Lesson 3: Powers of Powers of 10

Goals

- Generalize a process for finding a power raised to a power, and justify (orally and in writing) that $(10^n)^m = 10^{n \cdot m}$.

Learning Targets

- I can explain and use a rule for raising a power of 10 to a power.

Lesson Narrative

Students make use of repeated reasoning to discover the exponent rule $(10^n)^m = 10^{n \cdot m}$ (MP8). At this time, students develop rules for positive exponents. In subsequent lessons, students will extend the exponent rules to cases where the exponents are zero or negative. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them (MP2).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Preparation

Create a visual display for the rule $(10^m)^n = 10^{m \cdot n}$ to display for all to see throughout the unit. The display will be introduced during the discussion of“Taking Powers of Powers of 10” activity. For an example of how the rule works, consider showing $(10^2)^3 = (10 \cdot 10)(10 \cdot 10)(10 \cdot 10) = 10^6$ using colors or other visual aids to highlight that the result is 10^6 because there are three groups of 10^2. For example,

Rule	Example for Why it Works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$
	three groups of two factors that are ten = six factors that are ten

Student Learning Goals

Let's look at powers of powers of 10.

3.1 Big Cube

Warm Up: 5 minutes

The purpose of this warm-up is to introduce students the idea of raising a value with an exponent to another power. Computing the volume of a cube whose side lengths are themselves powers of 10 introduces the basic structure of a power to a power, which will lead to a general exponent rule during later activities.

Look out for different strategies used to compute $10{,}000^3$ so they can be discussed during whole-class discussion. Some students may count zeros to keep track of place value, while other students will write 10,000 as 10^4 and use exponent rules.

Addressing

- 8.EE.A.1

Building Towards

- 8.EE.A.4

Launch

Give students 3 minutes of quiet work time followed by a brief whole-class discussion.

Anticipated Misconceptions

Some students may want to compute by multiplying $(10{,}000) \cdot (10{,}000) \cdot (10{,}000)$. Ask these students whether they can use powers of 10 and the exponent rule from the last lesson to make their calculations easier.

Student Task Statement

What is the volume of a giant cube that measures 10,000 km on each side?

Student Response

1,000,000,000,000 km^3, because $10{,}000^3 = \left(10^4\right)^3 = 10^4 \cdot 10^4 \cdot 10^4$. Using the exponent rule from the previous lesson, this is equal to 10^{4+4+4} or just 10^{12}. The side length is given in km, so the volume will be in units of km^3.

Activity Synthesis

The purpose of discussion is to highlight the fact that $\left(10^4\right)^3$ is equal to 10^{12} as a way to transition to the next activity, where this pattern is generalized. Select previously identified students to share their strategies for computing $10{,}000^3$. Ask students what patterns they notice between $\left(10^4\right)^3$ and 10^{12}. If students mention the strategy of counting zeros to multiply powers of 10, connect it to the process of adding exponents. In other words, the total volume in km^3 is a 1 followed by the number of zeros in $(10{,}000) \cdot (10{,}000) \cdot (10{,}000)$ because $10^4 \cdot 10^4 \cdot 10^4 = 10^{4+4+4}$.

3.2 Raising Powers of 10 to Another Power

15 minutes

Students explore patterns to discover the property $(10^m)^n = 10^{m \cdot n}$ for values of m and n that are positive integers. Non-positive integer exponents will be explored in a subsequent lesson. Notice students who try to rush to complete the table without recognizing the patterns moving between columns in the table. As students work, ask them if they can explain what patterns they are finding. Select students who can explain the patterns they see to share during whole-class discussion.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table, then select 1–2 students to share and explain their solutions. When it is clear that students know how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 10–12 minutes to work before a brief whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Student Response

1. a.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	10^{10}
$(10^3)^4$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^{12}
$(10^4)^2$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$(10^8)^{11}$	skip	10^{88}

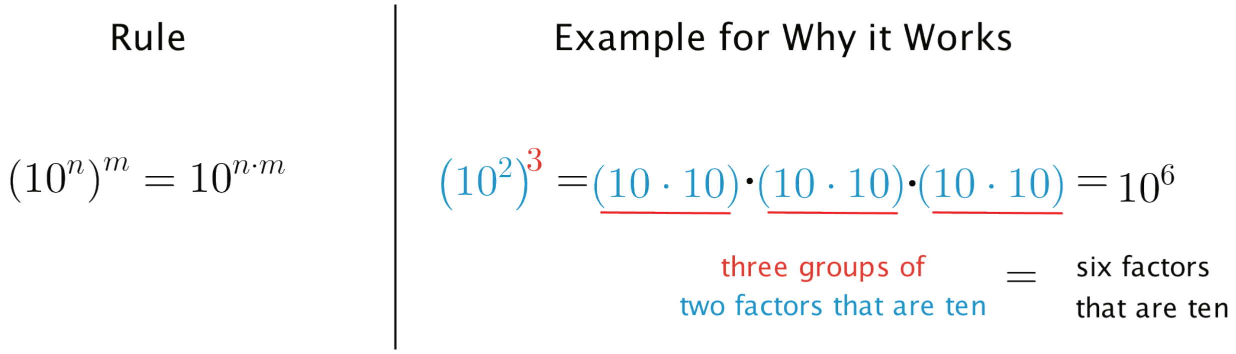
b. Skip the expanded column of $(10^8)^6$ because the table cannot fit 88 factors that are 10.

2. $(10^m)^n = 10^{m \cdot n}$ because there are n groups of m factors that are 10.

3. This is 10^9 cubic meters of oil because $(10^3)^3 = 10^9$. Answers vary. Sample response: This is closest to a lakeful of oil.

Activity Synthesis

Select students who can explain the patterns they noticed to share in a whole-class discussion. Create a visual display for the rule $(10^m)^n = 10^{m \cdot n}$ to display for all to see throughout the unit. For an example of how the rule works, consider showing $(10^2)^3 = (10 \cdot 10)(10 \cdot 10)(10 \cdot 10) = 10^6$ using colors or other visual aids to highlight that the result is 10^6 because there are three groups of 10^2. For example,



Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their explanation of the patterns they noticed in the task. Ask each student to meet with 2–3 other partners in a row for feedback. Provide student with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “Can you give an example?”, “Can you say that another way?”, “How do you know…?”, etc.). Students can borrow ideas and language from each partner to strengthen their final explanation.
Design Principle(s): Optimize output (for explanation)

3.3 How Do the Rules Work?

10 minutes
This activity presents two valid ways to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent, but where the execution of one of the ideas has a mistake. Thinking through this problem will reveal whether students understand the two exponent rules discussed in this lesson, providing opportunity for formative assessment.

Notice students who point out what was correct in both Andre and Elena’s responses. The eventual goal of learning exponent rules is to avoid expanding factors, but expect students at this stage to expand exponential expressions to test whether or not each step is correct.

Addressing

- 8.EE.A.1

Building Towards

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give 6–7 minutes to work followed by partner and whole-class discussions.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Continue to display, or provide a physical copy of the visual display for the rule $(10^n)^m = 10^{n \cdot m}$ from the previous activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, "When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$."
- Elena says, "10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$."

Do you agree with either of them? Explain your reasoning.

Student Response

Answers vary. Sample response: Andre uses the rule for multiplying values with the same base correctly. Elena is partially correct because $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3$, but she did not use the correct exponent rule. She should have written $(10^2)^3 = 10^{2 \cdot 3}$ which is also equal to 10^6.

Are You Ready for More?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Student Response

Since $4{,}096 = 2^{12}$, it can be broken down into other representations of the form $(2^m)^n$ so that $m \cdot n = 12$. For example, $(2^2)^6 = 4^6$, $(2^3)^4 = 8^4$, $(2^4)^3 = 16^3$, $(2^6)^2 = 64^2$, and $(2^{12})^1 = 4{,}096^1$.

Activity Synthesis

It is important to note in the discussion that Elena is not completely wrong. She recognizes that $10^2 \cdot 10^2 \cdot 10^2$ is equivalent to $(10^2)^3$, which shows good conceptual understanding of exponents as repeated multiplication. Select students to share any correct and incorrect steps that they and their partner noticed in Andre and Elena's work. Ask students:

- "Andre wrote $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2}$ and Elena wrote $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3$. How are these ways of thinking different? How are they the same?" (Both methods are combining three powers using properties of exponents without writing out all of the factors. Andre's method only considers the bases of the three powers to combine them while Elena's method notices that each power is actually the same.)
- "What is one way you could avoid making the kinds of mistake that happened in this problem?" (Even if I don't write out all the factors, I could think about how many there will be if I were to write them out.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this to amplify mathematical uses of language to communicate about multiplying powers, bases, and exponents. Invite students to use these words when stating their ideas, and restating the ideas of others. Ask students to chorally repeat the phrases that include these words in context.
Design Principle(s): Support sense-making; Optimize output (for explanation)

Lesson Synthesis

The purpose of the discussion is to check whether students understand why $(10^n)^m = 10^{n \cdot m}$. Consider recording student responses and displaying them for all to see.

Here are some questions for discussion:

- "We looked at repeated multiplication of powers of 10. How would you write $10^4 \cdot 10^4 \cdot 10^4$ with exponents instead of repeated multiplication?" ($\left(10^4\right)^3$)
- "Then how would you write $\left(10^4\right)^3$ using a single exponent?" (10^{12})
- "In general, why do you multiply the exponents when you write a power to a power with a single exponent? Give an example to show your reasoning." (You multiply the exponents because the inner exponent shows how many factors are in each group and the outer exponent shows how many groups of factors there are. For example, $\left(10^4\right)^3$ means that there are 3 groups of factors, and each group has 4 factors that are 10. So there are a total of $3 \cdot 4 = 12$ factors that are 10 altogether.)

3.4 Making a Million

Cool Down: 5 minutes

There are many ways to express a given power of 10 using the exponent sum and product rules. Generating many ways of expressing the same value encourages the student to think more deeply about the rules and how they work.

Addressing

- 8.EE.A.1

Student Task Statement

Here are some equivalent ways of writing 10^4:

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents and multiplication.

Student Response

Answers vary. Sample responses:

- 1,000,000
- A million
- $10^2 \cdot 10^4$
- $(10^3)^2$

Student Lesson Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$\left(10^4\right)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $\left(10^6\right)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Glossary

- base (of an exponent)

Lesson 3 Practice Problems

Problem 1

Statement

Write each expression with a single exponent:

a. $(10^7)^2$

b. $(10^9)^3$

c. $(10^6)^3$

d. $(10^2)^3$

e. $(10^3)^2$

f. $(10^5)^7$

Solution

a. 10^{14}

b. 10^{27}

c. 10^{18}

d. 10^6

e. 10^6

f. 10^{35}

Problem 2

Statement

You have 1,000,000 number cubes, each measuring one inch on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

Solution

a. The height of the tower would be 1,000,000 inches. That is about 83,333 feet ($1{,}000{,}000 \div 12 \approx 83{,}333$), which is almost 16 miles ($83{,}333 \div 5{,}280 \approx 15.78$).

b. The square would be 1,000 inches on a side, because $1{,}000 \cdot 1{,}000 = 1{,}000{,}000$. 1,000 inches is about 83 feet. This probably wouldn't fit in a classroom.

c. The cube would be 100 inches on a side because $100 \cdot 100 \cdot 100 = 1{,}000{,}000$. This is $8\frac{1}{3}$ feet.

Problem 3

Statement

An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

a. How many amoebas are there after 1 hour?

b. How many amoebas are there after 2 hours?

c. Write an expression for the number of amoebas after 6 hours.

d. Write an expression for the number of amoebas after 24 hours.

e. Why might exponential notation be preferable to answer these questions?

Solution

a. 2

b. 4

c. 2^6 (or 64)

d. 2^{24} (or 16,777,216)

e. Exponential notation is simpler to write than very large or small numbers, and the expression 2^{24} visibly includes the information that the amoebas have divided 24 times.

(From Unit 7, Lesson 1.)

Problem 4

Statement

Elena noticed that, nine years ago, her cousin Katie was twice as old as Elena was then. Then Elena said, "In four years, I'll be as old as Katie is now!" If Elena is currently e years old and Katie is k years old, which system of equations matches the story?

A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

D. $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

Solution

D

(From Unit 4, Lesson 15.)