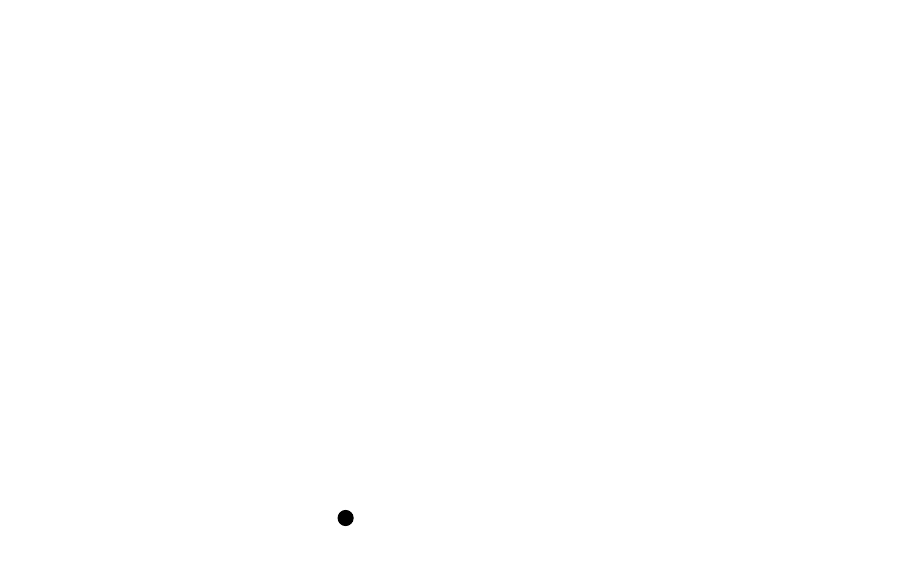
* Let’s prove the Pythagorean Theorem.

### 14.1: Notice and Wonder: Variable Version



What do you notice? What do you wonder?

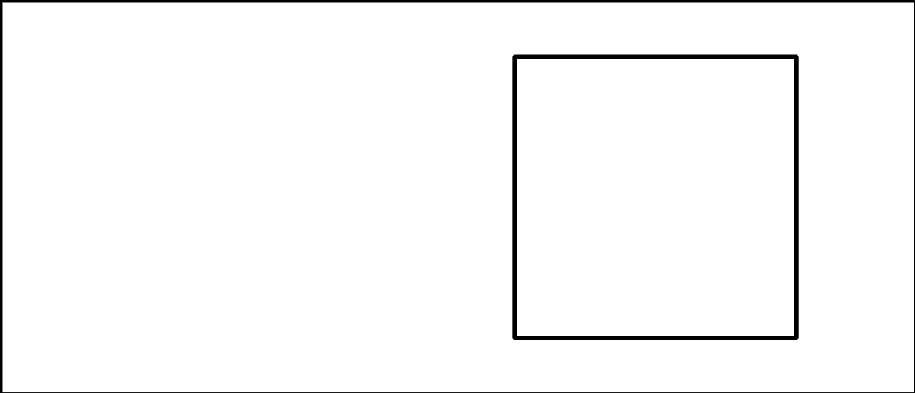
### 14.2: Prove Pythagoras Right



Elena is playing with the equivalent ratios she wrote in the warm-up. She rewrites . Diego notices and comments, “I got . The  and remind me of the Pythagorean Theorem.” Elena says, “The Pythagorean Theorem says that . I bet we could figure out how to show that.”

1. How did Elena get from ?
2. What equivalent ratios of side lengths did Diego use to get ?
3. Prove in a right triangle with legs length and and hypotenuse length .

### 14.3: An Alternate Approach



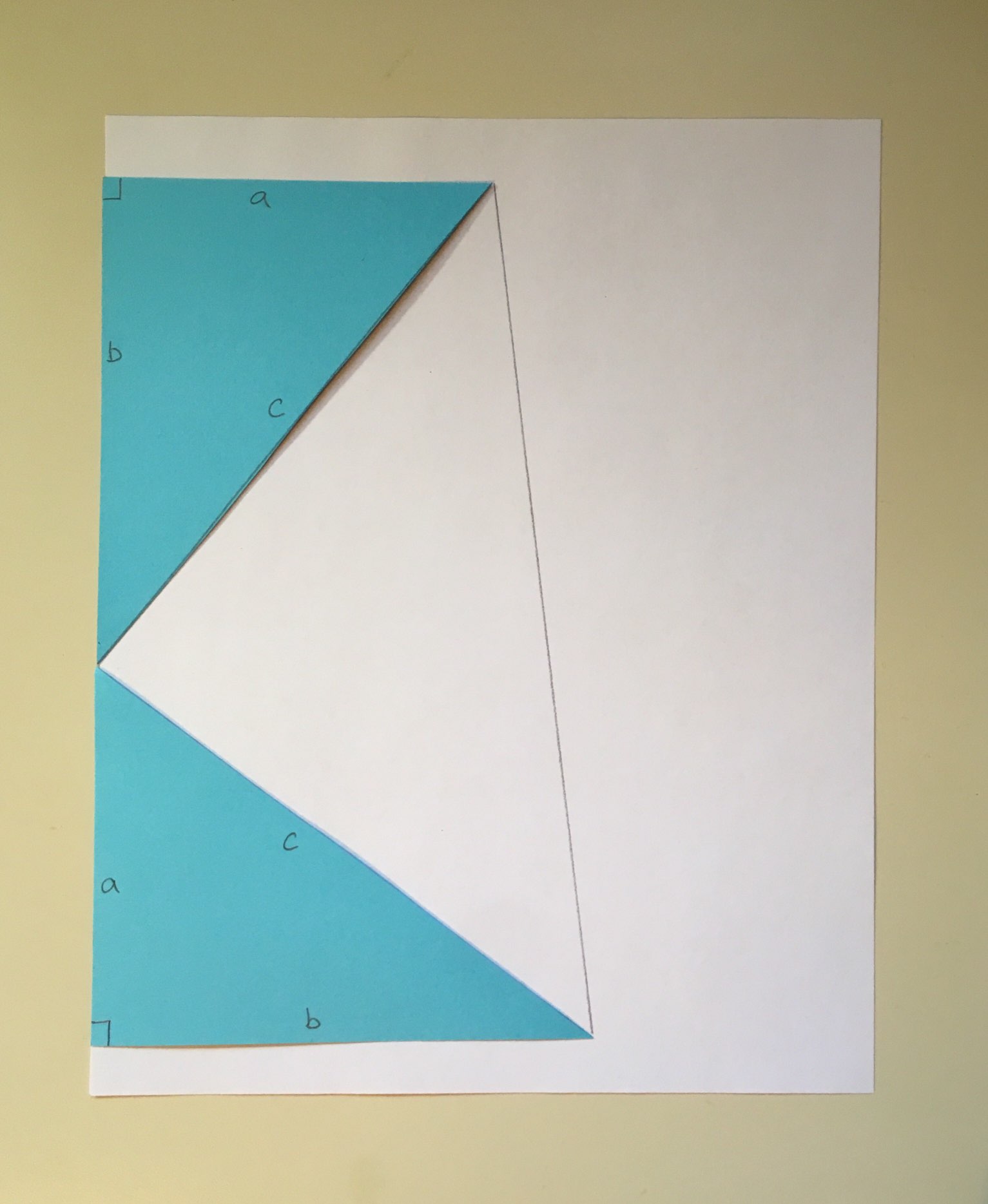
When Pythagoras proved his theorem he used the 2 images shown here. Can you figure out how he used these diagrams to prove in a right triangle with hypotenuse length ?

#### Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

* Cut out 2 congruent right triangles
* Label the long sides , the short sides  and the hypotenuses .
* Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

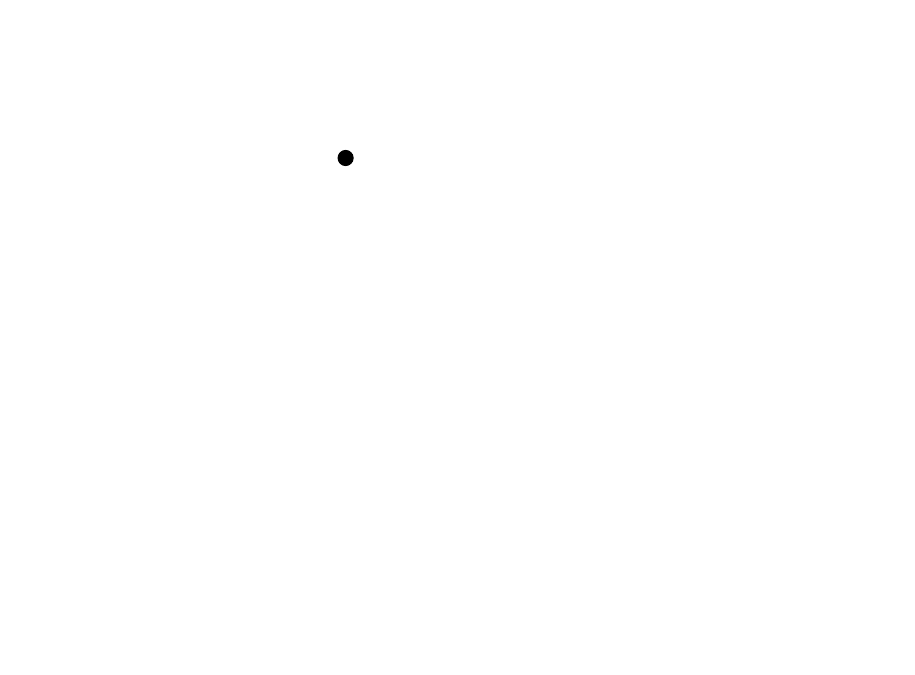
How does this diagram prove the Pythagorean Theorem?



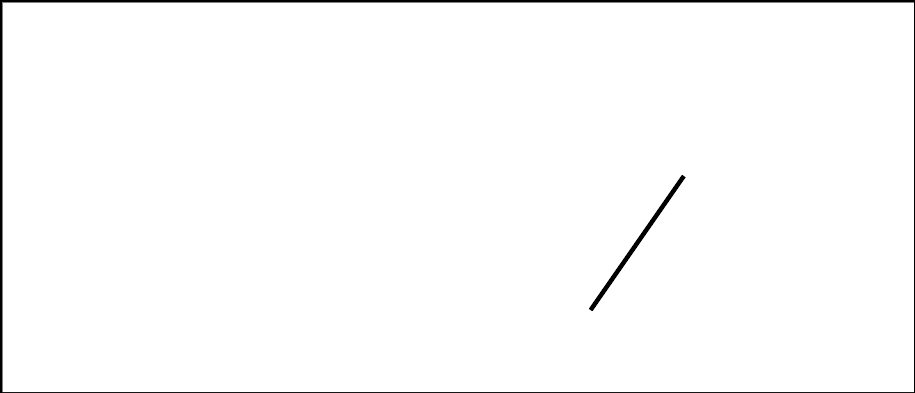
### Lesson 14 Summary

In any right triangle with legs and and hypotenuse , we know that . We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle, . Because the largest triangle is similar to the middle triangle, . We can rewrite these equations as and .

We can add the 2 equations to get that or . From the original diagram we can see that , so or .

Using the Pythagorean Theorem we can describe a triangle's angles without ever drawing it. For example, a triangle with side lengths 8, 15, and 17 is right because . A triangle with side lengths 8, 15, and 18 is obtuse because . A triangle with side lengths 8, 15, and 16 is acute because .



© CC BY 2019 by Illustrative Mathematics®