Lesson 18: Surface Area of a Cube

Goals

- Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as $6 \cdot s^2$.
- Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents.
- Write expressions, with or without exponents, to represent the surface area of a given cube.

Learning Targets

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson Narrative

In this lesson, students practice using exponents of 2 and 3 to express products and to write square and cubic units. Along the way, they look for and make use of structure in numerical expressions (MP7). They also look for and express regularity in repeated reasoning (MP8) to write the formula for the surface area of a cube. Students will continue this work later in the course, in the unit on expressions and equations.

Note: Students will need to bring in a personal collection of 10–50 small objects ahead of time for the first lesson of the next unit. Examples include rocks, seashells, trading cards, or coins.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's write a formula to find the surface area of a cube.

18.1 Exponent Review

Warm Up: 5 minutes
In this warm-up, students compare pairs of numerical expressions and identify the expression with the greater value. The task allows students to review what they learned about exponents and prompts them to look for and make use of structure in numerical expressions (MP7).

Students should do these without calculators and without calculating, although it is fine for them to check their answers with a calculator.

Addressing

- 6.EE.A.1

Launch

Give students 1–2 minutes of quiet think time. Ask them to answer the questions without multiplying anything or using a calculator, and to give a signal when they have an answer for each question and can explain their reasoning.

Anticipated Misconceptions

When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. Remind them about the meaning of the exponent notation. For example, show that $5 \cdot 3 = 15$, which is much smaller than $5 \cdot 5 \cdot 5$, which equals 125.

Student Task Statement

Select the greater expression of each pair without calculating the value of each expression. Be prepared to explain your choices.

- $10 \cdot 3$ or 10^3
- 13^2 or $12 \cdot 12$

- $97 + 97 + 97 + 97 + 97 + 97$ or $5 \cdot 97$

Student Response

- 10^3 is greater because it is 1,000.
- 13^2 is greater because it is $13 \cdot 13$, and this will be greater than $12 \cdot 12$.
- $97 + 97 + 97 + 97 + 97 + 97$ is greater because it is $6 \cdot 97$, which is greater than $5 \cdot 97$.

Activity Synthesis

Ask one or more students to explain their reasoning for each choice. If not mentioned in students' explanations, highlight the structures in the expressions that enable us to evaluate each one without performing any calculations.

Point out, for example, that since we know that 10^3 means $10 \cdot 10 \cdot 10$, we can tell that it is much larger than $10 \cdot 3$.

For the last question, remind students that we can think of repeated addition in terms of multiple groups (i.e., that the sum of six 97s can be seen as six groups of 97 or $6 \cdot 97$). The idea of using groups to write equivalent expressions will support students as they write expressions for the surface area of a cube later in the lesson (i.e., writing the areas of all square faces of a cube as $6s^2$) .

18.2 The Net of a Cube

20 minutes

This activity contains two sets of problems. The first set involves computations with simple numbers and should be solved numerically. Use students' work here to check that they are drawing a net correctly.

The second set encourages students to write expressions rather than to simplify them through calculations. The goal is to prepare students for the general rules s^3 and $6s^2$, which are more easily understood through an intermediate step involving numbers.

Note that students will be introduced to the idea that $5 \cdot x$ means the same as $5x$ in a later unit, so expect them to write $6 \cdot 17^2$ instead of $6(17^2)$. It is not critical that they understand that a number and a variable (or a number and an expression in parentheses) placed next to each other means they are being multiplied.

As students work on the second set, monitor the ways in which they write their expressions for surface area and volume. Identify those whose expressions include :

- products (e.g., $17 \cdot 17$ or $17 \cdot 17 \cdot 17$),
- sums of products (e.g., $(17 \cdot 17) + (17 \cdot 17) + \ldots$),
- combination of like terms (e.g., $6 \cdot (17 \cdot 17)$),

- exponents (e.g., $17^2 + 17^2 + \ldots$) or 17^3), and
- completed calculation (e.g., 289).

Select these students to share their work later. Notice the lengths of the expressions and sequence their explanations in order—from the longest expression to the most succinct.

Addressing

- 6.EE.A.1
- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students access to their geometry toolkits and 8-10 minutes of quiet work time. Tell students to try to answer the questions without using a calculator. Ask them to share their responses with their partner afterwards.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge about calculating surface area and volume. Share examples of expressions for a cube in a few different forms to illustrate how surface area and volume can be expressed. Allow continued access to concrete manipulatives such as snap cubes for students to view or manipulate.
Supports accessibility for: Visual-spatial processing; Conceptual processing

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Anticipated Misconceptions

Students might think the surface area is $(17 \cdot 17)^6$. Prompt students to write down how they would compute surface area step by step, before trying to encapsulate their steps in an expression. Dissuade students from using calculators in the last two problems and assure them that building an expression does not require extensive computation.

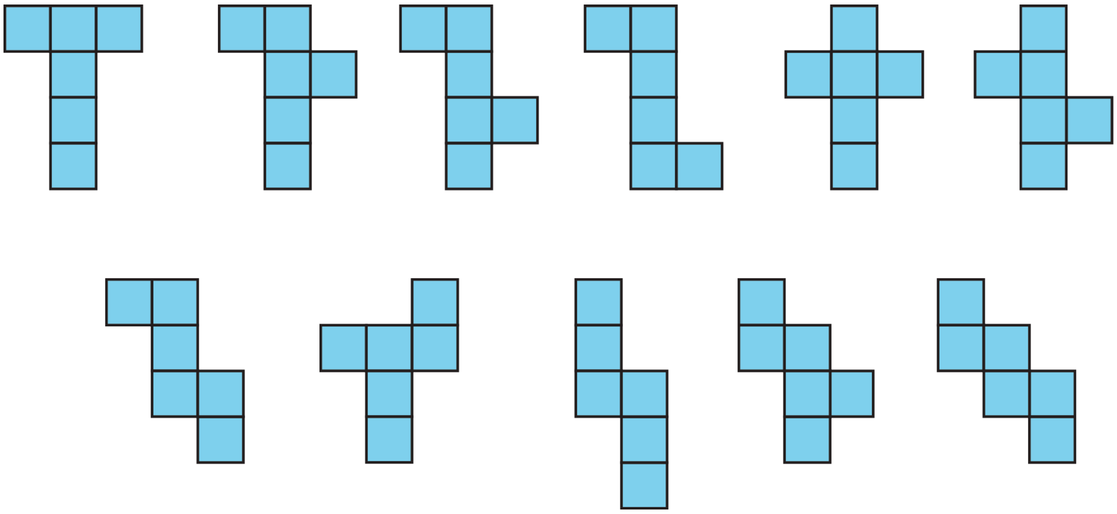
Students may think that refraining from using a calculator meant performing all calculations—including those of larger numbers—on paper or mentally, especially if they are unclear about the meaning of the term "expression." Ask them to refer to the expressions in the warm-up, or share examples of expressions in a few different forms, to help them see how surface area and volume can be expressed without computation.

Student Task Statement

1. A cube has edge length 5 inches.
   a. Draw a net for this cube, and label its sides with measurements.
   b. What is the shape of each face?
   c. What is the area of each face?
   d. What is the surface area of this cube?
   e. What is the volume of this cube?
2. A second cube has edge length 17 units.
   a. Draw a net for this cube, and label its sides with measurements.
   b. Explain why the area of each face of this cube is 17^2 square units.
   c. Write an expression for the surface area, in square units.
   d. Write an expression for the volume, in cubic units.

Student Response

1. For the cube that has edge length 5:
   a. Drawings vary. 11 unique nets are possible:



   b. A square
   c. 25 square inches
   d. 150 square inches
   e. 125 cubic inches
2. For the cube that has edge length 17:

a. Drawings vary, but should be one of the 11 nets shown in the previous problem.

b. Answers vary. Sample explanation: The side length of each square face is 17 units, so its area is $17 \cdot 17$ or 17^2 square units.

c. $6 \cdot 17^2$ (or equivalent)

d. 17^3 (or equivalent)

Activity Synthesis

After partner discussions, select a couple of students to present the solutions to the first set of questions, which should be straightforward.

Then, invite previously identified students to share their expressions for the last two questions. If possible, sequence their presentation in the following order. If any expressions are missing but needed to illustrate the idea of writing succinct expressions, add them to the lists.

Surface area:

- $(17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17)$
- $17^2 + 17^2 + 17^2 + 17^2 + 17^2 + 17^2$
- $6 \cdot (17 \cdot 17)$
- $6 \cdot (17^2)$
- $6 \cdot (289)$
- 1,734

Volume:

- $17 \cdot 17 \cdot 17$
- 17^3
- 4,913

Discuss how multiplication can simplify expressions involving repeated addition and exponents can do the same for repeated multiplication. While the last expression in each set above is the simplest to write, getting there requires quite a bit of computation. Highlight $6 \cdot 17^2$ and 17^3 as efficient ways to express the surface area and volume of the cube.

As the class discusses the different expressions, consider directing students' attention to the units of measurements. Remind students that, rather than writing $6 \cdot (17^2)$ *square units*, we can write $6 \cdot (17^2)$ $units^2$, and instead of 17^3 *cubic units*, we can write 17^3 $units^2$. Unit notations will appear again later in the course, so it can also be reinforced later.

If students are not yet ready for the general formula, which comes next, offer another example. For instance, say: "A cube has edge length 38 cm. How can we express its surface area and volume?"

Help students see that its surface area is $6 \cdot (38^2)$ cm^2 and its volume is 38^3 cm^3. The large number will discourage calculation and focus students on the form of the expressions they are building and the use of exponents.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite groups to create a visual display showing their strategy and calculations for the surface area and volume of a cube with an edge length of 17 units. Allow students time to quietly circulate and analyze the strategies in at least 2 other displays in the room. Give students quiet think time to consider what is the same and what is different. Next, ask students to return to their original group to discuss what they noticed. Listen for and amplify observations that highlight the advantages and disadvantages to each method and their level of succinctness. This will help students make connections between calculations of cubes, regardless of the edge length.
Design Principle(s): Optimize output; Cultivate conversation

18.3 Every Cube in the Whole World

10 minutes
In this activity, students build on what they learned earlier and develop the formulas for the surface area and the volume of a cube in terms of a variable edge length s.

Encourage students to refer to their work in the preceding activity as much as possible and to generalize from it. As before, monitor for different ways of writing expressions for surface area and volume. Identify students whose work includes the following:

- products (e.g., $s \cdot s$, or $s \cdot s \cdot s$),
- sums of products (e.g., $(s \cdot s) + (s \cdot s) + \ldots$),
- combination of like terms (e.g., $6 \cdot (s \cdot s)$), and
- exponents (e.g., $s^2 + s^2 + \ldots$, or s^3).

Select these students to share their work later. Again, notice the lengths of the expressions and sequence their explanations in order—from the longest expression to the most succinct.

Addressing

- 6.EE.A.2.a
- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Give students access to their geometry toolkits and 7–8 minutes of quiet think time. Tell students they will be answering on the same questions as before, but with a variable for the side length. Encourage them to use the work they did earlier to help them here.

Anticipated Misconceptions

If students are unclear or unsure about using the variable s, explain that we are looking for an expression that would work for any edge length, and that a variable, such as s, can represent any number. The s could be replaced with any edge length in finding surface area and volume.

To connect students' work to earlier examples, point to the cube with edge length 17 units from the previous activity. Ask: "If you wrote the surface area as $6 \cdot 17^2$ before, what should it be now?"

As students work, encourage those who may be more comfortable using multiplication symbols to instead use exponents whenever possible.

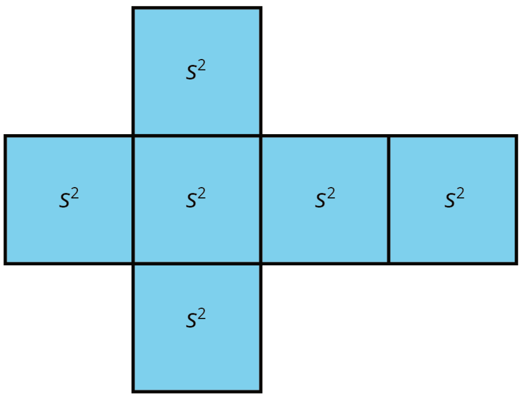
Student Task Statement

A cube has edge length s.

1. Draw a net for the cube.
2. Write an expression for the area of each face. Label each face with its area.
3. Write an expression for the surface area.
4. Write an expression for the volume.

Student Response

1. Drawings vary. Here is one possible labeled net (each face is a square whose side lengths are s):



2. The area of each face is s^2.

3. The surface area is $6 \cdot s^2$.
4. The volume is s^3.

Activity Synthesis

Discuss the problems in as similar a fashion as was done in the earlier activity involving a cube with edge length 17 units. Doing so enables students to see structure in the expressions (MP7) and to generalize through repeated reasoning (MP8).

Select previously identified students to share their responses with the class. If possible, sequence their presentation in the following order to help students see how the expressions $6 \cdot s^2$ and s^3 come about. If any expressions are missing but needed to illustrate the idea of writing succinct expressions, add them to the lists.

Surface area:

- $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$
- $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$
- $6(s \cdot s)$
- $6 \cdot (s^2)$ or $6 \cdot s^2$

Volume

- $s \cdot s \cdot s$
- s^3

Refer back to the example involving numerical side length (a cube with edge length 17 units) if students have trouble understanding where the most concise expression of surface area comes from.

Present the surface area as $6 \cdot s^2$. You can choose to also write it as $6s^2$.

Lesson Synthesis

Review the formulas for volume and surface area of a cube.

- The volume of a cube with edge length s is s^3.
- A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.

18.4 From Volume to Surface Area

Cool Down: 5 minutes

Addressing

- 6.EE.A.1
- 6.G.A.4

Student Task Statement

1. A cube has edge length 11 inches. Write an expression for its volume and an expression for its surface area.

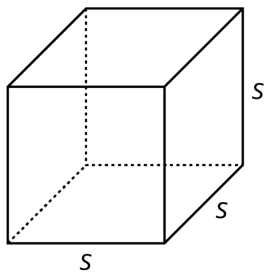
2. A cube has a volume of 7^3 cubic centimeters. What is its surface area?

Student Response

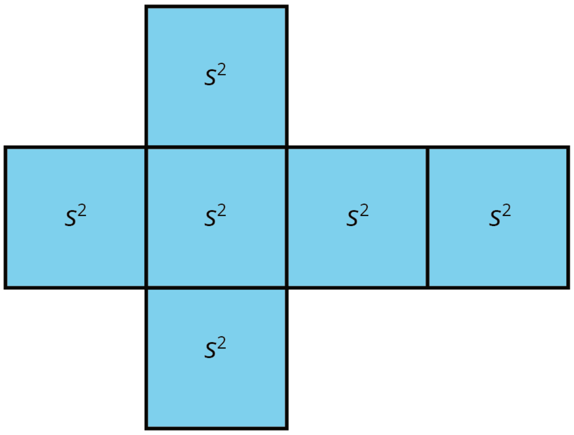
1. Volume: 11^3 or $11 \cdot 11 \cdot 11$. Surface area: $6 \cdot (11 \cdot 11)$ (or equivalent).
2. The surface area is $6 \cdot 7^2$, which is 294 square centimeters.

Student Lesson Summary

The volume of a cube with edge length s is s^3.



A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.



Lesson 18 Practice Problems

Problem 1

Statement

a. What is the volume of a cube with edge length 8 in?

b. What is the volume of a cube with edge length $\frac{1}{3}$ cm?

c. A cube has a volume of 8 ft^3. What is its edge length?

Solution

a. 512 cu in ($8 \cdot 8 \cdot 8 = 512$)

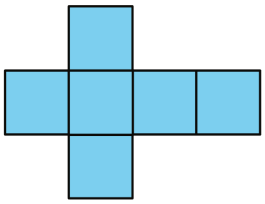
b. $\frac{1}{27}$ cu cm ($\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$)

c. 2 ft ($2 \cdot 2 \cdot 2 = 8$)

Problem 2

Statement

a. What three-dimensional figure can be assembled from this net?



b. If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

Solution

a. Cube

b. The surface area is $6 \cdot 61^2$ sq cm, and the volume is 61^3 cu cm.

Problem 3

Statement

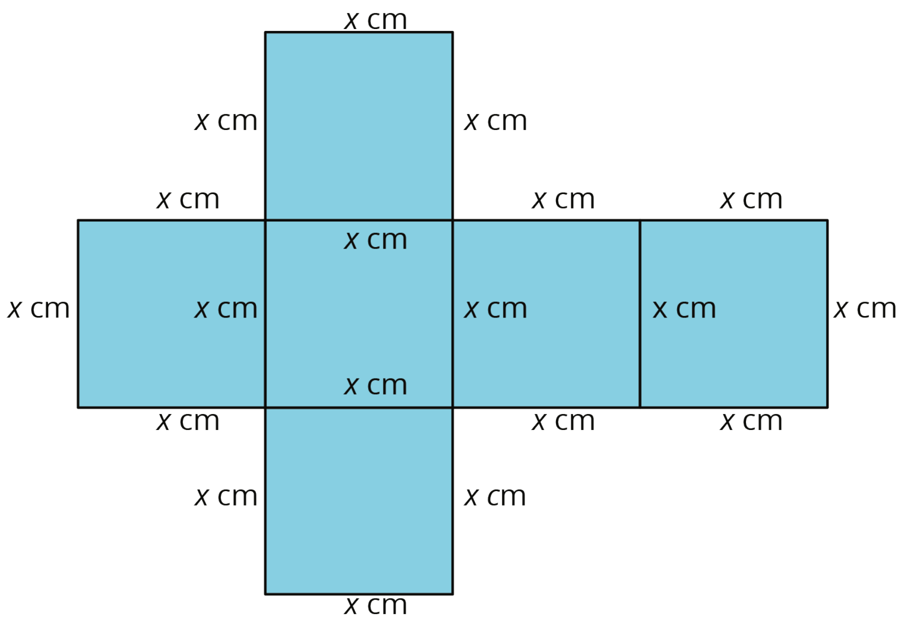
a. Draw a net for a cube with edge length x cm.

b. What is the surface area of this cube?

c. What is the volume of this cube?

Solution

a.



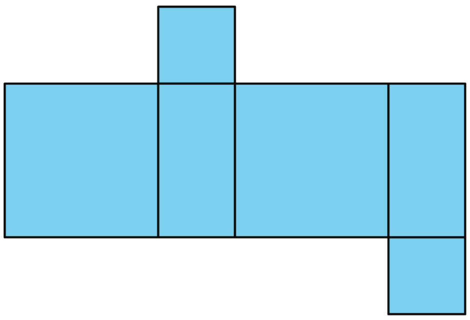
b. $6x^2$ sq cm (or equivalent)

c. $x \cdot x \cdot x$ cu cm (or equivalent)

Problem 4

Statement

Here is a net for a rectangular prism that was not drawn accurately.



a. Explain what is wrong with the net.

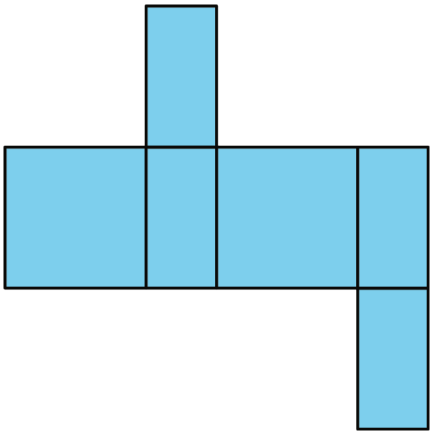
b. Draw a net that can be assembled into a rectangular prism.

c. Create another net for the same prism.

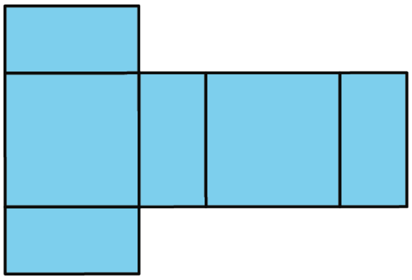
Solution

a. When the shape is folded, the two small squares are not the right size to close the three-dimensional figure. The small squares can be replaced with rectangles as in the picture, or the large squares can be the same size and shape as the two (non-square) rectangles in the net.

b.



c.



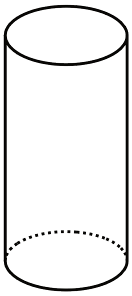
(From Unit 1, Lesson 14.)

Problem 5

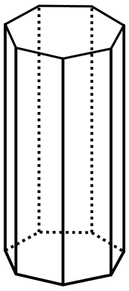
Statement

State whether each figure is a polyhedron. Explain how you know.

A



B



Solution

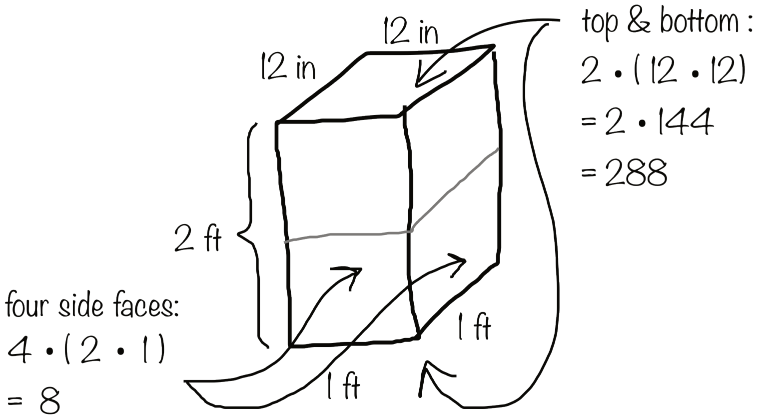
Figure A is not a polyhedron. It has a curved surface and there are faces that are not polygons. Figure B is a polyhedron. It is composed of polygons and each side of every polygon joins a side of another polygon.

(From Unit 1, Lesson 13.)

Problem 6

Statement

Here is Elena's work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

Solution

Disagree. Sample reasoning: Elena calculated the area of the top and bottom faces in square inches but the area of the side faces in square feet. The combined area of the top and bottom faces is 2 square feet, so the correct surface area is 10 square feet.

(From Unit 1, Lesson 12.)