Lesson 2: Multiplying Powers of Ten

Goals

- Generalize a process for multiplying exponential expressions with the same base, and justify (orally and in writing) that $10^n \cdot 10^m = 10^{n+m}$.

Learning Targets

- I can explain and use a rule for multiplying powers of 10.

Lesson Narrative

Students make use of repeated reasoning to discover the exponent rule $10^n \cdot 10^m = 10^{n+m}$ (MP8). At this time, students develop rules for positive exponents. In subsequent lessons, students will extend the exponent rules to cases where the exponents are zero or negative. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them (MP2).

Alignments

Building On

- 5.NBT.A.3.a: Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Building Towards

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

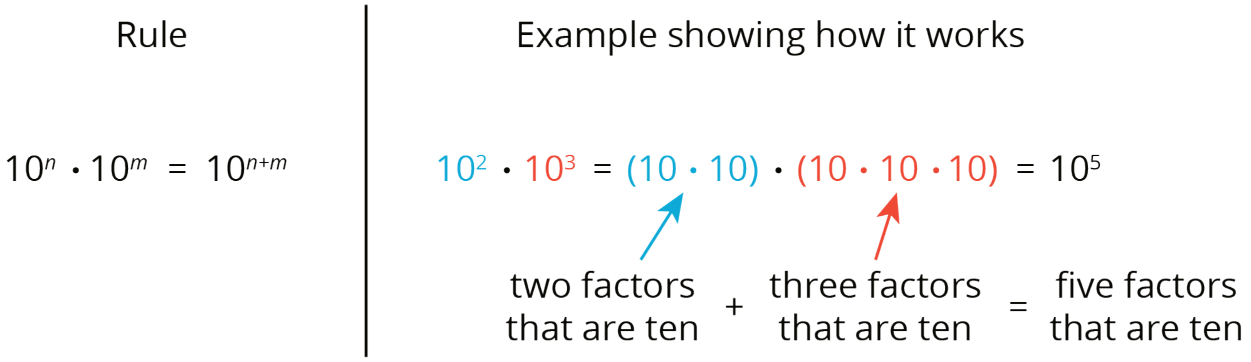
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- Think Pair Share

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create visual displays of the exponent rule $10^n \cdot 10^m = 10^{n+m}$ to be displayed for all to see throughout the unit. Here is a sample visual display:



Student Learning Goals

Let’s explore patterns with exponents when we multiply powers of 10.

2.1 100, 1, or $\frac{1}{100}$?

Warm Up: 5 minutes
This warm-up gives students a chance to think about different numbers that a diagram might represent. In later activities, students thinking about diagrams that represent different powers of 10.

Building On

- 5.NBT.A.3.a

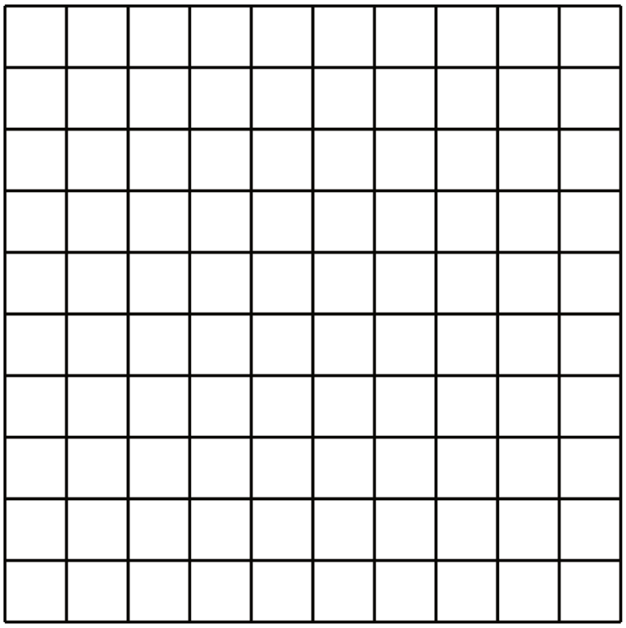
Building Towards

- 8.EE.A.1
- 8.EE.A.3

Launch

Give students 1 minute of quiet think time followed by 2 minutes of partner discussion.

Student Task Statement



Clare said she sees 100.

Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Who do you agree with?

Student Response

Answers vary. Sample response: I agree with all of them. There are 100 small squares and 1 big square. Each small square is $\frac{1}{100}$ of the large square.

Activity Synthesis

Poll the class to see who agrees with each person in turn. Then ask someone to explain in each case.

2.2 Picture a Power of 10

15 minutes

The purpose of this activity is for students to develop a sense of visual scale between powers of 10. Students should understand that multiplying by 10 corresponds to increasing the exponent by 1. Even though the notation for 10^{100} does not appear to be much different than 10^{98}, it is 100 times larger. Small changes in the exponent can result in large changes in the value of the expression.

Building Towards

- 8.EE.A.1
- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by 5 minutes to share their responses with their partner and a whole-class discussion.

Access for Students with Disabilities

Representation: Provide Access for Perception. Provide students with additional copies of the representations to draw on or highlight. Students may benefit from being able to mark each small square with the different powers of ten to develop a sense of visual scale between powers of 10.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Access for English Language Learners

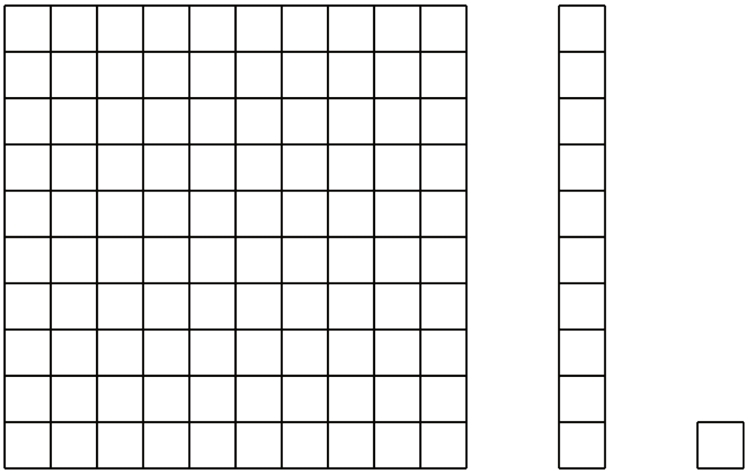
Conversing, Representing, Writing: MLR2 Collect and Display. Circulate and listen to students talk about the different powers of 10 representations during pair work or group work, and jot notes about important words or phrases (e.g., power of 10, multiply by 10) and expressions (e.g., 10^2 vs. $10 \cdot 10$), together with helpful sketches or diagrams of the respective rectangle and squares. Scribe students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson and unit. This will help students read and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may multiply the exponent by 10, for example 10^2, 10^{20}, 10^{200}. Ask these students to write out what 10^{20} would mean and whether that matches their intention.

Student Task Statement

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could you represent the large square as a power of 10?
2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?
3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?
4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

Student Response

1. The large square can be seen as 10^2 because there are 100 small squares.
2. The medium rectangle rectangle represents 10^3 because $10^2 \cdot 10 = 10^3$. The large square represents 10^4 because $10^2 \cdot 10^2 = 10^4$.
3. The small square represents 10^4 because $10^5 \div 10 = 10^4$. The large square represents 10^6 because $10^5 \cdot 10 = 10^6$.
4. The medium rectangle rectangle represents 10^{99} because $10^{100} \div 10 = 10^{99}$ and the small square represents 10^{98} because $10^{100} \div 10^2 = 10^{98}$.

Activity Synthesis

The key takeaway is that increasing the exponent of a power of 10 by 1 corresponds to multiplying or dividing by 10, and decreasing the exponent by 1 corresponds to dividing by 10.

Ask students to share their responses for the last several questions. Record and display their responses for all to see. If possible, reference the image to highlight the magnitude of change when the power of 10 increases or decreases by 1 or 2. If a student gives an answer such as, “the area of the medium rectangle is $10^2 + 10^2 + \ldots + 10^2$, ask for volunteers to help write the expression using a single power of 10.

If projection is available, consider sharing this applet, which illustrates animals measured in units that vary by powers of ten. https://ggbm.at/NhpASDzz

2.3 Multiplying Powers of Ten

15 minutes

The goal of this activity is to help students flexibly transition between different notations for powers of 10 and introduce the property of multiplication of values with the same base. Students observe that $10^n \cdot 10^m = 10^{n+m}$ for values of n and m that are positive integers. The second question hints at the reasoning that will extend exponent rules to include zero exponents, but students will investigate that more deeply in a later lesson.

Notice students who need help writing the general rule in in terms of n and m. You might ask, "What patterns did you notice with the exponents in the table? So if the exponents are n and m, how do you write what you did with the exponents?"

Addressing

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before asking 1–2 students to share and explain their answers. When it is clear that students know how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 7–8 minutes to work before a brief whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Student Response

1. a.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^7
$10^4 \cdot 10^4$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^3 \cdot 10^5$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^{18} \cdot 10^{23}$	skip	10^{41}

b. I chose to skip the expanded column of $10^{18} \cdot 10^{23}$ because there are too many factors that are 10 and they won't fit in the table.

2. a. $10^n \cdot 10^m = 10^{n+m}$ because multiplying n factors that are 10 with m factors that are 10 results in $n + m$ factors that are 10.

   b. 10^4 because $10^4 \cdot 10^0 = 10^{4+0}$. That means 10^0 must equal 1 for the rule to work.

3. There are 10^{20} bacteria because 10^7 people times 10^{13} bacteria per person is equal to 10^{20} total bacteria.

Are You Ready for More?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

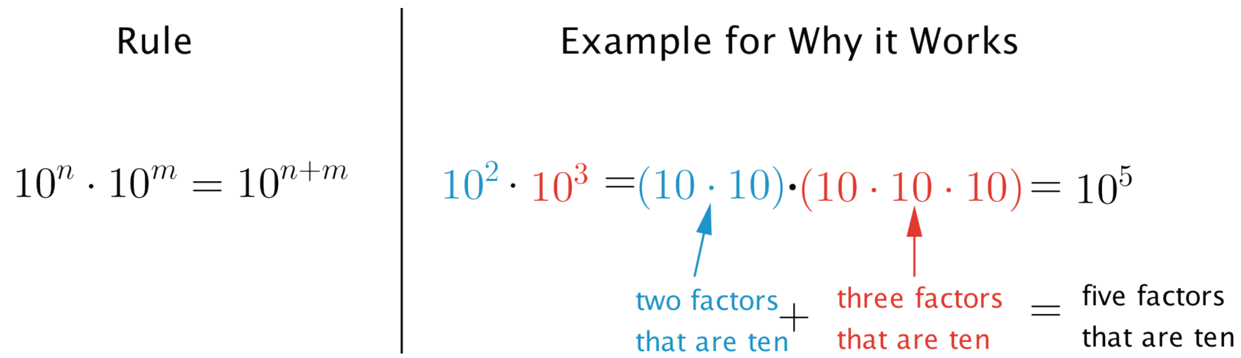
1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?
2. How about 10^7? 10^8?

Student Response

1. This question is equivalent to the question, “How many ways are there to write the number 6 as the sum of smaller positive whole numbers?” There are ten ways to do this.
2. 14 ways, 22 ways

Activity Synthesis

Create and post visual displays showing the exponent rules for reference throughout the unit, with one visual display for each rule. The visual display could include an example to illustrate how the rule works, along with visual aids and use of color. Here is a sample visual display:



Explain the visual display to students and display it for all to see throughout the unit.

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present a hypothetical student statement that represents a misunderstanding of the exponent rule, such as "$10^2 \cdot 10^3 = 10^6$ because $2 \cdot 3 = 6$." Prompt discussion by asking, "Do you agree with the statement? Why or why not?" Then, ask students to individually write an improved statement. Improved responses should include connections between the representations of a single expression, expanded form, and the single power of 10. This will help students evaluate, and improve on, the written mathematical arguments of others and highlight why $10^n \cdot 10^m = 10^{n+m}$.
Design Principle(s): Maximize meta-awareness.

Lesson Synthesis

The purpose of the discussion is to check whether students understand why $10^n \cdot 10^m = 10^{n+m}$. Consider recording student responses and displaying them for all to see.

Here are some questions for discussion:

- "How could you write $10^{15} \cdot 10^5$ using a single exponent without expanding all of the factors?" (The first part is 15 factors that are 10 and the second is 5 factors that are 10. This makes a total of 20 factors that are 10.)
- "In general, what is a rule for multiplying two powers of 10 together into a single power of 10?" (The exponents of the two powers of 10 are added together.)

2.4 That's a Lot of Dough, Though!

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.
2. Each year, roughly 10^6 computer programmers each make about \$$10^5$. How much money is this all together? Express your answer both as a power of 10 and as a dollar amount.

Student Response

1. 10^{38}, because $10^{32} \cdot 10^6 = 10^{32+6} = 10^{38}$.
2. This is \$100,000,000,000 (one hundred billion dollars). Each programmer makes 10^5 dollars, and there are 10^6 programmers. So we multiply $10^5 \cdot 10^6 = 10^{5+6}$, which is 10^{11}.

Student Lesson Summary

In this lesson, we developed a rule for multiplying powers of 10: multiplying powers of 10 corresponds to adding the exponents together. To see this, multiply 10^5 and 10^2. We know that 10^5 has five factors that are 10 and 10^2 has two factors that are 10. That means that $10^5 \cdot 10^2$ has 7 factors that are 10.

$$10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7.$$

This will work for other powers of 10 too. So $10^{14} \cdot 10^{47} = 10^{61}$.

This rule makes it easier to understand and work with expressions that have exponents.

Lesson 2 Practice Problems

Problem 1

Statement

Write each expression with a single exponent:

a. $10^3 \cdot 10^9$

b. $10 \cdot 10^4$

c. $10^{10} \cdot 10^7$

d. $10^3 \cdot 10^3$

e. $10^5 \cdot 10^{12}$

f. $10^6 \cdot 10^6 \cdot 10^6$

Solution

a. 10^{12}

b. 10^5

c. 10^{17}

d. 10^6

e. 10^{17}

f. 10^{18}

Problem 2

Statement

A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water.

a. What is the area of the surface of the water in the pool?

b. How much water does the pool hold?

c. Express your answers to the previous two questions as powers of 10.

Solution

a. 100,000 square feet

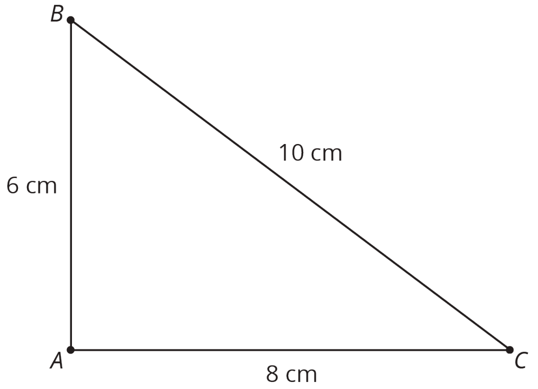
b. 1,000,000 cubic feet

c. 10^5 square feet, 10^6 cubic feet

Problem 3

Statement

Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



Solution

$DE = 3$ and $DF = 4$ (The scale factor is $\frac{1}{2}$, so each side length is half the corresponding side length of ABC.)

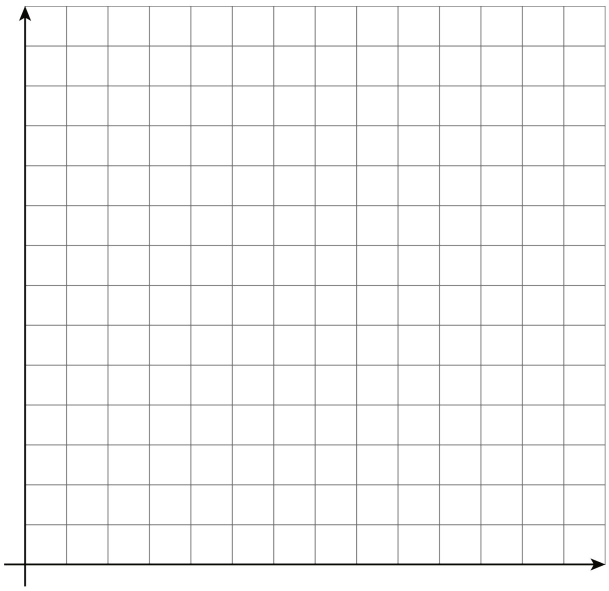
(From Unit 2, Lesson 7.)

Problem 4

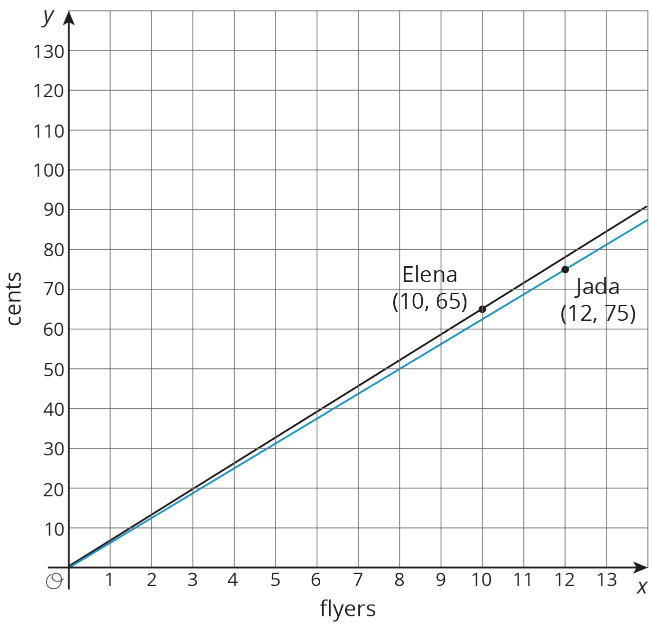
Statement

Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



Solution



Elena is paid more money after distributing 14 flyers because her graph is steeper than Jada's. For any number of flyers, the point on Elena's graph is higher than the point on Jada’s graph.

(From Unit 3, Lesson 3.)