Lesson 6: Strategic Solving

Goals

- Categorize (orally and in writing) linear equations in one variable based on their structure, and solve equations from each category.
- Describe (orally and in writing) features of linear equations that have one solution, no solution, or many solutions.
- Describe (orally) strategies for solving linear equations in one variable with different features or structures.

Learning Targets

- I can solve linear equations in one variable.

Lesson Narrative

In previous lessons students have started to acquire fluency with a general method of solving equations, and have seen that different solution paths are possible. In this lesson students learn to stop and think ahead strategically before plunging into a solution method. After a warm-up in which they construct their own equation to solve a problem, they look at equations with different structures and decide whether the solution will be positive, negative, or zero, without solving the equation. They judge which equations are likely to be easy to solve and which are likely to be difficult.

When students look for features of an equation that will tell them something about the solution or help them choose a solution path, they engage in MP7.

Alignments

Addressing

- 8.EE.C.7: Solve linear equations in one variable.
- 8.EE.C.7.b: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Poll the Class
- Think Pair Share

Student Learning Goals

Let's solve linear equations like a boss.

6.1 Equal Perimeters

Warm Up: 5 minutes
The purpose of this activity is for students to begin building linear equations and solving them.

Addressing

- 8.EE.C.7

Instructional Routines

- Think Pair Share

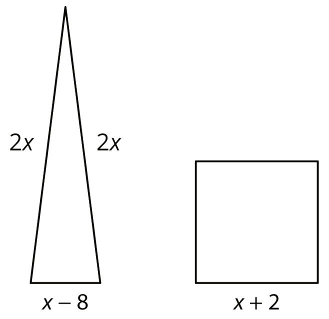
Launch

Arrange students in groups of 2. Give students 2 minutes quiet think time, then 2 minutes to discuss their solutions with a partner. Instruct groups to explain to each other how they came up with expressions and an equation to represent the situation.

Student Task Statement

The triangle and the square have equal perimeters.

1. Find the value of x.
2. What is the perimeter of each of the figures?



Student Response

1. $x = 16$. Since the perimeters are equal, the perimeter of the triangle must equal the perimeter of the square. The perimeter of the triangle is $2x + 2x + (x - 8)$, which is $5x - 8$, and the perimeter of the square is 4 times the side length, or $4(x + 2)$.
$$\begin{aligned} 5x - 8 &= 4(x+2) \\ 5x - 8 &= 4x + 8 \\ 5x &= 4x + 16 \\ x &= 16 \end{aligned}$$
2. $P = 72$. Since the perimeters are equal, we can use either expression to find the perimeter. From the square: $4(16 + 2) = 72$.

Activity Synthesis

Ask groups to share their strategies for solving the question. Consider asking some of the following questions:

- “What expression represents the perimeter of the triangle? The perimeter of the square?” (The expression for perimeter of the triangle is $5x - 8$, and the perimeter of the square is $4(x + 2)$.)
- “What was your strategy in making an equation?” (If both perimeters are the same, we can say their expressions are equal.)
- “What does x mean in the situation?” (It means an unknown value. None of the sides or perimeter is represented by x, so we cannot say it represents a specific thing on the figures.)
- “Looking at the figures, are there any values that x could not be? Explain your reasoning.” (Since the triangles have sides that are $2x$, x cannot be 0 or a negative value. Triangles cannot have sides with 0 or negative side lengths. Since the third side is $x - 8$, we can use this same reasoning to realize that x must actually be greater than 8.)
- “How does this information help when solving?” (If I make a mistake in my solution and get a value of x that is less than or equal to 8, then I know immediately that my answer is not reasonable and I can try to find my error.)

6.2 Predicting Solutions

10 minutes

The purpose of this activity is to shift the focus from solving an equation to thinking about what it means for a number to be a solution of an equation. Students inspect each equation, looking at the structure, the signs, and the operations in it to decide if the solution is positive, negative, or zero. Some questions are paired with another question (for example, the last two questions) so students can take advantage of their thinking from one to the next.

Addressing

- 8.EE.C.7

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2.

Display the equation $5x = 6x$ for all to see.

Ask students, "How might we know whether x is a positive number, negative number, or zero, without solving the equation?" (The variables can be combined into one term, but there are no constant terms. That means eventually the variable term has to equal 0, so x must be 0.)

Display the equation $5x = -16.5$ for all to see and ask the same question. (Without solving, we can see that a positive number of xs has to equal a negative value, so x must be a negative number.)

Instruct students to inspect each equation carefully and use reasoning to answer the questions in the activity rather than trying to solve each equation for a specific value. Give 5 minutes of quiet think time, and then ask students to compare their work with their partner. For any questions they disagree on, students should work to reach an agreement.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about operations with signed numbers.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking: MLR8 Discussion Supports. When partners are compare their work, provide sentence frames to help students describe reasons for whether an equation has a solution that is positive, negative, or zero. Displaying the following: “I know that equation _____ will have a positive/negative/zero solution because ____.”, “Some features of equations with a positive/negative/zero solution are ____.” and “When I look at structure of this equation I notice that ____.” This will help students practice talking about the structure of equations and the operations within them.
Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

Without solving, identify whether these equations have a solution that is positive, negative, or zero.

1. $\frac{x}{6} = \frac{3x}{4}$
2. $7x = 3.25$
3. $7x = 32.5$
4. $3x + 11 = 11$
5. $9 - 4x = 4$
6. $-8 + 5x = -20$
7. $-\frac{1}{2}(-8 + 5x) = -20$

Student Response

1. Zero. Sample reasoning: There are only x terms, so there is no constant term for the variable to equal, or the constant term is 0.

2. Positive. Sample reasoning: A positive amount of xs equals a positive value.
3. Positive. Sample reasoning: A positive amount of xs equals a positive value.
4. Zero. Sample reasoning: Since the constant terms on each side are equal, the final constant term is 0.
5. Positive. Sample reasoning: If you subtract 9 from each side, you will be left with a negative amount of xs equal to a negative number.
6. Negative. Sample reasoning: If you add 8 to each side, you will be left with a positive amount of xs equal to a negative number.
7. Positive. Sample reasoning: After distributing, we will have a negative amount of xs equal to a negative number.

Activity Synthesis

For each equation, invite groups to share how they decided if the solution was positive, negative, or zero. After each group shares, ask if any other group reasoned about the problem in a different way and invite them to share their reasoning.

The purpose of this discussion is for students to practice talking about equations, the operations within them, and use logical thinking. There is no need to try and formally generalize student thinking for all cases at this time. In later grades, students will continue the work started here looking for structure in equations.

6.3 Which Would You Rather Solve?

20 minutes

The purpose of this activity is for students to think about what they see as “least difficult” and “most difficult” when looking at equations and to practice solving equations. Students also discuss strategies for dealing with “difficult” parts of equations.

Addressing

- 8.EE.C.7.b

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Poll the Class

Launch

Keep students in the same groups of 2. Give students 3–5 minutes quiet think time to get started and then 5–8 minutes to discuss and work with their partner. Leave ample time for a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Here are a lot of equations:

A. $-\frac{5}{6}(8+5b)=75+\frac{5}{3}b$

B. $-\frac{1}{2}(t+3)-10=-6.5$

C. $\frac{10-v}{4}=2(v+17)$

D. $2(4k+3)-13=2(18-k)-13$

E. $\frac{n}{7}-12=5n+5$

F. $3(c-1)+2(3c+1)=-(3c+1)$

G. $\frac{4m-3}{4}=-\frac{9+4m}{8}$

H. $p-5(p+4)=p-(8-p)$

I. $2(2q+1.5)=18-q$

J. $2r+49=-8(-r-5)$

1. Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.

2. Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Student Response

A. $b=-14$

B. $t=-10$

C. $v=-14$

D. $k=3$

E. $n=-3.5$

F. $c=0$

G. $m=-\frac{1}{4}$

H. $p=-2$

I. $q=3$

J. $r = \frac{3}{2}$

1. Answers vary. Sample response: Equation A would be difficult to solve because there are some fractions on each side of the equation. Equation C would be easy to solve because there is only one fraction on one side of the equation.
2. Answers vary.

Are You Ready for More?

Mai gave half of her brownies, and then half a brownie more, to Kiran. Then she gave half of what was left, and half a brownie more, to Tyler. That left her with one remaining brownie. How many brownies did she have to start with?

Student Response

7 brownies. An equation that represents this scenario is $\frac{x+1}{2} + \frac{x - \frac{x+1}{2} + 1}{2} = x - 1$, where x is the number of brownies Mai started with.

Activity Synthesis

The purpose of this discussion is for students to discuss strategies for solving different types of equations. Some students may have thought that an equation was in the "least difficult" category, while others thought that the same equation was in the "most difficult" category. Remind students that once you feel confident about the strategies for solving an equation, it may move into the "least difficult" category, and recognizing good strategies takes practice and time.

Poll the class for each question as to whether they placed it in the "most difficult" category, "least difficult" category, or if it was somewhere in the middle. Record and display the results of the poll for all to see.

For questions with a split vote, have a group share something that was difficult about it and something that made it seem easy. If there are any questions that everyone thought would be more difficult or everyone thought would be less difficult, ask students why it seemed that way. Ask students, "Were there any equations that were more difficult to solve than you expected? Were there any that were less difficult to solve than you expected?"

Consider asking some of the following questions to further the discussion:

- "For equation A, what could we do to eliminate the fraction?" (Multiply each side by the common denominator of 6. Then the terms will all have integer coefficients.)
- "Which other equations could we use this strategy for?" (Any equations that had fractions, such as B, C, E, and G.)
- "What steps do you need to do to solve equation D? Which other equations are like this one?" (There is a lot of distributing and collecting like terms. F and H also have to distribute several times.)

- “What other strategies or steps did you use in solving the equations?”

Access for English Language Learners

Representing: MLR3 Clarify, Critique, Correct. After polling students about the difficulty of each problem, choose the question that was ranked as “most difficult” for students to solve, and display a possible incorrect solution. For example, display $2(2q + 1.5) = 4q + 1.5$, to invite discussion about the distributive property. Invite students to share what makes the question difficult and to name strategies or moves that can be used to solve the equation. Invite students to work with a partner to revise and correct the response. Let them know that they should each be prepared to explain their work. When pairs share their responses with the class, draw students' attention to the language used to describe the steps they took, to clarify their strategies. This will support student understanding of strategies they can use to deal with “difficult” parts of equations.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Instruct students to write an equation with a variable and a constant term on each side that they would look at and consider difficult to solve.

Select several students' equations to display for all to see. Discuss with students:

- “What are some things these equations have in common that might be considered difficult to solve?” (Answers vary, but students may use fractions, decimals, distribution, and negatives to increase level of difficulty.)
- “What strategies do we know for solving equations that have each of these things?” (For fractions, we can find a common denominator and multiply each side of the equation by the denominator to eliminate the fractions. For distribution, we can make sure to collect like terms before we do other steps. For use of decimals and negatives, we can make sure we perform calculations carefully.)

Choose one of the displayed equations for students to solve. Have them compare solutions with a partner. Ask students, “Did any of you use different strategies for solving this equation than your partner? How many of you followed the same solution path?” Share a correct solution with the class so they can compare their solutions.

6.4 Think Before You Step

Cool Down: 5 minutes
Addressing

- 8.EE.C.7

Student Task Statement

1. Without solving, identify whether this equation has a solution that is positive, negative, or zero:

$$3x - 5 = \text{-}3$$

2. Solve the equation.

$$x - 5(x - 1) = x - (2x - 3)$$

Student Response

1. If $3x - 5 = \text{-}3$, then the x must be positive. If x is negative, then subtracting 5 from $3x$ would result in a number less than -3. For similar reasons, x cannot be zero.
2. $x = \frac{2}{3}$

Student Lesson Summary

Sometimes we are asked to solve equations with a lot of things going on on each side. For example,

$$x - 2(x + 5) = \frac{3(2x - 20)}{6}$$

This equation has variables on each side, parentheses, and even a fraction to think about. Before we start distributing, let's take a closer look at the fraction on the right side. The expression $2x - 20$ is being multiplied by 3 and divided by 6, which is the same as just dividing by 2, so we can re-write the equation as

$$x - 2(x + 5) = \frac{2x - 20}{2}$$

But now it’s easier to see that all the terms on the numerator of right side are divisible by 2, which means we can re-write the right side again as

$$x - 2(x + 5) = x - 10$$

At this point, we could do some distribution and then collect like terms on each side of the equation. Another choice would be to use the structure of the equation. Both the left and the right side have something being subtracted from x. But, if the two sides are equal, that means the "something" being subtracted on each side must also be equal. Thinking this way, the equation can now be re-written with less terms as

$$2(x + 5) = 10$$

Only a few steps left! But what can we tell about the solution to this problem right now? Is it positive? Negative? Zero? Well, the 2 and the 5 multiplied together are 10, so that means the 2 and the x multiplied together cannot have a positive or a negative value. Finishing the steps we have:

$$\begin{aligned} 2(x+5) &= 10 & \\ x+5 &= 5 & \text{Divide each side by 2} \\ x &= 0 & \text{Subtract 5 from each side} \end{aligned}$$

Neither positive nor negative. Just as predicted.

Lesson 6 Practice Problems

Problem 1

Statement

Solve each of these equations. Explain or show your reasoning.

$2b + 8 - 5b + 3 = -13 + 8b - 5$

$2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$

$2c - 3 = 2(6 - c) + 7c$

Solution

a. $b = \frac{29}{11}$. Responses vary. Sample response: Collect like terms on each side, add 18 to each side, add $3x$ to each side, then divide each side by 11.

b. $x = 0$. Responses vary. Sample response: Collect like terms on the left side, distribute and collect like terms on the right side, add $3x$ to each side, subtract 15 from each side, then divide each side by 9.

c. $c = -5$. Responses vary. Sample response: Distribute and collect like terms on each side, subtract 12 from each side, subtract $2x$ from each side, then divide each side by 3.

Problem 2

Statement

Solve each equation and check your solution.

$-3w - 4 = w + 3$

$3(3 - 3x) = 2(x + 3) - 30$

$\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$

Solution

a. $w = \frac{-7}{4}$

b. $x = 3$

c. $z = 8$

Problem 3

Statement

Elena said the equation $9x + 15 = 3x + 15$ has no solutions because $9x$ is greater than $3x$. Do you agree with Elena? Explain your reasoning.

Solution

Elena is incorrect. Responses vary. Sample response: $9x > 3x$ when $x > 0$, but $9x < 3x$ when $x < 0$ and $9x = 3x$ when $x = 0$. The solution to the equation is $x = 0$.

Problem 4

Statement

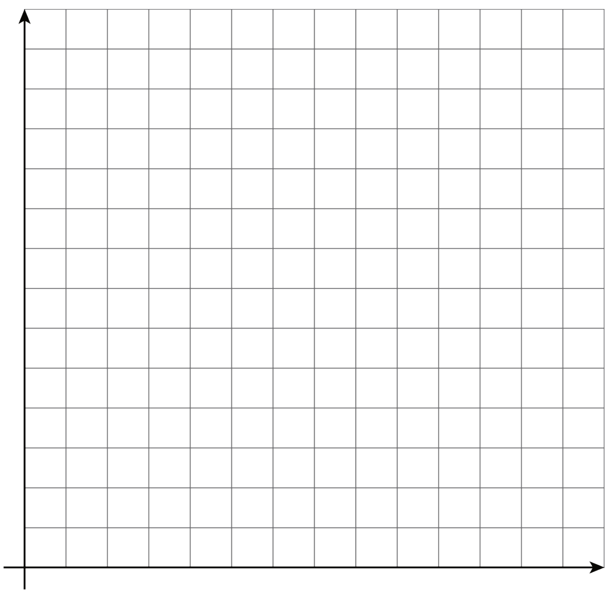
The table gives some sample data for two quantities, x and y, that are in a proportional relationship.

a. Complete the table.

b. Write an equation that represents the relationship between x and y shown in the table.

c. Graph the relationship. Use a scale for the axes that shows all the points in the table.

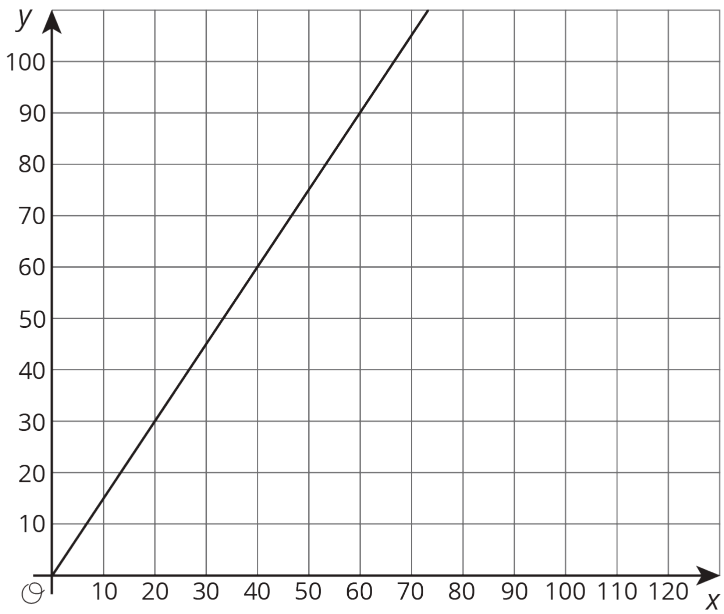
x	y
14	21
64	
	39
1	



Solution

x	y
14	21
64	96
26	39
1	$\frac{3}{2}$

a. $y = \frac{3}{2}x$ (or equivalent)

b. 

(From Unit 3, Lesson 3.)