## Lesson 12: Arithmetic with Complex Numbers

* Let’s work with complex numbers.

### 12.1: Math Talk: Telescoping Sums

Find the value of these expressions mentally.

$2−2+20−20+200−200$

$100−50+10−10+50−100$

$3+2+1+0−1−2−3$

$1+2+4+8+16+32−16−8−4−2−1$

### 12.2: Adding Complex Numbers

1. This diagram represents $(2+3i)+(-8−8i)$.
* 
	1. How do you see $2+3i$ represented?
	2. How do you see $-8−8i$ represented?
	3. What complex number does $A$ represent?
	4. Add “like terms” in the expression $(2+3i)+(-8−8i)$. What do you get?
1. Write these sums and differences in the form $a+bi$, where $a$ and $b$ are real numbers.
	1. $(-3+2i)+(4−5i)$ (Check your work by drawing a diagram.)
	2. $(-37−45i)+(11+81i)$
	3. $(-3+2i)−(4−5i)$
	4. $(-37−45i)−(11+81i)$

### 12.3: Multiplication on the Complex Plane

1. Draw points to represent 2, 22, 23, and 24 on the real number line.
* 
	1. Write $2i$, $(2i)^{2}$, $(2i)^{3}$, and $(2i)^{4}$ in the form $a+bi$.
	2. Plot $2i$, $(2i)^{2}$, $(2i)^{3}$, and $(2i)^{4}$ on the complex plane.
	+ 

#### Are you ready for more?

1. If $a$ and $b$ are positive numbers, is it true that $\sqrt{ab}=\sqrt{a}\sqrt{b}$? Explain how you know.
2. If $a$ and $b$ are negative numbers, is it true that $\sqrt{ab}=\sqrt{a}\sqrt{b}$? Explain how you know.

### Lesson 12 Summary

When we add a real number with an imaginary number, we get a complex number. We usually write complex numbers as:

$a+bi$

where $a$ and $b$ are real numbers. We say that $a$ is the real part and $bi$ is the imaginary part.

To add (or subtract) two complex numbers, we add (or subtract) the real parts and add (or subtract) the imaginary parts. For example:

$(2+3i)+(4+5i)=(2+4)+(3i+5i)=6+8i$

$(2+3i)−(4+5i)=(2−4)+(3i−5i)=-2−2i$

In general:

$(a+bi)+(c+di)=(a+c)+(b+d)i$

and:

$(a+bi)−(c+di)=(a−c)+(b−d)i$

When we raise an imaginary number to a power, we can use the fact that $i^{2}=-1$ to write the result in the form $a+bi$. For example, $(4i)^{3}=4i⋅4i⋅4i$. We can group the $i$ factors together to see how to rewrite this.

$\begin{matrix}4i⋅4i⋅4i&=(4⋅4⋅4)⋅(i⋅i⋅i)\\&=64⋅(i^{2}⋅i)\\&=64⋅-1⋅i\\&=-64i\end{matrix}$

So in this example, $a$ is 0 and $b$ is -64.



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