Lesson 4: More Balanced Moves

Goals

- Calculate a value that is a solution for a linear equation in one variable, and compare and contrast (orally) solution strategies with others.
- Critique (in writing) the reasoning of others in solving a linear equation in one variable.

Learning Targets

- I can make sense of multiple ways to solve an equation.

Lesson Narrative

In this lesson, students continue to reinforce the circle connecting three fundamental ideas: a solution to an equation is a number that makes the equation true, performing the same operation on each side of an equation maintains the equality in the equation, and therefore two equations related by such a move have the same solutions. In the warm-up, students are given an equation and then asked whether each of four other equations has the same solution as the given one. They see the move connecting each of the four to the original and conclude that they all have the same solutions. In the next activity, they compare two correct solution paths for an equation, and then two incorrect solution paths, identifying the mistakes made. Then they get some practice solving equations choosing their own paths.

Alignments

Addressing

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
- 8.EE.C.7: Solve linear equations in one variable.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

> **Student Learning Goals**
>
> Let's rewrite some more equations while keeping the same solutions.

4.1 Different Equations?

Warm Up: 5 minutes

The purpose of this warm-up is to for students to use the structure of equations to recognize when they are the same without having to solve for the specific x value that makes the equations true.

Monitor for students who:

- solve each equation for x, then compare the solutions
- solve Equation 1 for x, then substitute it into Equations A–D
- manipulate Equations A–D to look like Equation 1 and vice versa

Addressing

- 8.EE.C

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Give students 2–3 minutes quiet think time, then whole-class discussion.

Student Task Statement

Equation 1

$$x - 3 = 2 - 4x$$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

Equation A	Equation B	Equation C	Equation D
$2x - 6 = 4 - 8x$	$x - 5 = -4x$	$2(1 - 2x) = x - 3$	$-3 = 2 - 5x$

Student Response

All of the other equations have the same solutions as the first equation, $x = 1$.

Equation A: If you multiply each side of Equation 1 by 2, the result is Equation A. So if x makes Equation 1 true then it makes Equation A true as well.

Equation B: If you subtract 2 from each side of Equation 1, the result is Equation B, so if Equation 1 is true then Equation B is true.

Equation C: If you switch everything to the left of the equal sign and everything to the right of the equal sign on Equation C, then rewrite the expression $2(1 - 2x)$ as $2 - 4x$ using the distributive property, then the result is Equation 1, so if Equation 1 is true then Equation C is true.

Equation D: If you subtract x from each side of Equation 1, the result is Equation D, so if Equation 1 is true then Equation D is true.

Activity Synthesis

Select students previously identified to share how they determined whether each equation had the same solution as Equation 1 in the sequence listed in the Activity Narrative. Point out that the question did not ask students what the solution was, only whether each equation had the same solution.

To help students make connections between the different methods their classmates used to solve the warm-up, ask:

- “Which method of answering the question was most efficient? After seeing all these ways to answer the question, which would you choose?”
- “What is an advantage of changing the equation to look like Equation 1? What is a disadvantage?” (An advantage is that I could see quickly whether it would be the same as Equation 1, and I didn't have to keep going to actually figure out the value of x. A disadvantage would be that I never discovered what the value for x is that makes the equations true.)
- “How is this method (manipulating the equation to look like Equation 1) similar to what we did in previous lessons with the balance hangers?” (In order to keep the hangers balanced, I had to make sure to do the same thing to each side of the hanger. In order to have each equation still be true, I have to make sure to do the same thing to each side of an equation.)

By showing that two equations are related by a move (or series of moves), we know they must have the same solution.

If time allows, have students create another equation with the same solution as Equation 1 and trade with a partner. They should then explain the step(s) necessary to make it look like Equation 1 to each other.

4.2 Step by Step by Step by Step

15 minutes

Before students work on solving complex equations on their own, in this activity they examine the work (both good and bad) of others. The purpose of this activity is to build student fluency solving equations by examining the solutions of others for both appropriate and inappropriate strategies (MP3).

Encourage students to use precise language when discussing the different steps made by the four students in the problem (MP6). For example, if a student says Clare distributed to move from $12x + 3 = 3(5x + 9)$ to $3(4x + 1) = 3(5x + 9)$, ask them to be more specific about how Clare used the distributive property to help the whole class follow along. (Clare used the distributive property to re-write $12x + 3$ as $3(4x + 1)$.)

Addressing

- 8.EE.C.7

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give 4–5 minutes of quiet work time and ask students to pause after the first two problems for a partner discussion. Give 2–3 minutes for partners to work together on the final problem followed by a whole-class discussion. Refer to MLR 3 (Clarify, Critique, Correct) to guide students in using language to describe the wrong steps.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Demonstrate for students how to use an index card or scrap piece of paper to cover and then unveil the steps one at a time. Invite students to make comparisons at each step. *Supports accessibility for: Organization; Attention*

Student Task Statement

Here is an equation, and then all the steps Clare wrote to solve it:

$$\begin{aligned} 14x-2x+3 &= 3(5x+9) \\ 12x+3 &= 3(5x+9) \\ 3(4x+1) &= 3(5x+9) \\ 4x+1 &= 5x+9 \\ 1 &= x+9 \\ \text{-}8 &= x \end{aligned}$$

Here is the same equation, and the steps Lin wrote to solve it:

$$\begin{aligned} 14x-2x+3 &= 3(5x+9) \\ 12x+3 &= 3(5x+9) \\ 12x+3 &= 15x+27 \\ 12x &= 15x+24 \\ \text{-}3x &= 24 \\ x &= \text{-}8 \end{aligned}$$

1. Are both of their *solutions* correct? Explain your reasoning.
2. Describe some ways the steps they took are alike and different.
3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.

Mai:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 7x + 3 &= 3(9) \\ 7x + 3 &= 27 \\ 7x &= 24 \\ x &= \frac{24}{7} \end{aligned}$$

Noah:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 15x + 27 \\ 27x + 3 &= 27 \\ 27x &= 24 \\ x &= \frac{24}{27} \end{aligned}$$

Student Response

1. Both solutions are correct. The solution of $x = -8$ is the only value of x that makes the equation true.

2. Answers vary. Sample response: Both students combined like terms from line one to line two. Clare used the distributive property to re-write the left side as $3(4x + 1)$ moving from line two to line three, while Lin used the same property to distribute the 3 on the left side moving from line two to line three.

3. Mai made an error moving from line two to line three by subtracting $5x$ from each side of the equation before multiplying by 3 on the right hand side of the equation. Noah made an error moving from line two to line three by adding $15x$ to each side of the equation instead of adding $-15x$ to each side of the equation.

Activity Synthesis

Begin the discussion by asking, “How do you know when a solution to an equation is correct?” (One way to know it is correct is by substituting the value of x into the original equation and seeing if it makes the equation true.)

Display Clare and Lin’s solutions for all to see. Poll the class to see which solution they prefer. It is important to draw out that neither solution is better than the other, they are two ways of accomplishing the same task: solving for x. Invite groups to share ways the steps Clare and Lin took are alike and different while annotating the two solutions with students’ observations. If none of the groups say it, point out that while the final steps may look different for Clare and Lin, their later steps worked to reduce the total number of terms until only an x-term and a number remained on either side of the equal sign.

Display Mai and Noah’s incorrect solutions for all to see. Invite groups to share an incorrect step they found and what advice they would give to Mai and Noah for checking their work in the future.

Access for English Language Learners

Conversing: MLR7 Compare and Connect. During the analysis of Clare and Lin’s solutions, ask students first to identify what is similar and what is different about each of the approaches. Then ask students to connect the approaches by asking questions about the related mathematical operations (e.g., “Why does this approach include multiplication, and this one does not?”). Emphasize language used to make sense of strategies used to calculate lengths, areas, and volumes. This will help students make sense of different ways to solve the same equation that both lead to a correct solution.
Design Principle(s): Optimize output (for comparison); Cultivate conversation

4.3 Make Your Own Steps

15 minutes
The purpose of this lesson is to increase fluency in solving equations. Students will solve equations individually and then compare differing, though accurate, solution paths in order to compare their work with others. This will help students recognize that while the final solution will be the same, there is more than one path to the correct answer that uses principles of balancing equations learned in previous lessons.

Addressing

- 8.EE.C.7

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 3–4. Give students quiet think time to complete the activity and then tell groups to share how they solved the equations for x and discuss the similarities and differences in their solution paths.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about solving equations. Encourage students to use previously solved equations as guidelines to determine appropriate steps.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Representing, Conversing: MLR2 Collect and Display. As groups discuss their work, circulate and listen for the language students describe the similarities and differences in their solution paths. Write down different solution paths that led to the same result in a visual display. Consider grouping words and phrases used for each step in different areas of the display (e.g., "multiply first", "divide first", "subtract x from the right", "add x from the left"). Continue to update the display as students move through the activity, and remind them to borrow from the display while discussing with their group. This will help students develop their mathematical language around explaining different solution paths to solving equations.
Design Principle(s); Maximize meta-awareness

Student Task Statement

Solve these equations for x.

1. $\frac{12+6x}{3} = \frac{5-9}{2}$
2. $x - 4 = \frac{1}{3}(6x - 54)$
3. $\text{-}(3x - 12) = 9x - 4$

Student Response

1. $x = \text{-}3$. Solutions vary. Possible solution path:

$$\begin{aligned} \frac{12+6x}{3} &= \frac{5-9}{2} \\ 4 + 2x &= \text{-}2 \\ 2x &= \text{-}6 \\ x &= \text{-}3 \end{aligned}$$

2. $x = 14$. Solutions vary. Possible solution path:

$$\begin{aligned} x - 4 &= \frac{1}{3}(6x - 54) \\ x - 4 &= 2x - 18 \\ \text{-}4 &= x - 18 \\ 14 &= x \end{aligned}$$

3. $x = \frac{4}{3}$. Solutions vary. Possible solution path:

$$
\begin{aligned}
-(3x - 12) &= 9x - 4 \\
-3x + 12 &= 9x - 4 \\
12 &= 12x - 4 \\
16 &= 12x \\
\frac{16}{12} &= x \\
\frac{4}{3} &= x
\end{aligned}
$$

Are You Ready for More?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

Student Response

$n + (n + 1) + (n + 2) = 24$ so $n = 7$ therefore the cups have 7, 8, and 9 pencils in them.

Activity Synthesis

Students should take away from this activity the importance of using valid steps to solve an equation over following a specific solution path. Invite students to share what they discussed in their groups. Consider using some of the following prompts:

- "How many different ways did your group members solve each problem?"
- "When you compared solution paths, did you still come up with the same solution?" (Yes, even though we took different paths, we ended up with the same solutions.)
- "How can you make sure that the path you choose to solve an equation is a valid path?" (I can use the steps we discovered earlier when we were balancing: adding the same value to each side, multiplying (or dividing) by the same value to each side, distributing and collecting like terms whenever it is needed.)
- "What are some examples of steps that will not result in a valid solution?" (Performing an action to only one side of an equation and distributing incorrectly will give an incorrect solution.)

Lesson Synthesis

Display the following prompts one at a time and after each ask students if the move described maintains the equality of an equation:

- subtract a number from each side (maintains)
- subtract $4x$ from each side (maintains)
- dividing each side of the equation by 7 (maintains)
- adding $5x$ to one side and 10 to the other (maintains equality only if $x = 2$)

- add 4 to one side and add 5 to the other (does not maintain equality)

Ask students to write an equation and a solution to the equation that contains an error. Then, tell students to swap with a partner and try to find the error in their partner's solution.

4.4 Mis-Steps

Cool Down: 5 minutes

Addressing

- 8.EE.C.7

Student Task Statement

Lin solved the equation $8(x-3)+7=2x(4-17)$ incorrectly. Find the errors in her solution. What should her answer have been?

Lin's solution:

$$\begin{aligned} 8(x-3)+7&=2x(4-17)\\ 8(x-3)+7&=2x(13)\\ 8x-24+7&=26x\\ 8x-17&=26x\\ \text{-}17&=34x\\ \text{-}\frac{1}{2}&=x \end{aligned}$$

Student Response

Lin's errors:

$$\begin{aligned} 8(x-3)+7&=2x(4-17) && \\ 8(x-3)+7&=2x(13) && 4-17 \text{ is -13}\\ 8x-24+7&=26x && \\ 8x-17&=26x && \\ \text{-}17&=34x && \text{Lin should have subtracted } 8x \text{ from each side.}\\ \text{-}\frac{1}{2}&=x && \text{Testing } x=\text{-}\frac{1}{2} \text{ will show that the equation is not true.} \end{aligned}$$

Lin's solution should have been $x=\frac{1}{2}$.

Student Lesson Summary

How do we make sure the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an x from one line to the next–there are many possible mistakes to watch out for!

Fortunately, each step we take solving an equation results in a new equation with the same solution as the original. This means we can check our work by substituting the value of the solution into the original equation. For example, say we solve the following equation:

$$\begin{aligned} 2x &= -3(x+5) \\ 2x &= -3x + 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Substituting 3 in place of x into the original equation,

$$\begin{aligned} 2(3) &= -3(3+5) \\ 6 &= -3(8) \\ 6 &= -24 \end{aligned}$$

we get a statement that isn't true! This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3. Fixing it, we now have

$$\begin{aligned} 2x &= -3(x+5) \\ 2x &= -3x - 15 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

Substituting -3 in place of x into the original equation to make sure we didn't make another mistake:

$$\begin{aligned} 2(-3) &= -3(-3+5) \\ -6 &= -3(2) \\ -6 &= -6 \end{aligned}$$

This equation is true, so $x = -3$ is the solution.

Lesson 4 Practice Problems

Problem 1

Statement

Mai and Tyler work on the equation $\frac{2}{5}b + 1 = -11$ together. Mai's solution is $b = -25$ and Tyler's is $b = -28$. Here is their work. Do you agree with their solutions? Explain or show your reasoning.

Mai:

$\frac{2}{5}b + 1 = -11$

$\frac{2}{5}b = -10$

$b = -10 \cdot \frac{5}{2}$

$b = -25$

Tyler:

$\frac{2}{5}b + 1 = -11$

$2b + 1 = -55$

$2b = -56$

$b = -28$

Solution

No, they both have errors in their solutions. Explanations vary. Sample response: Mai added -1 on the left side and 1 on the right side of the equation. Tyler multiplied both sides of the equation by 5 but forgot to multiply the 1 by 5.

Problem 2

Statement

Solve $3(x - 4) = 12x$

Solution

$x = -\frac{4}{3}$. One way to solve is to distribute, subtract $3x$ from each side, and divide by 9. Another way is to first divide each side by 3, subtract x for each side, then divide each side by 3.

Problem 3

Statement

Describe what is being done in each step while solving the equation.

a. $2(-3x + 4) = 5x + 2$

b. $-6x + 8 = 5x + 2$

c. $8 = 11x + 2$

d. $6 = 11x$

e. $x = \frac{6}{11}$

Solution

a. Original equation

b. Distributive property

c. Add $6x$ to each side

d. Subtract 2 from each side

e. Multiply each side by $\frac{1}{11}$

Problem 4

Statement

Andre solved an equation, but when he checked his answer he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Andre's mistake and what is the solution to the equation?

$$\begin{aligned} -2(3x-5) &= 4(x+3)+8 \\ -6x+10 &= 4x+12+8 \\ -6x+10 &= 4x+20 \\ 10 &= -2x+20 \\ -10 &= -2x \\ 5 &= x \end{aligned}$$

Solution

Andre's mistake occurred in the transition from the 3rd line to the 4th line. He added $6x$ on the left side but subtracted $6x$ on the right side. The correct solution is $x = -1$.

Problem 5

Statement

Choose the equation that has solutions $(5, 7)$ and $(8, 13)$.

A. $3x - y = 8$

B. $y = x + 2$

C. $y - x = 5$

D. $y = 2x - 3$

Solution

D

(From Unit 3, Lesson 12.)

Problem 6

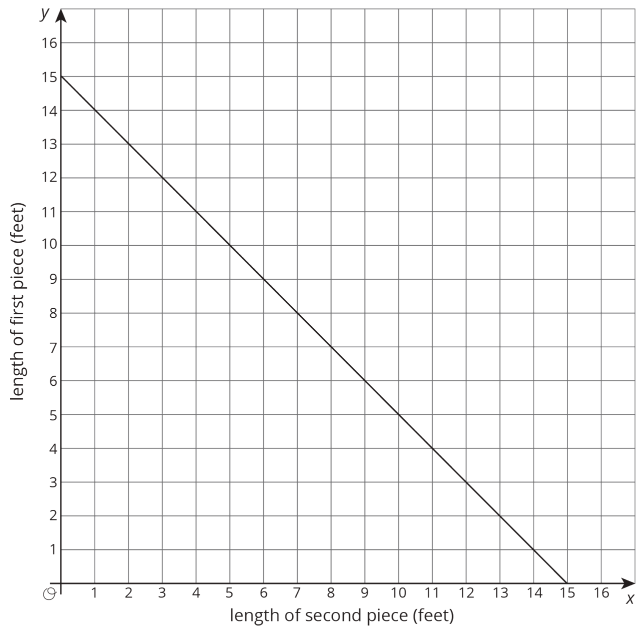
Statement

A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the second piece, x, for each length of the first piece, y.

a. How long is the ribbon? Explain how you know.

b. What is the slope of the line?

c. Explain what the slope of the line represents and why it fits the story.



Solution

a. 15 feet. Explanations vary. Sample response: When the second piece is 0 feet long, the first is 15 feet long, so that is the length of the ribbon.

b. -1

c. Answers vary. Sample response: The slope shows the change in length of one piece for every 1 foot increase in length of the other piece. If one piece is 1 foot longer, the other is 1 foot shorter because the total of the two lengths is constant.

(From Unit 3, Lesson 9.)