## Lesson 12: Arithmetic with Complex Numbers

* Let’s work with complex numbers.

### 12.1: Math Talk: Telescoping Sums

Find the value of these expressions mentally.

$2−2+20−20+200−200$

$100−50+10−10+50−100$

$3+2+1+0−1−2−3$

$1+2+4+8+16+32−16−8−4−2−1$

### 12.2: Adding Complex Numbers

1. This diagram represents $\left(2+3i\right)+\left(-8−8i\right)$.
* 
	1. How do you see $2+3i$ represented?
	2. How do you see $-8−8i$ represented?
	3. What complex number does $A$ represent?
	4. Add “like terms” in the expression $\left(2+3i\right)+\left(-8−8i\right)$. What do you get?
1. Write these sums and differences in the form $a+bi$, where $a$ and $b$ are real numbers.
	1. $\left(-3+2i\right)+\left(4−5i\right)$ (Check your work by drawing a diagram.)
	2. $\left(-37−45i\right)+\left(11+81i\right)$
	3. $\left(-3+2i\right)−\left(4−5i\right)$
	4. $\left(-37−45i\right)−\left(11+81i\right)$

### 12.3: Multiplication on the Complex Plane

1. Draw points to represent 2, 22, 23, and 24 on the real number line.
* 
	1. Write $2i$, $\left(2i\right)^{2}$, $\left(2i\right)^{3}$, and $\left(2i\right)^{4}$ in the form $a+bi$.
	2. Plot $2i$, $\left(2i\right)^{2}$, $\left(2i\right)^{3}$, and $\left(2i\right)^{4}$ on the complex plane.
	+ 

#### Are you ready for more?

1. If $a$ and $b$ are positive numbers, is it true that $\sqrt{ab}=\sqrt{a}\sqrt{b}$? Explain how you know.
2. If $a$ and $b$ are negative numbers, is it true that $\sqrt{ab}=\sqrt{a}\sqrt{b}$? Explain how you know.

### Lesson 12 Summary

When we add a real number with an imaginary number, we get a complex number. We usually write complex numbers as:

$a+bi$

where $a$ and $b$ are real numbers. We say that $a$ is the real part and $bi$ is the imaginary part.

To add (or subtract) two complex numbers, we add (or subtract) the real parts and add (or subtract) the imaginary parts. For example:

$\left(2+3i\right)+\left(4+5i\right)=\left(2+4\right)+\left(3i+5i\right)=6+8i$

$\left(2+3i\right)−\left(4+5i\right)=\left(2−4\right)+\left(3i−5i\right)=-2−2i$

In general:

$\left(a+bi\right)+\left(c+di\right)=\left(a+c\right)+\left(b+d\right)i$

and:

$\left(a+bi\right)−\left(c+di\right)=\left(a−c\right)+\left(b−d\right)i$

When we raise an imaginary number to a power, we can use the fact that $i^{2}=-1$ to write the result in the form $a+bi$. For example, $\left(4i\right)^{3}=4i⋅4i⋅4i$. We can group the $i$ factors together to see how to rewrite this.

$\begin{matrix}4i⋅4i⋅4i&=\left(4⋅4⋅4\right)⋅\left(i⋅i⋅i\right)\\&=64⋅\left(i^{2}⋅i\right)\\&=64⋅-1⋅i\\&=-64i\end{matrix}$

So in this example, $a$ is 0 and $b$ is -64.



© CC BY 2019 by Illustrative Mathematics®