Lesson 8: Combining Bases

Goals

- Generalize a process for multiplying expressions with different bases having the same exponent, and justify (orally and in writing) that $(ab)^n = a^n \cdot b^n$.

Learning Targets

- I can use and explain a rule for multiplying terms that have different bases but the same exponent.

Lesson Narrative

Previously, students saw that the exponent rules thus far only apply when the bases are the same. In this lesson, students explore what happens when bases are different. This leads to the rule $a^n b^n = (a \cdot b)^n$. Students make use of structure when decomposing numbers into their constituent factors and regrouping them (MP7). Students create viable arguments and critique the reasoning of others when they generate expressions equivalent to 3,600 and $\frac{1}{200}$ using exponent rules and determine the validity of other teams' expressions (MP3).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for the rule $(a \cdot b)^n = a^n \cdot b^n$. As a guiding example, consider $2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$.

Student Learning Goals

Let's multiply expressions with different bases.

8.1 Same Exponent, Different Base

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to relate expressions of the form $a^n \cdot b^n$ to $(a \cdot b)^n$ by exploring the structure of the factors (MP7). Students should notice that the factors in the expanded form of $5^3 \cdot 2^3$ can be rearranged and multiplied to show the factors in the expanded form of 10^3. Evaluating and expanding expressions with exponents helps prepare students for the next activity in which they more generally explore products of bases with the same exponent.

Addressing

- 8.EE.A.1

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

> ### Student Task Statement
>
> 1. Evaluate $5^3 \cdot 2^3$
> 2. Evaluate 10^3

Student Response

1. $5^3 \cdot 2^3 = 125 \cdot 8 = 1{,}000$.
2. $10^3 = 1{,}000$.

Activity Synthesis

Consider asking some of the following questions to focus the conversation on the common exponents:

- "What connections do you see between the two expressions?" (The product of the bases in the first expression is equal to the base in the second expression: $2 \cdot 5 = 10$. The exponents are the same in both expressions.)
- "Is there a way to tell just by looking at the expressions that they would be equal? How?" (Since there are 3 factors that are 5 and 3 factors that are 2, group the 2s and 5s together to get 3 factors that are 10.)

Highlight student explanations that clearly show the connection between $2^3 \cdot 5^3$ and 10^3 by inspecting their factors.

8.2 Power of Products

15 minutes

Students use repeated reasoning to discover the rule $(a \cdot b)^n = a^n \cdot b^n$ (MP8).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Encourage students to share their reasoning with their partner as they work to complete the table. Give students 10–12 minutes of work time followed by a whole-class discussion.

Access for Students with Disabilities

Engagement: Internalize Self Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, check in with select students after they have completed the first 2-3 rows.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may write $2x^4$ instead of $(2x)^4$. Similarly, students may write $a \cdot b^n$ instead of $(a \cdot b)^n$. Ask these students to explain the difference between $3 \cdot 4^2$ and $(3 \cdot 4)^2$.

Student Task Statement

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? What happens if neither the exponents nor the bases are the same? Explain or show your reasoning.

Student Response

1.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$	$(3 \cdot 3) \cdot (7 \cdot 7) = (3 \cdot 7)(3 \cdot 7) = 21 \cdot 21$	21^2
$2^4 \cdot 3^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$ $= (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$ $= 6 \cdot 6 \cdot 6 \cdot 6$	6^4
$3^3 \cdot 5^3$	$15 \cdot 15 \cdot 15 = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$ $= (3 \cdot 3 \cdot 3)(5 \cdot 5 \cdot 5)$	15^3
Answers vary. Sample: $3^4 \cdot 10^4$	Answers vary. Sample: $30 \cdot 30 \cdot 30 \cdot 30$ $= (3 \cdot 10)(3 \cdot 10)(3 \cdot 10)(3 \cdot 10)$ $= (3 \cdot 3 \cdot 3 \cdot 3)(10 \cdot 10 \cdot 10 \cdot 10)$	30^4
$2^4 \cdot x^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x \cdot x)$ $= (2 \cdot x)(2 \cdot x)(2 \cdot x)(2 \cdot x)$ $= 2x \cdot 2x \cdot 2x \cdot 2x$	$(2x)^4$
$a^n \cdot b^n$	$(a \cdot \ldots \cdot a)(b \cdot \ldots b) = (a \cdot b) \cdot \ldots \cdot (a \cdot b)$	$(a \cdot b)^n$
$7^4 \cdot 2^4 \cdot 5^4$	$(7 \cdot 7 \cdot 7 \cdot 7)(2 \cdot 2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5 \cdot 5)$ $= (7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)$ $= 70 \cdot 70 \cdot 70 \cdot 70$	70^4

2. Answers vary. Sample response: No, it is not possible to write $2^3 \cdot 3^4$ with a single exponent because regrouping it into factors that are 6 will leave an extra factor of 3. If the exponents are not the same, the factors cannot be grouped together evenly.

Activity Synthesis

Ask students to share their reasoning about whether $2^3 \cdot 3^4$ can be written with a single exponent. The key takeaway for the discussion should be that the exponents need to be the same to combine the bases into a single base with that exponent.

Introduce a visual display for the rule $(a \cdot b)^n = a^n \cdot b^n$. Display it for all to see throughout the unit.

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to “What happens if neither the exponents nor the bases are the same?” Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them to strengthen their ideas and clarify their language (e.g., “What exponent rule are you using?”, “Can you give an example?”, “How do you know...?”, etc.). Students can borrow ideas and language from each partner to strengthen their final version.
Design Principle(s): Optimize output (for generalization)

8.3 How Many Ways Can You Make 3,600?

Optional: 15 minutes
At this point, students have worked with many different patterns involving exponents. This activity gives students an opportunity to deepen their thinking by generating different equivalent expressions using the rules of exponents. The process of generating different expressions requires students to understand the numerous ways numbers can be broken into factors and how to combine those factors and express the result using exponents.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2–3. Provide students with tools for creating a visual display. There will be several rounds in which students will have to generate multiple expressions equivalent to a specific number. For the first round, students have to generate expressions that are equal to 3,600. Show students how to set up their display with an example, such as the one shown here. As a whole class, come up with an expression equivalent to 3,600 using each of the three rules. If time allows, discuss using prime factorization as a strategy. For an example using the first rule: $3{,}600 = (600 \cdot 6) = (2^3 \cdot 3^1 \cdot 5^2)(2^1 \cdot 3^1) = 2^{3+1} \cdot 3^{1+1} \cdot 5^2 = 2^4 \cdot 3^2 \cdot 5^2$. Some simpler examples include:

$a^n \cdot a^m = a^{n+m}$	$\frac{a^n}{a^m} = a^{n-m}$	$a^n \cdot b^n = (a \cdot b)^n$
$60^1 \cdot 60^1 = 60^2$	$\frac{60^5}{60^3} = 60^2$	$6^2 \cdot 10^2 = 60^2$

Display these examples for all to see while students are working to generate their own expressions equivalent to 3,600. Set a timer for 1 minute (or other amount, depending on time available) and let students work.

When time is up, pair each group with another group for scoring. It would be beneficial to choose 2 groups to use as examples and demonstrate this process for the whole class:

- A group gets 1 point for every *unique* expression they found that is equivalent to 3,600. (If the two groups found the same expression, neither group gets a point for it.)
- 2 points for every unique expression that uses negative exponents.
- Students can challenge the other group's expressions if they think they don't really equal 3,600, or if the group didn't use any of the three rules.

In the second round, shift the students' attention to the number $\frac{1}{200}$ and have them create another visual display (perhaps on the back of their first visual display). Again, the group gets 1 point for each unique expression equivalent to $\frac{1}{200}$ except if it uses negative exponents, in which case it gets 2 points.

Play as many rounds of this game as time allows. In subsequent rounds, groups pair up with a different opponent. Consider using the following numbers in different rounds if time permits: 810,000; $\frac{1}{64}$; 3,375. Leave a few minutes for a brief whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a whole-class think aloud to demonstrate the steps of the game. Keep the worked-out calculations on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. Demonstrate how to play the game. To do this, select a group of students (and include yourself) to begin the game while the rest of the class observes. This will help clarify the expectations of the task, invite more student participation, and facilitate meta-awareness of the language involving exponent rules.
Design Principle(s): Support sense-making; Maximize meta-awareness.

Student Task Statement

Your teacher will give your group tools for creating a visual display to play a game. Divide the display into 3 columns, with these headers:

$a^n \cdot a^m = a^{n+m}$ $\frac{a^n}{a^m} = a^{n-m}$ $a^n \cdot b^n = (a \cdot b)^n$

How to play:

When the time starts, you and your group will write as many expressions as you can that equal a specific number using one of the exponent rules on your board. When the time is up, compare your expressions with another group to see how many points you earn.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rule you claimed.
- If an expression uses negative exponents, you get 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the three exponent rules.

Student Response

- 3,600: Answers vary. Sample responses: 60^2, $6^2 \cdot 10^2$, $6^{-3} \cdot 6^5 \cdot \frac{10^8}{10^6}$.
- $\frac{1}{200}$: Answers vary. Sample responses: $2^{-3} \cdot 5^{-2}$, $\frac{2^6}{2^9} \cdot \frac{5^4}{5^6}$.

Are You Ready for More?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Student Response

1. Answers vary. Sample answers: 10, 13, 16
2. Answers vary. Sample answers: 8, 11, 14
3. This is true for multiples of 3 and numbers that are one more than a multiple of 3, but not for numbers that are one less than a multiple of 3.

Activity Synthesis

In a whole-class discussion, ask “Explain in your own words: What did you learn about exponents from this activity?”

The main point is that numbers can be broken down into their factors in many ways and the exponent rules can be used to express the same value in many ways.

Lesson Synthesis

In this lesson, students saw that it is possible to combine bases together as long as the exponent is the same. Students also practiced using all of the rules they know to write many equivalent exponential expressions. The goal of the discussion is mainly to check that students understand why the exponent rule $a^n \cdot b^n = (a \cdot b)^n$ works. If there is time and interest, students can also share their observations about what they learned by trying to generate as many equivalent exponential expressions as they can.

Here are questions for discussion:

- “Is it possible to write $4^5 \cdot 5^5$ using a single exponent?” (Yes, $4^5 \cdot 5^5 = 20^5$.)
- “What about $4^3 \cdot 5^5$?” (No. You could combine 3 factors that are 4 and 3 factors that are 5 to make 3 factors that are 20, but there are still 2 factors that are 5 left over.)
- “When is it possible to combine bases together in a single exponent?” (It is only possible when both bases have the same exponent.)
- “What are some patterns or strategies you saw in the ‘How Many Ways?’ game?”

To close the discussion, it may help to mention that looking for patterns in the factors of numbers has a long tradition in mathematics, with many applications for building better computers and devices. Many ways that computers have been programmed to think are based on patterns of factors. Both hacking and protecting computer networks from hackers are rooted in patterns of factors at a fundamental level. This might be a topic that interested students can further explore.

8.4 Help an Absent Student

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

> ### Student Task Statement
>
> Using words and equations, explain what you learned about exponents in this lesson so that someone who was absent could read what you wrote and understand the lesson. Consider using an example like $2^4 \cdot 3^4 = 6^4$.

Student Response

Answers vary. Sample response: Today we learned about how to multiply numbers with exponents together. For example, when you multiply $2^4 \cdot 3^4$, you can rearrange the factors to get 6^4. To see this, notice that $2^4 \cdot 3^4 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = 6^4$. The bases only pair up like this if the exponents are the same. If the exponents aren't the same, then there will be some unpaired factors. For example, $2^3 \cdot 3^4 = (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot 3 = 6^3 \cdot 3$. There is an extra factor that is 3 in this case.

Student Lesson Summary

Before this lesson, we made rules for multiplying and dividing expressions with exponents that only work when the expressions have the *same* base. For example,

$$10^3 \cdot 10^2 = 10^5$$

or

$$2^6 \div 2^2 = 2^4$$

In this lesson, we studied how to combine expressions with the same exponent, but *different* bases. For example, we can write $2^3 \cdot 5^3$ as $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$. Regrouping this as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ shows that

$$\begin{aligned} 2^3 \cdot 5^3 &= (2 \cdot 5)^3 \\ &= 10^3 \end{aligned}$$

Notice that the 2 and 5 in the previous example could be replaced with different numbers or even variables. For example, if a and b are variables then $a^3 \cdot b^3 = (a \cdot b)^3$. More generally, for a positive number n,

$$a^n \cdot b^n = (a \cdot b)^n$$

because both sides have exactly n factors that are a and n factors that are b.

Lesson 8 Practice Problems

Problem 1

Statement

Select **all** the true statements:

A. $2^8 \cdot 2^9 = 2^{17}$

B. $8^2 \cdot 9^2 = 72^2$

C. $8^2 \cdot 9^2 = 72^4$

D. $2^8 \cdot 2^9 = 4^{17}$

Solution

["A", "B"]

Problem 2

Statement

Find x, y, and z if $(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 \cdot (2 \cdot 5)^7 = 2^x \cdot 3^y \cdot 5^z$.

Solution

$x = 12$, $y = 9$, $z = 11$

Problem 3

Statement

Han found a way to compute complicated expressions more easily. Since $2 \cdot 5 = 10$, he looks for pairings of 2s and 5s that he knows equal 10. For example, $3 \cdot 2^4 \cdot 5^5 = 3 \cdot 2^4 \cdot 5^4 \cdot 5 = (3 \cdot 5) \cdot (2 \cdot 5)^4 = 15 \cdot 10^4 = 150{,}000$. Use Han's technique to compute the following:

a. $2^4 \cdot 5 \cdot (3 \cdot 5)^3$

b. $\frac{2^3 \cdot 5^2 \cdot (2 \cdot 3)^2 \cdot (3 \cdot 5)^2}{3^2}$

Solution

a. 270,000

b. 180,000

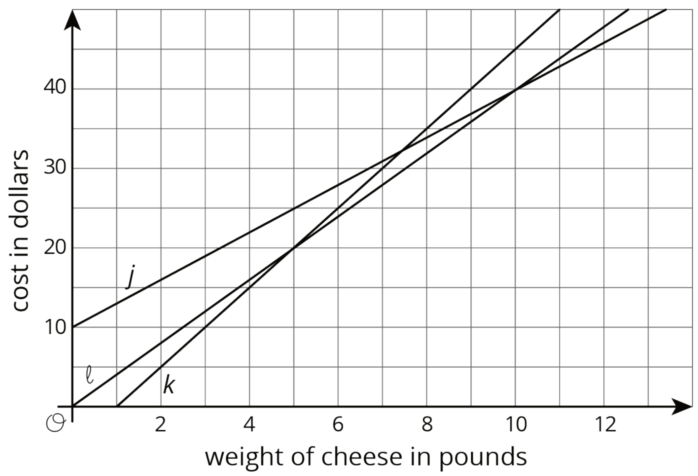
Problem 4

Statement

The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

- Store A sells the cheese for a dollars per pound.
- Store B sells the same cheese for b dollars per pound and a customer has a coupon for $5 off the total purchase at that store.
- Store C is an online store, selling the same cheese at c dollar per pound, but with a $10 delivery fee.

This graph shows the price functions for stores A, B, and C.



a. Match Stores A, B, and C with Graphs j, k, and ℓ.

b. How much does each store charge for the cheese per pound?

c. How many pounds of cheese does the coupon for Store B pay for?

d. Which store has the lowest price for a half a pound of cheese?

e. If a customer wants to buy 5 pounds of cheese for a party, which store has the lowest price?

f. How many pounds would a customer need to order to make Store C a good option?

Solution

a. Store A: Graph ℓ
Store B: Graph k
Store C: Graph j

b. Store A charges $4 per pound, Store B changes $5 per pound, and Store C charges $3 per pound.

c. 1 pound of cheese.

d. Store B

e. Store A or Store B would both charge the same amount for 5 lbs of cheese.

f. If a customer orders more than 10 pounds of cheese, Store C has the lowest price.

(From Unit 5, Lesson 8.)