## Lesson 8: Cubes and Cube Roots

* Let’s compare equations with cubes and cube roots.

### 8.1: Put Your Arm(s) Up

How are these graphs the same? How are they different?





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### 8.2: Finding Cube Roots with a Graph

How many solutions are there to each of the following equations? Estimate the solution(s) from the graph of $y=x^{3}$. Check your estimate by substituting it back into the equation.

1. $x^{3}=8$
2. $x^{3}=2$
3. $x^{3}=0$
4. $x^{3}=-8$
5. $x^{3}=-2$



### 8.3: Cube Root Equations



1. Use the graph of $y=\sqrt[3]{x}$ to estimate the solution(s) to $\sqrt[3]{x}=-4$.
2. Use the meaning of cube roots to find an exact solution to the equation $\sqrt[3]{x}=-4$. How close was your estimate?
3. Find the solution of the equation $\sqrt[3]{x}=3.5$ using the meaning of cube roots. Use the graph to check that your solution is reasonable.

### 8.4: Solve These Equations With Cube Roots in Them

Here are a lot of equations:

* $\sqrt[3]{t+4}=3$
* $-10=-\sqrt[3]{a}$
* $\sqrt[3]{3−w}−4=0$
* $\sqrt[3]{z}+9=0$
* $\sqrt[3]{r^{3}−19}=2$
* $5−\sqrt[3]{k+1}=-1$
* $\sqrt[3]{p+4}−2=1$
* $6−\sqrt[3]{b}=0$
* $\sqrt[3]{2n}+3=-5$
* $4+\sqrt[3]{-m}+4=6$
* $-\sqrt[3]{c}=5$
* $\sqrt[3]{s−7}+3=0$
1. Without solving, identify 3 equations that you think would be the least difficult to solve and 3 equations that you think would be the most difficult to solve. Be prepared to explain your reasoning.
2. Choose 4 equations and solve them. At least one should be from your “least difficult” list and at least one should be from your “most difficult” list.

#### Are you ready for more?

All of these equations were equivalent to equations that could be written in the form $\sqrt[3]{ax+b}+c=0$ for some constants $a$, $b$, and $c$. Find a formula that would solve any such equation for $x$ in terms of $a$, $b$, and $c$.

### Lesson 8 Summary

Every number has exactly one cube root. You can see this by looking at the graph of $y=x^{3}$.

If $y$ is any number, for example, -4, then we can see that $y=-4$ crosses the graph in one and only one place, so the equation $x^{3}=-4$ will have the solution $-\sqrt[3]{4}$. This is true for any number $a$: $y=a$ will cross the graph in exactly one place, and $x^{3}=a$ will have one solution, $\sqrt[3]{a}$.



In an equation like $\sqrt[3]{t}+6=0$, we can isolate the cube root and cube each side:

$\begin{matrix}\sqrt[3]{t}+6&=0\\\sqrt[3]{t}&=-6\\t&=(-6)^{3}\\t&=-216\end{matrix}$

While cubing each side of an equation won’t create an equation with solutions that are different than the original equation, it is still a good idea to always check solutions in the original equation because little mistakes can creep in along the way.



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