Lesson 8: How Much in Each Group? (Part 1)

Goals

- Compare and contrast (orally) strategies for solving problems about "how many groups?" and "how much in 1 group?"
- Create a tape diagram to represent and solve a problem asking "How much in 1 group?" where the dividend, divisor, and quotient may be fractions, and explain (orally) the solution method.
- Write multiplication and division equations to represent a problem asking "How much in 1 group?"

Learning Targets

- I can tell when a question is asking for the amount in one group.
- I can use diagrams and multiplication and division equations to represent and answer "how much in each group?" questions.

Lesson Narrative

Previously, students looked at division situations in which the number of groups (or the fraction of a group) was unknown. They interpreted division expressions as a way to find out that number (or fraction) of groups. In this lesson, students encounter situations where the number of groups is known but the size of each group is not. They interpret division expressions as a way to answer "how much in a group?" questions.

Students use the same tools—multiplication and division equations and tape diagrams—and the same structure of equal-sized groups to reason about "how much in a group?" questions (MP7). They also continue to relate their reasoning in quantitative contexts to their reasoning on abstract representations (MP2). Students find both whole-number and non-whole-number quotients, recognizing that, like the number of groups, the amount in one group can also be a whole number or a fraction.

Alignments

Addressing

- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's look at division problems that help us find the size of one group.

8.1 Inventing a Situation

Warm Up: 5 minutes
By now students have written many division equations based on verbal descriptions of situations. This warm-up prompts them to go in the other direction: to interpret a division expression and write a fitting question the expression could help answer. Then, they trade descriptions with a partner and reason about the value of the same expression in someone else's story.

As students write their descriptions, look out for scenarios that are unclear and ask their authors clarifying questions. Select 2–3 students whose descriptions clearly match the expression to share later.

Students may struggle to answer their partner's question because the descriptions are confusing or do not match the given expression. Encourage them to ask the questions they have about the description, but still try to find the answer to the given expression.

Addressing

- 6.NS.A.1

Launch

Arrange students in groups of 2. Explain to students the two halves of this activity: to write a situation and a question that can be represented by the equation $12 \div \frac{2}{3} = ?$, and to trade papers with their partner and find the answer to their partner's question.

Give 2 minutes of quiet think time for the first question, another minute for the second question, and follow with a whole-class discussion.

Anticipated Misconceptions

Some students may struggle writing a context. Ask them to reflect on the work they have done and what the division expression means for them, or suggest that they write related multiplication expressions to help them understand the division expression.

Student Task Statement

1. Think of a situation with a question that can be represented by the equation $12 \div \frac{2}{3} = ?$ Describe the situation and the question.
2. Trade descriptions with your partner, and answer your partner's question.

Student Response

1. Answers vary. Possible response: I have 12 feet of ribbon. The bows I am making each require $\frac{2}{3}$ ft of ribbon. How many bows can I make with the ribbon I have?
2. 18

Activity Synthesis

Ask selected students to share their stories with the whole group. After each student shares, have another student explain how the story matches the division equation. Afterward, ask these and other students for the value of the given expression, and write and display the completed division equation for all to see.

If not already mentioned by students, highlight that the answer is the same for all scenarios because they are all based on the same division expression.

8.2 How Much in One Batch?

15 minutes

In this task, students explore division situations (in the context of baking cookies) where the number of groups and a total amount are given, but the size of 1 group is unknown. They write multiplication equations in which the missing factor answers the question "how much in each group?" instead of "how many groups?"

Students continue to use equations, diagrams, and the connection between multiplication and division in their reasoning. No grid is provided here, but allow students to use graph paper to support their diagram drawing if desired or needed.

As students work, monitor how they start their diagrams and which quantity they represent first. If they are not quite sure how to show a particular quantity, ask them to refer to earlier diagrams and notice how the number of groups, the size of each group, and the total amount were represented on a single diagram. Select students whose diagrams would be instructive to others to share later.

Addressing

- 6.NS.A.1

Launch

Arrange students in groups of 3–4. Give groups 5–6 minutes of quiet work time and 2 minutes to share their responses in their group. Provide access to geometry toolkits (especially graph paper and colored pencils).

Anticipated Misconceptions

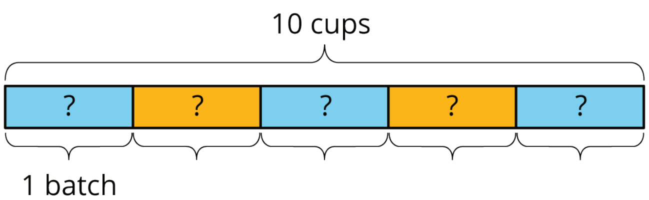
When writing multiplication equations, some students might simply use the smaller number as a factor and the larger number as the product. Or, when writing division equations, they might simply divide the larger number by the smaller number without attending to what the numbers mean. Encourage them to think about what the numbers represent. To help them see the connection between verbal statements and mathematical equations, support their reasoning with simple examples, e.g., "To make 2 batches of cookies, we need 6 cups of flour. How many cups are needed for 1 batch?" This can be interpreted as "2 groups of what equals 6?" and then $2 \cdot ? = 6$.

Student Task Statement

To make 5 batches of cookies, 10 cups of flour are required. Consider the question: How many cups of flour does each batch require?

We can write equations and draw a diagram to represent this situation.

$5 \cdot ? = 10$
$10 \div 5 = ?$



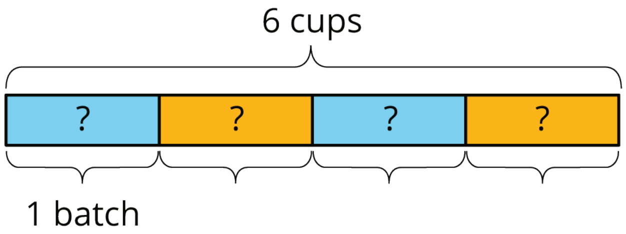
This helps us see that each batch requires 2 cups of flour.

For each question, write a multiplication equation and a division equation, draw a diagram, and find the answer.

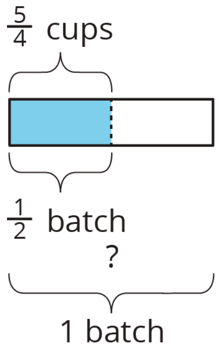
1. To make 4 batches of cupcakes, it takes 6 cups of flour. How many cups of flour are needed for 1 batch?
2. To make $\frac{1}{2}$ batch of rolls, it takes $\frac{5}{4}$ cups of flour. How many cups of flour are needed for 1 batch?
3. Two cups of flour make $\frac{2}{3}$ batch of bread. How many cups of flour make 1 batch?

Student Response

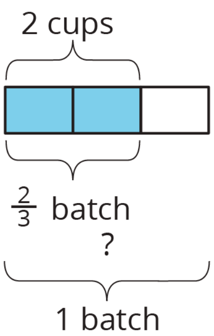
1. Multiplication equation: $4 \cdot ? = 6$ (or $? \cdot 4 = 6$), division equation: $6 \div 4 = ?$ (or $6 \div ? = 4$). Each batch takes $\frac{6}{4}$ (or $1\frac{1}{2}$) cups of flour.



2. Multiplication equation: $\frac{1}{2} \cdot ? = \frac{5}{4}$ (or $? \cdot \frac{1}{2} = \frac{5}{4}$), division equation: $\frac{5}{4} \div \frac{1}{2} = ?$ (or $\frac{5}{4} \div ? = \frac{1}{2}$). Each batch takes $\frac{10}{4}$ (or $2\frac{1}{2}$) cups of flour.

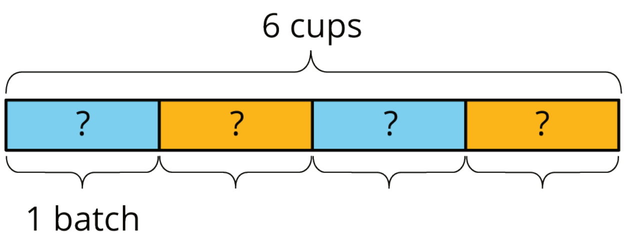


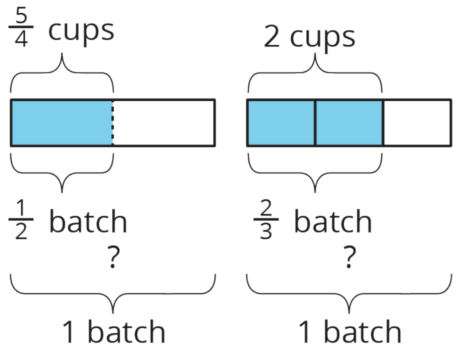
3. Multiplication equation: $\frac{2}{3} \cdot ? = 2$ (or $? \cdot \frac{2}{3} = 2$), division equation: $2 \div \frac{2}{3} = ?$ (or $2 \div ? = \frac{2}{3}$). Each batch takes 3 cups of flour.



Activity Synthesis

Select previously identified students to share their diagrams and reasoning, or display the ones here for all to see. If other students used alternative approaches, invite a couple of them to share.





To highlight some key ideas in this lesson and in the unit, discuss:

- “How are these diagrams like those in previous lessons? How are they different?” (Alike: We can use them to show a total amount, the number of groups, the size of 1 group. Different: We don't know the value of 1 group here. In past diagrams, we were given the size of 1 group, but needed to find the number of groups.)

- “How are these equations and the ones in previous lessons alike and different?” (Alike: The multiplication equations have one unknown factor. The division equations have the same setup. Different: The unknown factor here represents a different quantity than in past lessons.)

- “How are the diagrams for the second and third questions different from the one in the first question?” (In the first one, the given amount is greater than the size of 1 batch. In the other two, the given amount is less than the size of 1 batch or is a fraction of a batch.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate connections between representations. As students share their diagrams and reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.
Supports accessibility for: Visual-spatial processing; Conceptual processing

8.3 One Container and One Section of Highway

15 minutes

In this activity, students continue to reason about the size of 1 group in situations involving equal-sized groups. In the first part, the given total amount is a whole number. In the second, the given amount is a fraction. The visual representations for both parts are very similar, allowing students to notice the structure of the relationships and generalize their observations (MP7).

Addressing

- 6.NS.A.1

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Launch

Display the two sets of images in the activity. Give students a minute to observe them. Ask them to be prepared to share at least one thing they notice and one thing they wonder. Then, solicit a few observations and questions from students.

Diagram 1

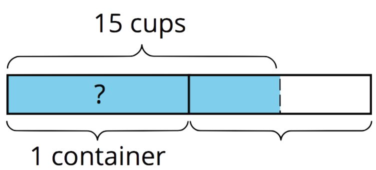


Diagram 2

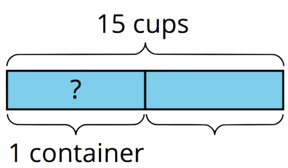
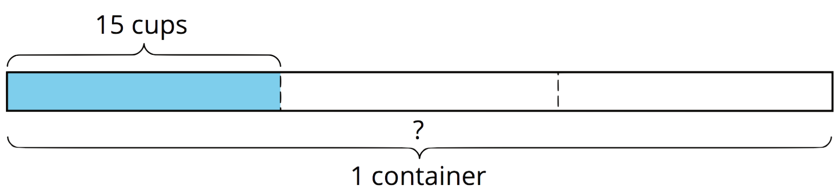
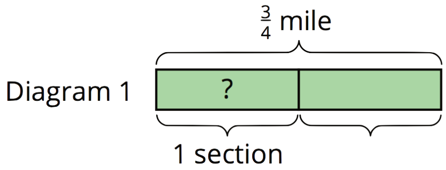
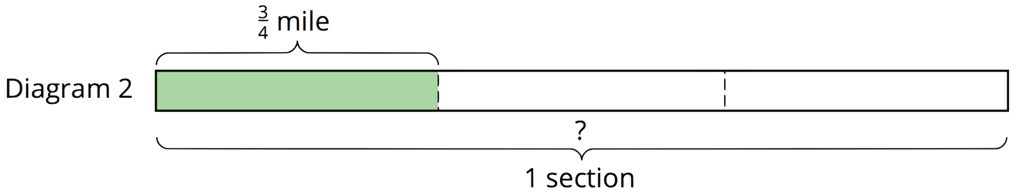
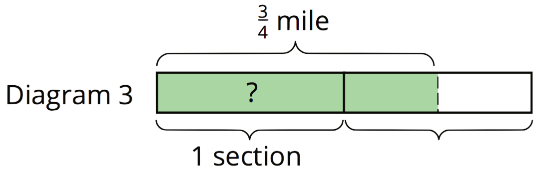


Diagram 3









Keep students in groups of 3–4. Give students 3–4 minutes of quiet think time for the first set of questions (about water in containers) and ask them to pause for a brief group discussion before continuing on to the second set (about highways). Tell students to compare their responses and discuss any questions or discrepancies until they reach an agreement. Afterward, give them another 4–5 minutes of quiet think time to complete the remaining questions and, if possible, time to discuss their responses in groups.

Access for English Language Learners

Speaking, Conversing: MLR8 Discussion Supports. Display sentence frames that will help students produce verbal justifications. For example, "Diagram ___ represents situation ___ because . . .", "A multiplication equation that represents this situation is _____ because . . ." and "A division equation that represents this situation is _____ because . . . ."
Design Principle(s): Support sense-making, Optimize output (for justification)

Student Task Statement

Here are three tape diagrams that represent situations about filling containers of water.

Diagram 1

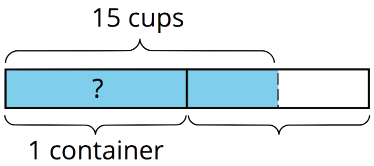


Diagram 2

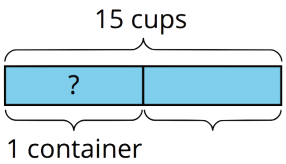
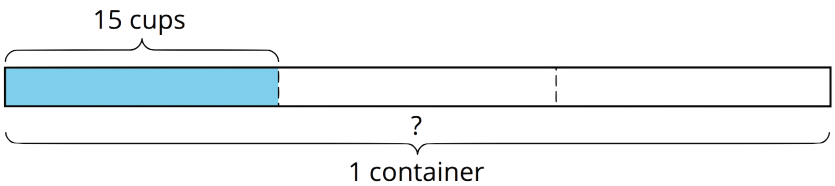


Diagram 3



Match each situation to a diagram and use the diagram to help you answer the question. Then, write a multiplication equation and a division equation to represent the situation.

1. Tyler poured a total of 15 cups of water into 2 equal-sized bottles and filled each bottle. How much water was in each bottle?

2. Kiran poured a total of 15 cups of water into equal-sized pitchers and filled $1\frac{1}{2}$ pitchers. How much water was in the full pitcher?

3. It takes 15 cups of water to fill $\frac{1}{3}$ pail. How much water is needed to fill 1 pail?

   Here are tape diagrams that represent situations about cleaning sections of highway.

Diagram 1

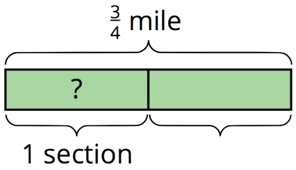


Diagram 2

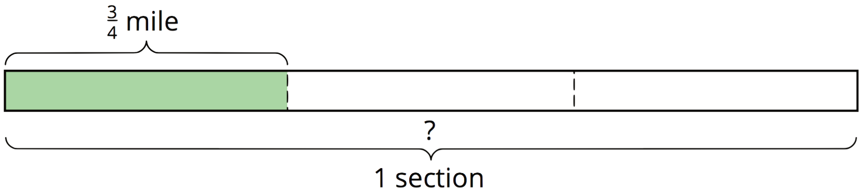
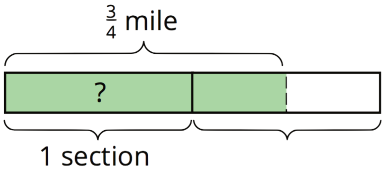


Diagram 3



Match each situation to a diagram and use the diagram to help you answer the question. Then, write a multiplication equation and a division equation to represent the situation.

4. Priya's class has adopted two equal sections of a highway to keep clean. The combined length is $\frac{3}{4}$ of a mile. How long is each section?

5. Lin's class has also adopted some sections of highway to keep clean. If $1\frac{1}{2}$ sections are $\frac{3}{4}$ mile long, how long is each section?

6. A school has adopted a section of highway to keep clean. If $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long, how long is the section?

Student Response

1. Diagram 2, multiplication equation: $2 \cdot ? = 15$ (or $? \cdot 2 = 15$), division equation: $15 \div 2 = ?$ (or $15 \div ? = 2$), answer: $7\frac{1}{2}$ cups.

2. Diagram 1, multiplication equation: $(1\frac{1}{2}) \cdot ? = 15$ (or $? \cdot (1\frac{1}{2}) = 15$), division equation: $15 \div 1\frac{1}{2} = ?$ (or $15 \div ? = 1\frac{1}{2}$), answer: 10 cups.

3. Diagram 3, multiplication equation: $\frac{1}{3} \cdot ? = 15$ (or $? \cdot \frac{1}{3} = 15$), division equation: $15 \div \frac{1}{3} = ?$ (or $15 \div ? = \frac{1}{3}$), answer: 45 cups.

4. Diagram 1, multiplication equation: $2 \cdot ? = \frac{3}{4}$ (or $? \cdot 2 = \frac{3}{4}$), division equation: $\frac{3}{4} \div 2 = ?$ (or $\frac{3}{4} \div ? = 2$), answer: $\frac{3}{8}$ mile.

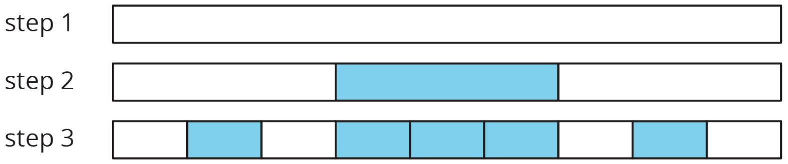
5. Diagram 3, multiplication equation: $(1\frac{1}{2}) \cdot ? = \frac{3}{4}$ (or $? \cdot (1\frac{1}{2}) = \frac{3}{4}$), division equation: $\frac{3}{4} \div 1\frac{1}{2} = ?$ (or $\frac{3}{4} \div ? = 1\frac{1}{2}$), answer: $\frac{1}{2}$ mile.

6. Diagram 2, multiplication equation: $\frac{1}{3} \cdot ? = \frac{3}{4}$ (or $? \cdot \frac{1}{3} = \frac{3}{4}$), division equation: $\frac{3}{4} \div \frac{1}{3} = ?$ (or $\frac{3}{4} \div ? = \frac{1}{3}$), answer: $\frac{9}{4}$ miles.

Are You Ready for More?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is step 1.
- Color in the middle third of the tape diagram. This is step 2.
- Do the same to each remaining segment that is not colored in. This is step 3.
- Keep repeating this process.



1. How much of the diagram is colored in after step 2? Step 3? Step 10?

2. If you continue this process, how much of the tape diagram will you color?

3. Can you think of a different process that will give you a similar result? For example, color the first fifth instead of the middle third of each strip.

Student Response

1. Step 2: $\frac{1}{3}$, Step 3: $\frac{5}{9}$, Step 10: $\frac{19,171}{19,683}$

2. The whole thing.

3. Answers vary.

Activity Synthesis

Most of the discussions are to take place in small groups. Reconvene as a class to discuss any unresolved disagreements or common misconceptions, and to highlight the following points:

- "In both sets of questions, what information was missing?" (The size of 1 group.)
- "How were the division equations in the two contexts different?" (In the water context, the dividend, which is the given amount of water, is a whole number and the quotients are greater than 1. In the highway context, the dividend is a fraction and the quotient could be greater or less than 1.)

Lesson Synthesis

Remind students that there are two multiplication equations that correspond to $7 \div \frac{1}{2} = ?$. We can write:

- $? \cdot \frac{1}{2} = 7$, which can be interpreted as: "how many groups of $\frac{1}{2}$ are in 7?"
- $\frac{1}{2} \cdot ? = 7$, which can be interpreted as: "$\frac{1}{2}$ of what number is 7?"

Because both multiplication equations are related to $7 \div \frac{1}{2} = ?$, we can use division to answer both questions. In this lesson, we looked at the second interpretation, where we know the number of groups but not the amount in 1 group.

To help students synthesize these meanings of division, consider asking them to draw a tape diagram for each interpretation of $7 \div \frac{1}{2} = ?$ and show that in both cases, the quotient is 14.

8.4 Funding a Camping Trip

Cool Down: 5 minutes

Addressing

- 6.NS.A.1

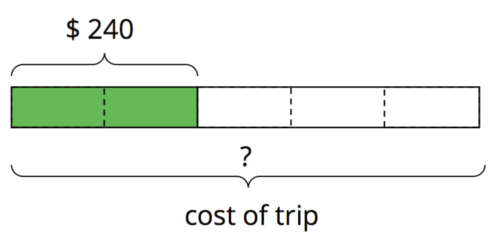
Student Task Statement

Consider the problem: Students in a sixth-grade class are raising money for an end-of-year camping trip. So far, they have raised \$240. This is $\frac{2}{5}$ of the cost of the trip. How much does the trip cost?

Write a multiplication equation and a division equation and draw a diagram to represent the situation. Then, find the answer and show your reasoning.

Student Response

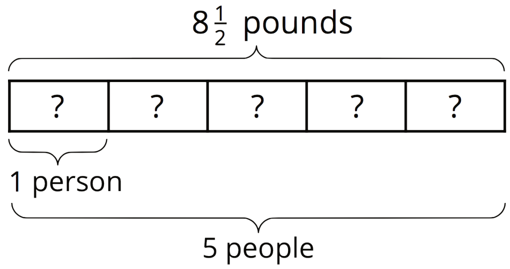
$\frac{2}{5} \cdot ? = 240$ (or $? \cdot \frac{2}{5} = 240$) and $240 \div \frac{2}{5} = ?$ (or $240 \div ? = \frac{2}{5}$). The trip costs \$600. If $\frac{2}{5}$ of the cost is \$240, then each $\frac{1}{5}$ of the cost is \$120, so the cost would be 5 times \$120, which is \$600.



Student Lesson Summary

Sometimes we know the amount for *multiple* groups, but we don't know how much is in one group. We can use division to find out.

For example, if 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?



We can represent this situation with a multiplication equation and a division equation:

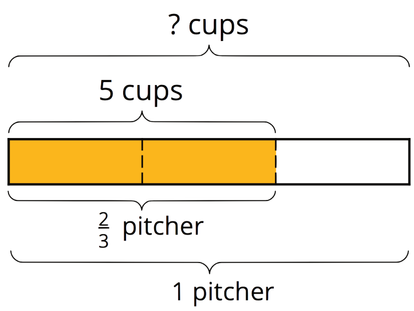
$$5 \cdot ? = 8\frac{1}{2}$$

$$8\frac{1}{2} \div 5 = ?$$

$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. This means each person gets $1\frac{7}{10}$ pounds.

Other times, we know the amount for *a fraction* of a group, but we don't know the size of one whole group. We can also use division to find out.

For example, Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?



We can represent this situation with a multiplication equation and a division equation:

$$\frac{2}{3} \cdot ? = 5$$

$$5 \div \frac{2}{3} = ?$$

The diagram can help us reason about the answer. If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $(3 \cdot \frac{5}{2})$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both situations have the same structures.

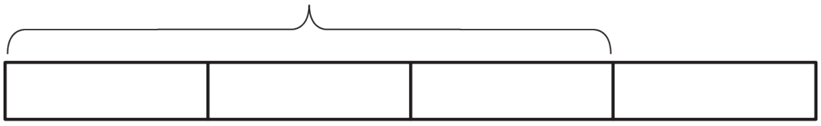
Lesson 8 Practice Problems

Problem 1

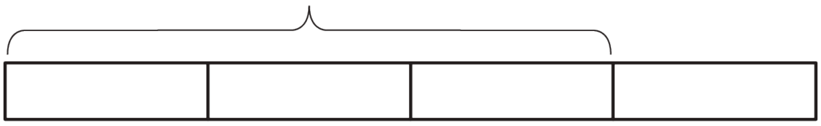
Statement

For each situation, complete the tape diagram to represent and answer the question.

a. Mai has picked 1 cup of strawberries for a cake, which is enough for $\frac{3}{4}$ of the cake. How many cups does she need for the whole cake?

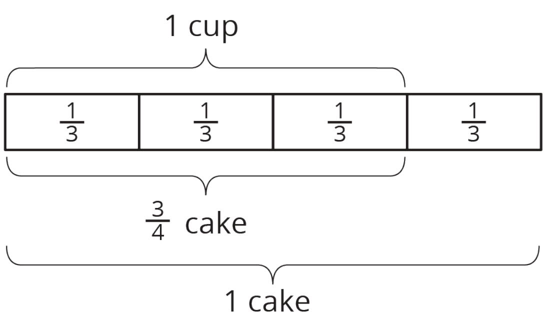


b. Priya has picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a cake. How many cups does she need for the whole cake?

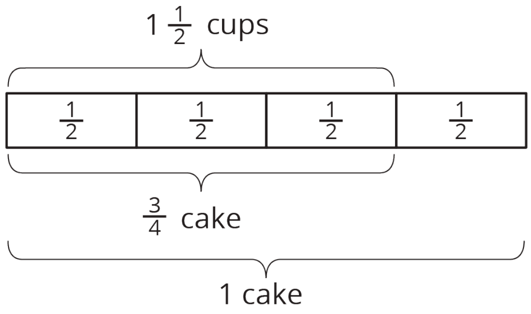


Solution

a. $1\frac{1}{3}$ cups of strawberries



b. 2 cups of raspberries



Problem 2

Statement

Consider the problem: Tyler painted $\frac{9}{2}$ square yards of wall area with 3 gallons of paint. How many gallons of paint does it take to paint each square yard of wall?

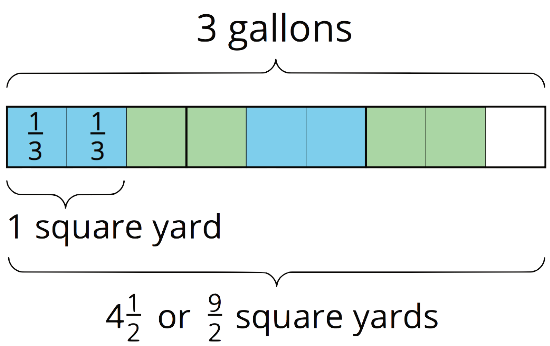
a. Write multiplication and division equations to represent the situation.

b. Draw a diagram to represent and answer the question.

Solution

a. Multiplication: $\frac{9}{2} \cdot ? = 3$, division: $3 \div \frac{9}{2} = ?$

b. Diagrams vary. Sample diagram:



It takes $\frac{2}{3}$ gallons of paint for each square yard of wall. (The answer is correct because $\frac{9}{2} \cdot \frac{2}{3} = 3$.)

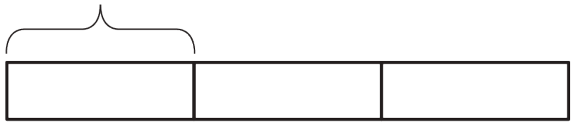
Problem 3

Statement

Consider the problem: After walking $\frac{1}{4}$ mile from home, Han is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?

a. Write multiplication and division equations to represent this situation.

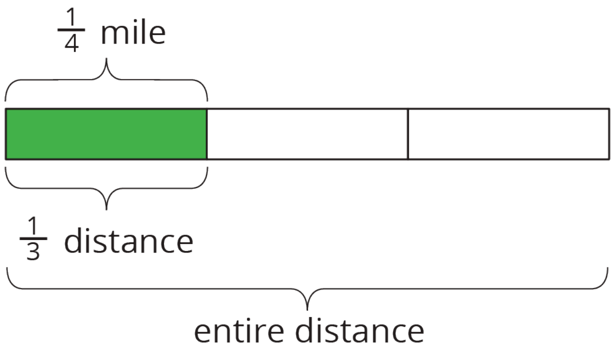
b. Complete the diagram to represent and answer the question.



Solution

a. Multiplication: $\frac{1}{3} \cdot ? = \frac{1}{4}$, division: $\frac{1}{4} \div \frac{1}{3} = ?$

b. Diagrams vary. Sample diagram:



$\frac{3}{4}$ mile ($\frac{1}{4} \div \frac{1}{3} = \frac{3}{4}$)

Problem 4

Statement

Here is a division equation: $\frac{4}{5} \div \frac{2}{3} = ?$

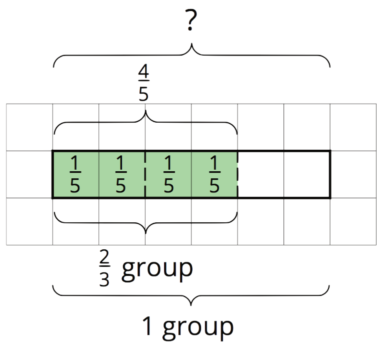
a. Write a multiplication equation that corresponds to the division equation.

b. Draw a diagram to represent and answer the question.

Solution

a. $\frac{2}{3} \cdot ? = \frac{4}{5}$ (or equivalent)

b. $\frac{6}{5}$. Sample diagram:



(From Unit 4, Lesson 7.)

Problem 5

Statement

Consider the problem: A set of books that are each 1.5 inches wide are being organized on a bookshelf that is 36 inches wide. How many books can fit on the shelf?

a. Write multiplication and division equations to represent the situation.

b. Find the answer. Draw a diagram, if needed.

c. Use the multiplication equation to check your answer.

Solution

a. $? \cdot (1.5) = 36$ (or equivalent), $36 \div 1.5 = ?$ (or equivalent)

b. 24 books can fit on the shelf.

c. $24 \cdot (1.5) = 36$

(From Unit 4, Lesson 3.)

Problem 6

Statement

a. Without calculating, order the quotients from smallest to largest.

$56 \div 8$ $\qquad$ $56 \div 8{,}000{,}000$ $\qquad$ $56 \div 0.000008$

b. Explain how you decided the order of the three expressions.

c. Find a number n so that $56 \div n$ is greater than 1 but less than 7.

Solution

a. $56 \div 8{,}000{,}000$, $56 \div 8$, $56 \div 0.000008$

b. Since the dividend is the same for all three expressions, the larger the divisor, the smaller the quotient.

c. Answers vary. Possible response: $n = 10$ would work since $56 \div 8 = 7$ and $56 \div 56 = 1$. (Any number between $n = 8$ and $n = 56$ would work.)

(From Unit 4, Lesson 1.)