Lesson 7: Practice with Rational Bases

Goals

- Identify (orally) misapplications of exponent rules to expressions with multiple bases (orally and in writing).
- Use exponent rules to rewrite exponential equations involving negative exponents to have a single positive exponent, and explain (orally) the strategy.

Learning Targets

- I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
- I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

Lesson Narrative

In this lesson, students practice all of the exponent rules they have learned so far and begin to look at expressions with multiple bases. The first activity asks students to reflect on their own conceptual understanding and procedural fluency with the exponent rules they have learned so far. The second activity asks students to analyze the structure of exponents to make sense of expressions with multiple bases, paving the way towards the rule $a^n \cdot b^n = (a \cdot b)^n$ in the next lesson (MP7).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Which One Doesn’t Belong?

Student Learning Goals

Let's practice with exponents.

7.1 Which One Doesn’t Belong: Exponents

Warm Up: 5 minutes

This warm-up prompts students to compare four exponential expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another. To allow all students to access the activity, each expression has one obvious reason it does not belong. Encourage students to move past the obvious reasons and find reasons based on mathematical properties. During the discussion, listen for important ideas and terminology that will be helpful in the upcoming work of the unit.

Addressing

- 8.EE.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the expression for all to see. Ask students to indicate when they have noticed one expression that doesn't belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression doesn't belong and together find at least one reason each expression doesn't belong.

Anticipated Misconceptions

In the last question, students may think $\frac{10^8}{5^5}$ is equal to 10^3 or $\left(\frac{10}{5}\right)^3$. Encourage these students to think about expanding the exponents into their repeated factors.

Student Task Statement

Which expression doesn't belong?

$$\frac{2^8}{2^5} \qquad \left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$$

$$\left(4^{-5}\right)^8 \qquad \frac{10^8}{5^5}$$

Student Response

Answers vary. Sample responses:

- $\frac{2^8}{2^5}$ doesn't belong because it is the only one where the bases are the same and the exponents are both positive.
- $\left(4^{-5}\right)^8$ doesn't belong because it is the only one where it is a single power of a power.
- $\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$ doesn't belong because it is the only one that has fractions in the base.
- $\frac{10^8}{5^5}$ doesn't belong because it is the only one with different bases.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as "base" or "exponent." Also, press students on unsubstantiated claims. For example: "$\left(4^{-5}\right)^8$ is a really large number." (It isn't, because of the negative exponent.) Or, "$\frac{2^8}{2^5}$ and $\frac{10^8}{5^5}$ are whole numbers." (They are, but how do we know?)

7.2 Exponent Rule Practice

15 minutes

This activity develops procedural fluency with exponent rules and encourages students to think about their own learning. Students choose 6 of 12 possible problems to solve, thereby identifying problems that they consider more difficult versus less difficult. Notice which problems students choose more than others, and which problems are skipped more than others. The first set of problems checks whether students can apply the exponent rules procedurally. The next set of problems checks whether students understand what negative exponents mean. The last set of problems asks students to evaluate exponents to check whether they understand the meaning of the zero exponent and the definition of exponents as repeated multiplication (by the base, or by the reciprocal of the base in the case of negative exponents).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Encourage students to work together with their partners. Encourage partners to choose mostly the same problems, but if they differ, partners should check one another's work. Encourage students to explain their reasoning by referencing the visual displays for the exponent rules. Give students 10–12 minutes to work followed by a brief whole-class discussion. Problems that many students chose to skip can be assigned as additional practice.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about exponents. Maintain a display of important terms and vocabulary. During the launch take time to review the visual display of rules for exponents. Some students may benefit from their own physical copy of the display.
Supports accessibility for: Memory; Language

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support small-group discussion, invite students to explain how they used exponent rules to solve at least 3 of the problems. Display sentence frames for students to use such as: "First I ____ , then I ____ ." and "I used _____ rule, because ______ ."
Design Principle(s): Optimize output for (explanation)

Student Task Statement

1. Choose 6 of the equations to write using a single exponent:
   - $7^5 \cdot 7^6$
   - $3^{-3} \cdot 3^8$
   - $2^{-4} \cdot 2^{-3}$
   - $\left(\frac{5}{6}\right)^4 \left(\frac{5}{6}\right)^5$
   - $\frac{3^5}{3^{28}}$
   - $\frac{2^{-5}}{2^4}$
   - $\frac{6^5}{6^{-8}}$
   - $\frac{10^{-12}}{10^{-20}}$
   - $\left(7^2\right)^3$
   - $\left(4^3\right)^{-3}$
   - $\left(2^{-8}\right)^{-4}$
   - $\left(6^{-3}\right)^5$
2. Which problems did you want to skip in the previous question? Explain your thinking.
3. Choose 3 of the following to write using a single, *positive* exponent:
   - 2^{-7}
   - 3^{-23}
   - 11^{-8}
   - 4^{-9}
   - 2^{-32}
   - 8^{-3}
4. Choose 3 of the following to evaluate:

- $\frac{10^5}{10^5}$
- $\left(\frac{2}{3}\right)^3$
- $2^8 \cdot 2^{-8}$
- $\left(\frac{5}{4}\right)^2$
- $\left(3^4\right)^0$
- $\left(\frac{7}{2}\right)^2$

Student Response

1.
    a. 7^{11}
    b. 3^5
    c. 2^{-7}
    d. $\left(\frac{5}{6}\right)^9$

    a. 3^{-23}
    b. 2^{-9}
    c. 6^{13}
    d. 10^8

    a. 7^6
    b. 4^{-9}
    c. 2^{32}
    d. 6^{-15}

2. Answers vary. Sample response: I wanted to skip questions with negative exponents because I'm least comfortable with those.
3.
    a. $\frac{1}{2^7}$
    b. $\frac{1}{3^{23}}$
    c. $\frac{1}{11^8}$

    a. $\frac{1}{4^9}$
    b. $\frac{1}{2^{32}}$
    c. $\frac{1}{8^3}$

4.
    a. 1
    b. $\frac{8}{27}$
    c. 1

    a. $\frac{25}{16}$
    b. 1
    c. $\frac{49}{4}$

Activity Synthesis

The goal of the discussion is to get a general sense of how fluent students have become with the exponent rules. Poll the class about how successful they felt while working on the problems. Here are some questions for discussion:

- "Which problems would you assign to your best friend? Why?"
- "Which problems really made you think? Why?"
- "What are some resources you could use to get more comfortable with the problems you are uncomfortable with?"

7.3 Inconsistent Bases

15 minutes

In this activity, students analyze powers that involve different bases. The goal is for them to recognize that exponents can be added (or subtracted) only when the powers being multiplied (or divided) have the same base. It is expected that students compute the value of the expressions on the left and right sides of the equation to show they are not actually equal. The last problem alludes to the rule $a^n \cdot b^n = (a \cdot b)^n$ which will be explored further in the next lesson.

As students work, notice those who check for equality by computing the value on either side of each equation or by expanding each power into its factors. Invite them to share later.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Display the false equation $2^3 \cdot 5^2 = 10^{3+2} = 10^5$ for all to see. Ask students whether they think the equation is true or false, and choose a few students to explain their reasoning. If not mentioned by students, expand $2^3 \cdot 5^2$ and 10^5 to show their repeated factors. Give students 10 minutes to work followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of multiplication and repeated factors. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Mark each equation as true or false. What could you change about the false equations to make them true?

1. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^6$

2. $3^2 \cdot 5^3 = 15^5$

3. $5^4 + 5^5 = 5^9$

4. $\left(\frac{1}{2}\right)^4 \cdot 10^3 = 5^7$

5. $3^2 \cdot 5^2 = 15^2$

Student Response

1. True because there are 6 factors that are $\frac{1}{3}$ on each side of the equation.
2. False because $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 < 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15$. Sample change: $3^5 \cdot 5^5 = 15^5$.
3. False because $5^4 + 5^5 < 5^9$. Sample change: $5^4 \cdot 5^5 = 5^9$.
4. False because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 10 \cdot 10 \cdot 10 < 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$. Sample change: $(\frac{1}{2})^3 \cdot 10^3 = 5^3$.
5. True because $15^2 = 15 \cdot 15 = (3 \cdot 5)(3 \cdot 5) = 3 \cdot 3 \cdot 5 \cdot 5 = 3^2 \cdot 5^2$.

Are You Ready for More?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

Student Response

$x = \text{-}13$. Explanations vary. Sample explanation: Since $9 = 3^2$, the right side of the equation becomes $(3^2)^{x+4}$, or 3^{2x+8}. This means that $x - 5 = 2x + 8$, so $x = \text{-}13$.

Activity Synthesis

The important take-away from this activity is that the exponent rules work because they capture patterns of repeated multiplication of a *single* base. The equation in the launch erroneously applies an exponent rule to a situation that involves multiple bases. This fails because with multiple bases, there are not the same patterns of repeated multiplication. Ask students to share their responses and display them for all to see. For the final question, ask students whether they think it is a coincidence that the equation is true, or if there is another, more general explanation. It is not necessary to dwell on this point since it will be addressed more fully in the next lesson. Consider involving more students in a whole-class discussion with the following questions:

- “Who can restate __’s reasoning in a different way?”
- “Why do the exponent rules we have looked at so far only work when looking at one particular base rather than mixing different bases together?” ($3^2 \cdot 3^3 = 3^5$ because there are 5 factors that are 3 on the left side, but $3^2 \cdot 4^3$ isn't 12^5 because there are not 5 factors that are 12.)

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Display an incorrect statement that represents a common misunderstanding of the exponent rule. For example, "$3^2 \cdot 5^3 = 15^5$ because $3 \cdot 2 = 15$ and $2 + 3 = 5$." Ask pairs of students to critique the reasoning by asking, "Do you agree? Why or why not?" Invite students to work with a partner to identify any errors and write an improved statement. Listen for students who notice the relationships between exponents, expanded form and exponent rules. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Maximize meta-awareness, Optimize output (for explanation)

Lesson Synthesis

In this lesson, students honed their skills working with exponent rules and discovered where the rules break down when looking at expressions with mismatching bases. Here are some questions for discussion:

- "Why is the equation $2^5 \cdot 2^3 = 2^{15}$ false?" (Multiplying 5 factors that are 2 by 3 factors that are 2 results in a total of 8 factors that are 2. Multiplying the exponents doesn't make sense in this case.)
- "Why is the equation $\frac{3^5}{3^2} = 3^3$ true?" (Expanding the left side, we get $\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$, which is equal to $1 \cdot 3 \cdot 3 \cdot 3$ or just 3^3.)
- "Why is the equation $\frac{6^5}{3^2} = 2^3$ false? Why might someone make this mistake?" (Expanding the left side, we get $\frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}$, which is equal to $2 \cdot 2 \cdot 6 \cdot 6 \cdot 6$. Someone might make this mistake because they divide 6 by 3 and use the exponent rule for division to subtract the exponent in the denominator from the exponent in the numerator.)

7.4 Working with Exponents

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

1. Rewrite each expression using a single, positive exponent:
   a. $\frac{9^3}{9^9}$
   b. $14^{-3} \cdot 14^{12}$

2. Diego wrote $6^4 \cdot 8^3 = 48^7$. Explain what Diego's mistake was and how you know the equation is not true.

Student Response

1. a. $\frac{1}{9^6}$

   b. 14^9

2. Explanations vary. Sample explanation: Diego multiplied the bases and added their exponents. The equation is not true because 4 repeated factors that are 6 multiplied by 3 repeated factors that are 8 is much smaller than 7 repeated factors that are 48.

Student Lesson Summary

In the past few lessons, we found rules to more easily keep track of repeated factors when using exponents. We also extended these rules to make sense of negative exponents as repeated factors of the **reciprocal** of the base, as well as defining a number to the power of 0 to have a value of 1. These rules can be written symbolically as:

$$x^n \cdot x^m = x^{n+m},$$

$$(x^n)^m = x^{n \cdot m},$$

$$\frac{x^n}{x^m} = x^{n-m},$$

$$x^{-n} = \frac{1}{x^n},$$

and

$$x^0 = 1,$$

where the base x can be any positive number. In this lesson, we practiced using these exponent rules for different bases and exponents.

Glossary

- reciprocal

Lesson 7 Practice Problems

Problem 1

Statement

Write with a single exponent:

a. $\frac{7^6}{7^2}$

b. $(11^4)^5$

c. $4^2 \cdot 4^6$

d. $6 \cdot 6^8$

e. $(12^2)^7$

f. $\frac{3^{10}}{3}$

g. $(0.173)^9 \cdot (0.173)^2$

h. $\frac{0.87^5}{0.87^3}$

i. $\frac{(\frac{5}{2})^8}{(\frac{5}{2})^6}$

Solution

a. 7^4

b. 11^{20}

c. 4^8

d. 6^9

e. 12^{14}

f. 3^9

g. 0.173^{11}

h. 0.87^2

i. $(\frac{5}{2})^2$

Problem 2

Statement

Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

a. Do you agree with Noah? Explain or show your reasoning.

b. Do you agree with Tyler? Explain or show your reasoning.

Solution

a. Disagree. Reasoning varies. Sample reasoning: $2^4 \cdot 3^2 = 16 \cdot 9 = 144$, but 6^6 is much bigger than 144.

b. Agree. Reasoning varies. Sample reasoning: $2^4 = 16$ and $4^2 = 16$, so $2^4 \cdot 4^2$ should equal $16 \cdot 16$ or 16^2.