## Lesson 17: Completing the Square and Complex Solutions

* Let’s find complex solutions to quadratic equations by completing the square.

### 17.1: Creating Quadratic Equations

Match each equation in standard form to its factored form and its solutions.

1. $x^{2}−25=0$
2. $x^{2}−5=0$
3. $x^{2}+25=0$
* $\left(x−5i\right)\left(x+5i\right)=0$
* $\left(x−5\right)\left(x+5\right)=0$
* $\left(x−\sqrt{5}\right)\left(x+\sqrt{5}\right)=0$
* $\sqrt{5}$, $-\sqrt{5}$
* 5, -5
* $5i$, ​​​​$-5i$

### 17.2: Sometimes the Solutions Aren't Real Numbers

What are the solutions to these equations?

1. $\left(x−5\right)^{2}=0$
2. $\left(x−5\right)^{2}=1$
3. $\left(x−5\right)^{2}=-1$

### 17.3: Finding Complex Solutions

Solve these equations by completing the square.

1. $x^{2}−8x+13=0$
2. $x^{2}−8x+19=0$

#### Are you ready for more?

For which values of $a$ does the equation $x^{2}−8x+a=0$ have two real solutions? One real solution? No real solutions? Explain your reasoning.

### 17.4: Can You See the Solutions on a Graph?

1. How many real solutions does each equation have? How many non-real solutions?
	1. $x^{2}−8x+13=0$
	2. $x^{2}−8x+16=0$
	3. $x^{2}−8x+19=0$
2. How do the graphs of these functions help us answer the previous question?
	1. $f\left(x\right)=x^{2}−8x+13$
	2. $g\left(x\right)=x^{2}−8x+16$
	3. $h\left(x\right)=x^{2}−8x+19$

### Lesson 17 Summary

Sometimes quadratic equations have real solutions, and sometimes they do not. Here is a quadratic equation with $x^{2}$ equal to a negative number (assume $k$ is positive):

$x^{2}=-k$

This equation will have imaginary solutions $i\sqrt{k}$ and $-i\sqrt{k}$. By similar reasoning, an equation of the form:

$\left(x−h\right)^{2}=-k$

will have non-real solutions if $k$ is positive. In this case, the solutions are $h+i\sqrt{k}$ and $h−i\sqrt{k}$.

It isn’t always clear just by looking at a quadratic equation whether the solutions will be real or not. For example, look at this quadratic equation:

$x^{2}−12x+41=0$

We can always complete the square to find out what the solutions will be:

$\begin{matrix}x^{2}−12x+36+5&=0\\\left(x−6\right)^{2}+5&=0\\\left(x−6\right)^{2}&=-5\\x−6&=\pm i\sqrt{5}\\x&=6\pm i\sqrt{5}\end{matrix}$

This equation has non-real, complex solutions $6+i\sqrt{5}$ and $6−i\sqrt{5}$.



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