Lesson 10: Representing Large Numbers on the Number Line

Goals

- Compare large numbers using powers of 10, and explain (orally) the solution method.
- Use number lines to represent (orally and in writing) large numbers as multiples of powers of 10.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals.
- I can write a large number as a multiple of a power of 10.

Lesson Narrative

In this lesson, students use number lines to visualize powers of 10, compare very large numbers, and make sense of orders of magnitude (MP2). They use the structure of a number line that is subdivided into 10 equal intervals to express large numbers as multiples of a power of 10, which naturally leads to the idea of scientific notation, which will be introduced in subsequent lessons (MP7).

In these materials, "multiple of a power of 10" does not necessarily mean an *integer* multiple of a power of 10. Students explore numbers of the form $b \cdot 10^n$, where b is some decimal number. Eventually, when students are formally introduced to scientific notation, b is restricted to values between 1 and 10.

Alignments

Addressing

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share

> **Student Learning Goals**
>
> Let’s visualize large numbers on the number line using powers of 10.

10.1 Labeling Tick Marks on a Number Line

Warm Up: 5 minutes

This warm-up prompts students to reason about values on a number line that end in a power of 10. It enables them to visualize and make sense of numbers expressed as a product of a single digit and a power of 10, which prepares them to begin working with scientific notation.

Expect student responses to include a variety of incorrect or partially-correct ideas. It is not important that all students understand the correct notation at this point, so it is not necessary to extend the time for this reason.

During the partner discussions, identify and select students who have partially-correct responses to share during the whole-class discussion.

Addressing

- 8.EE.A.3

Instructional Routines

- Think Pair Share

Launch

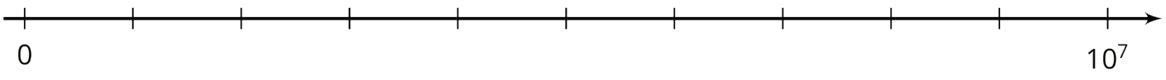
Arrange students in groups of 2. Give students 1 minute of quiet work time and then 2 minutes to compare their number line with their partner. Tell the partners to try and come to an agreement on the values on the number line. Follow with a whole-class discussion.

Anticipated Misconceptions

- Some students may count the tick marks instead of the segments and think 10^7 is being divided into 9 or 11 parts. Consider asking these students to mark each segment with a highlighter to count them.
- Some students may try to label the tick marks 10^1, 10^2, 10^3, etc., and as a result, they may say that the 10^7 is shown in the wrong place. Ask these students how many equal parts 10^7 is divided into, and how they would write that as a division problem or with a fraction.
- Some students may label the tick marks as 10^6, 20^6, 30^6, etc. Ask these students to expand 20^6 and 10^6 into their repeated factors and compare them so they see that 20^6 is not twice as much as 10^6.
- Some students may work with the idea of $\frac{1}{2}$ and label the middle tick mark as $10^{3.5}$. Explain that this tick mark would have a value of $\frac{1}{2} \cdot 10^7$ and ask these students to use what they know about powers of 10 to find the value of the first tick mark after 0.

Student Task Statement

Label the tick marks on the number line. Be prepared to explain your reasoning.



Student Response

The tick marks should be labeled:
$0, 1 \cdot 10^6, 2 \cdot 10^6, 3 \cdot 10^6, 4 \cdot 10^6, 5 \cdot 10^6, 6 \cdot 10^6, 7 \cdot 10^6, 8 \cdot 10^6, 9 \cdot 10^6, 10^7$.

Activity Synthesis

Ask selected students to explain how they labeled the number line. Record and display their responses on the number line for all to see. As students share, use their responses, correct or incorrect, to guide students to the idea that the first tick mark is $1 \cdot 10^6$, the second is $2 \cdot 10^6$, etc. This gives students the opportunity to connect the number line representation with the computational rules they developed in previous lessons. For example, if a student claims that the second tick mark is 20^6, they can check whether 20^6 is equal to $2 \cdot 10^6$ by expanding both expressions.

If not uncovered in students' explanations, ask the following questions to make sure students see how to label the number line correctly:

- “How many equal parts is 10^7 being divided into?” (10)
- “If the number at the end of this number line were 20, how would we find the value of each tick mark?” (Divide 20 by 10)

- “Can we use the same reasoning with 10^7 at the end?” (Yes)
- “What is $10^7 \div 10$?” (10^6) “What does this number represent?” (The distance between two tick marks)
- “Can we write 10^6 as $1 \cdot 10^6$?” (Yes).
- “If the first tick mark is $1 \cdot 10^6$, then what is the second tick mark?” ($2 \cdot 10^6$) “

10.2 Comparing Large Numbers with a Number Line

10 minutes (there is a digital version of this activity)
This activity encourages students to use the number line to make sense of powers of 10 and think about how to rewrite expressions in the form $b \cdot 10^n$, where b is between 1 and 10 (as in the case of scientific notation). It prompts students to use the structure of the number line to compare numbers, and to extend their use to estimate relative sizes of other numbers when no number lines are given.

As students work, notice the ways in which they compare expressions that are not written as multiples of 10^6. Highlight some of these methods in the discussion.

Addressing

- 8.EE.A.3

Building Towards

- 8.EE.A.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet time to work on the first problem, followed by 1–2 minutes to exchange and discuss their work with their partner (second problem).

Then, tell students that representing numbers as a single digit times a power of 10 is useful for making rough comparisons. Give an example: $9 \cdot 10^{11}$ is roughly 200 times as large as $4 \cdot 10^9$, because 10^{11} is 100 times as much as 10^9, and 9 is roughly twice as much as 4. Give students the remaining time to answer the last question. Follow with a brief whole-class discussion.

Classes using the digital version have an interactive applet. Students need to drag the points, marked with open circles and their coordinates, to their proper places on the number line. When all five points are on the line, feedback is available. Note: labels are placed above or below the points only to avoid crowding on the number line.

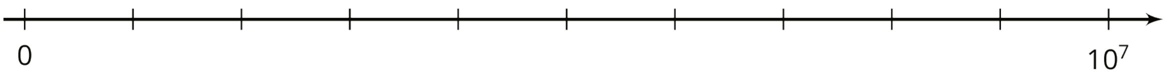
Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about number lines. Keep the display from the warm-up visible for students to reference.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may misplace expressions like $(0.6) \cdot 10^7$ or $75 \cdot 10^5$ on the number line. For $75 \cdot 10^5$, point out that 75 is the same as $(7.5) \cdot 10$, so $75 \cdot 10^5$ is equivalent to $(7.5) \cdot 10 \cdot 10^5$. For $(0.6) \cdot 10^7$, point out that 0.6 is the same as $6 \cdot 10^{-1}$, so $(0.6) \cdot 10^7$ is equivalent to $6 \cdot 10^{-1} \cdot 10^7$. Alternatively, tell the student to think of $(0.6) \cdot 10^7$ as between 0 and 10^7 in the same way that 0.6 is between 0 and 1 to guide them to the correct placement on the number line.

Student Task Statement



1. Place the numbers on the number line. Be prepared to explain your reasoning.
    a. 4,000,000
    b. $5 \cdot 10^6$
    c. $5 \cdot 10^5$
    d. $75 \cdot 10^5$
    e. $(0.6) \cdot 10^7$
2. Trade number lines with a partner, and check each other's work. How did your partner decide how to place the numbers? If you disagree about a placement, work to reach an agreement.
3. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.

Student Response

1. a. 4,000,000 is on the 4th tick mark because it's equal to $4 \cdot 10^6$.
    b. $5 \cdot 10^6$ is on the 5th tick mark.
    c. $5 \cdot 10^5$ is between 0 and the 1st tick mark because it is equal to $(0.5) \cdot 10^6$.
    d. $75 \cdot 10^5$ is between the 7th and 8th tick marks because it is equal to $(7.5) \cdot 10^6$.
    e. $0.6 \cdot 10^7$ is on the 6th tick mark because it is equal to $6 \cdot 10^6$.
2. No answer required.

3. $75 \cdot 10^5$ (or $(7.5) \cdot 10^6$) is about twice as large as 4,000,000 (or $4 \cdot 10^6$), because 7.5 is roughly twice as large as 4.

Activity Synthesis

Ask students, “How could you change 4,000,000, $75 \cdot 10^5$, and $(0.6) \cdot 10^7$ so that all the expressions have the same power of 10?” Highlight the main idea that it’s always possible to rewrite an expression that is a multiple of a power of 10 so that the leading factor is between 1 and 10. For example, 75 can be written as $(7.5) \cdot 10$, and 0.6 can be written as $6 \cdot 10^{-1}$.

If time allows, consider presenting a problem that allows students to use powers of 10 to estimate the relative sizes of large numbers and use them to answer a question in context:

- The population of the United States is roughly $3 \cdot 10^8$ people. The global population is roughly $7 \cdot 10^9$ people. Estimate how many times larger the global population is than the U.S. population. ($7 \cdot 10^9$ is roughly 20 times as large as $3 \cdot 10^8$, because 7 is roughly twice as large as 3, and 10^9 is 10 times as large as 10^8.)

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Why do you think _____ is larger?”, “How did you compare the two terms?”, “Can you represent your thinking in another way?”, etc.). Students can borrow ideas and language from each partner to strengthen their final version.
Design Principle(s): Optimize output (for explanation)

10.3 The Speeds of Light

20 minutes (there is a digital version of this activity)
This activity guides students to thinking in terms of scientific notation while investigating the properties of light. A number line that shows a power of 10 partitioned into 10 equal intervals is again used to illustrate the base-ten structure. Plotting numbers along it gives a clearer meaning to expressions that are a product of a single digit and a power of 10.

To distinguish more easily between the different speeds of light through various materials, the interval between $2 \cdot 10^8$ and $3 \cdot 10^8$ is magnified on the number line. This illustrates numbers with more decimal places and allows students to see how they are expressed in scientific notation.

Once a number line is labeled with powers of 10 and its structure is understood, numbers given in scientific notation can be placed on the number line fairly straightforwardly. This encourages students to look for ways to write the other numbers in scientific notation.

Addressing

- 8.EE.A.3
- 8.EE.A.4

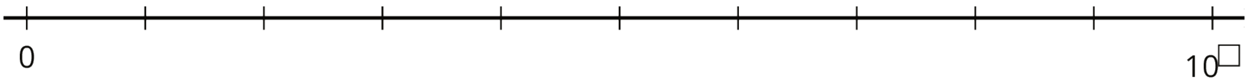
Instructional Routines

- MLR5: Co-Craft Questions

Launch

Display the following number line for all to see. Explain to students that as light moves through different materials, it slows down. The speed of light through empty space, with nothing in its way, is roughly 300,000,000 meters per second. The speed of light through olive oil is much slower at roughly 200,000,000 meters per second.

Ask students to decide what the power of 10 to use for the label of the rightmost tick mark on the number line so that the speed of light through space and through olive oil can be plotted. Give 1 minute of quiet think time before asking 1–2 students to share their responses. Make sure students see that 10^9 is appropriate because for 200,000,000 (which is $2 \cdot 10^8$) to be plotted between 0 and the last tick mark, the last power of 10 needs to be greater than 10^8).



Next, give students 10–12 minutes to work followed by a whole-class discussion.

Students using the digital materials can use the applet to plot the numbers. The magnifying glass allows them to zoom into any interval between two tick marks and plot numbers to an additional decimal place.

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display only the table and ask pairs of students to write possible questions that could be answered by the data in the table. Select 2–3 groups to share their questions with the class. Highlight questions that ask students to compare quantities. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in different ways (i.e., powers of 10, expanded form) prior to being asked to solve questions based on the values.
Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students may struggle to rewrite a number written using one power of 10 as a number with a different power, for example writing $125 \cdot 10^6$ using 10^8. Some may know the relationship between the powers of 10 (say, between 10^6 and 10^8), but may not know how expressing one in terms of the other affects the other factor. Help students make sense of the rewriting process with a series of questions such as these:

- “What do we need to multiply 10^6 by to get 10^8?” (100 or 10^2)
- “If we multiply 10^6 by 10^2, what must we also do to maintain the value of the expression? (Divide the other factor in the expression by 10^2.)
- “What is the resulting expression?” ($(125 \div 10^2) \cdot (10^6 \cdot 10^2) = (1.25) \cdot 10^8$)
- “How do we know that the two expressions are equivalent?” (We multiplied the expression by 10^2 and then divided it by 10^2, which is equal to multiplying it by 1.)

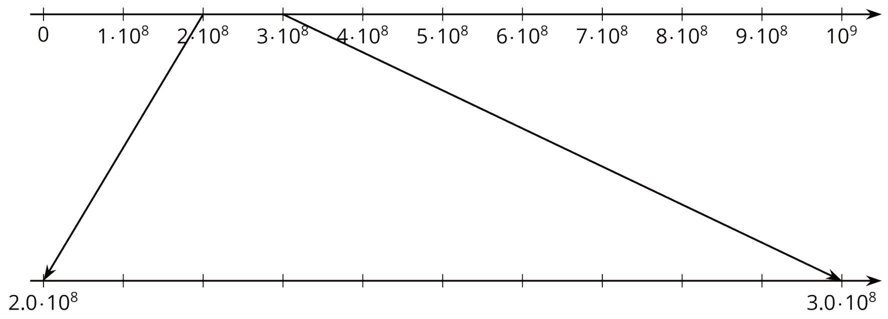
Student Task Statement

The table shows how fast light waves or electricity can travel through different materials.

material	speed (meters per second)
space	300,000,000
water	$(2.25) \cdot 10^8$
copper wire (electricity)	280,000,000
diamond	$124 \cdot 10^6$
ice	$(2.3) \cdot 10^8$
olive oil	200,000,000

1. Which is faster, light through diamond or light through ice? How can you tell from the expressions for speed?

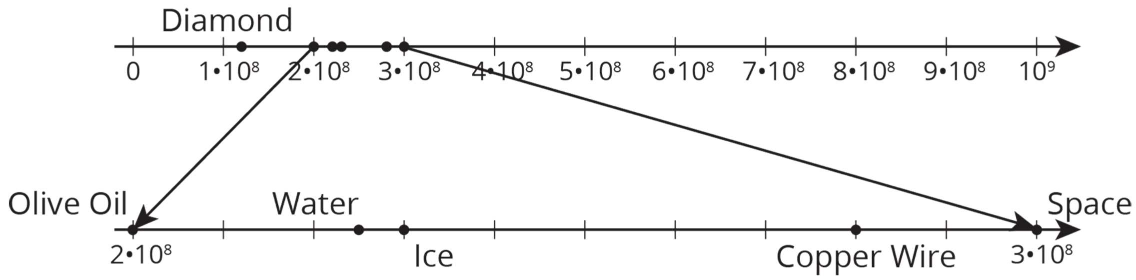
Let’s zoom in to highlight the values between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.



2. Label the tick marks between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.

3. Plot a point for each speed on both number lines, and label it with the corresponding material.
4. There is one speed that you cannot plot on the bottom number line. Which is it? Plot it on the top number line instead.
5. Which is faster, light through ice or light through diamond? How can you tell from the number line?

Student Response

1. Light is faster through ice than through diamond. You can tell because the speed of light through diamond is $(1.24) \cdot 10^8$ meters per second compared to through ice which is $(2.3) \cdot 10^8$ meters per second.
2. The zoomed-in part of the number line should be labeled $(2.1) \cdot 10^8$ through $(2.9) \cdot 10^8$.
3. 

4. Diamond. See number lines in problem 3.
5. Light through ice is faster because it is further to the right on the number line.

Are You Ready for More?

Find a four-digit number using only the digits 0, 1, 2, or 3 where:

- the first digit tells you how many zeros are in the number,
- the second digit tells you how many ones are in the number,
- the third digit tells you how many twos are in the number, and
- the fourth digit tells you how many threes are in the number.

The number 2,100 is close, but doesn't quite work. The first digit is 2, and there are 2 zeros. The second digit is 1, and there is 1 one. The fourth digit is 0, and there are no threes. But the third digit, which is supposed to count the number of 2's, is zero.

1. Can you find more than one number like this?
2. How many solutions are there to this problem? Explain or show your reasoning.

Student Response

The two possible solutions are 1,210 and 2,020. Explanations vary. Sample explanation: Since this is a four-digit number and the digits of this number count how many occurrences of each digit there are, the sum of the digits must be four. There cannot be any 3’s in the number, because that would mean some number needs to occur three times. But 3,000 doesn’t work, nor do 2,322 or 1,131. Numbers of the form _333, 3_33, or 33_3 can’t work, either, because the sum of the digits is more than four. Therefore, we are looking for combinations of the numbers 0, 1, and 2 that add up to 4, knowing that the last digit must be 0. At this point, there are not many choices left, and we can test them all.

Activity Synthesis

Ask students to explain how they were able to compare the speeds of light. Students do not yet know the definition of scientific notation, but this activity should help them see that expressing values in this format allows us to more easily compare them. Consider using the applet to further illustrate how each number could be plotted on the number line.

Tell students, “We saw that the speed of light through ice can be written as $(2.3) \cdot 10^8$ meters per second, and the speed of electricity can be written as $(2.8) \cdot 10^8$ meters per second. When you write them both the same way like this, it makes it much easier to compare them.”

Lesson Synthesis

The purpose of the discussion is to check that students understand how to express a large number as a multiple of a power of 10, find the value of a given multiple of a power of 10, and compare different large numbers by expressing them as multiples of the same power of 10.

It is important students understand that “multiple of a power of 10” does not mean *integer* multiple, necessarily. Tell students that they will be asked to express numbers as “multiples of a power of 10,” which might mean writing 52,000 as $(5.2) \cdot 10^4$, for example.

Here are some questions for discussion:

- “What are some ways you came up with to write 230,000,000 using powers of 10?” (Since the value of 230,000,000 will stay the same if it is multiplied by 10 and then divided by 10, we can think of it as:
$230{,}000{,}000 = 23{,}000{,}000 \cdot 10 = 2{,}300{,}000 \cdot 10^2 = \ldots = 23 \cdot 10^7 = (2.3) \cdot 10^8$.)
- “What are some of the ways you came up with to find the value of $(5.4) \cdot 10^5$?” (The value of $(5.4) \cdot 10^5$ is 540,000 because $(5.4) \cdot 10^5 = 54 \cdot 10^4 = 540 \cdot 10^3 = \ldots = 540{,}000$.)
- “How did you compare which was faster—the speed of light through diamond and the speed of light through ice?” (The speed of light through diamond was given as $124 \cdot 10^6$ meters per second, and the speed of light through ice was given as $(2.3) \cdot 10^8$ meters per second. The speed through diamond could be rewritten as $(1.24) \cdot 10^8$ meters per second, which makes it clear that it is slower than the speed of light through ice because $1.24 < 2.3$.)

10.4 Describe the Point

Cool Down: 5 minutes

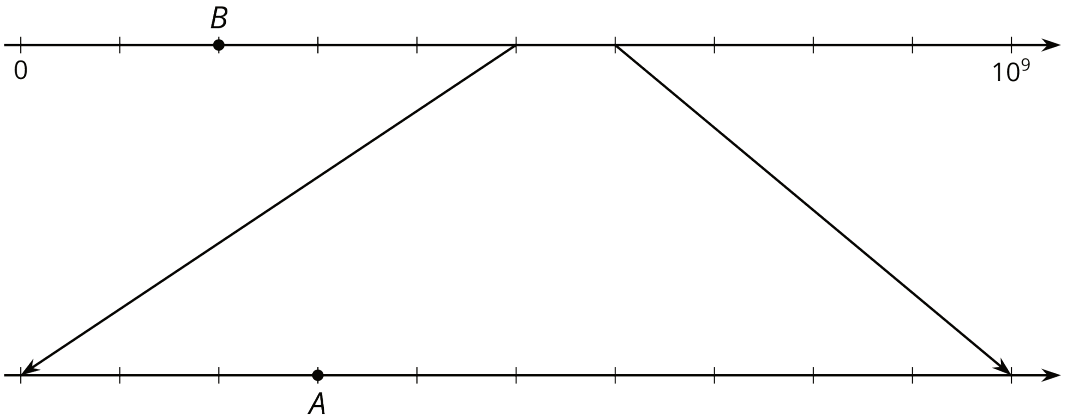
This cool-down measures whether students are able to conceptualize numbers written as a multiple of a positive power of 10 on the number line.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Student Task Statement

We described numbers in this lesson using both powers of 10 and using standard decimal value. For example, the speed of light through ice can be written as a multiple of a power of 10, such as $(2.3) \cdot 10^8$ meters per second, or as a value, such as 230,000,000 meters per second. Use the number line to answer questions about points A and B.



1. Describe point B as:
    a. A multiple of a power of 10
    b. A value
2. Describe point A as:
    a. A multiple of a power of 10
    b. A value
3. Plot a point C that is greater than B and less than A. Describe point C as:
    a. A multiple of a power of 10
    b. A value

Student Response

1. a. Point B represents $2 \cdot 10^8$.

    b. Point B has a value of 200,000,000.

2. a. Point A represents $(5.3) \cdot 10^8$.

   b. Point A has a value of 530,000,000.

3. Answers vary.

Student Lesson Summary

There are many ways to compare two quantities. Suppose we want to compare the world population, about 7.4 billion

to the number of pennies the U.S. made in 2015, about

8,900,000,000

There are many ways to do this. We could write 7.4 billion as a decimal, 7,400,000,000, and then we can tell that there were more pennies made in 2015 than there are people in the world! Or we could use powers of 10 to write these numbers:

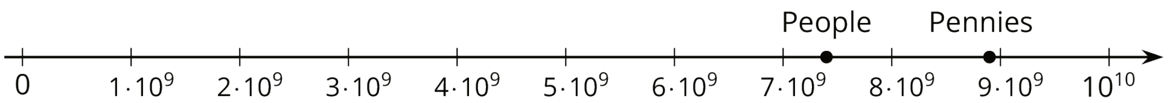
$$7.4 \cdot 10^9$$

for people in the world and

$$8.9 \cdot 10^9$$

for the number of pennies.

For a visual representation, we could plot these two numbers on a number line. We need to carefully choose our end points to make sure that the numbers can both be plotted. Since they both lie between 10^9 and 10^{10}, if we make a number line with tick marks that increase by one billion, or 10^9, we start the number line with 0 and end it with $10 \cdot 10^9$, or 10^{10}. Here is a number line with the number of pennies and world population plotted:



Lesson 10 Practice Problems

Problem 1

Statement

Find three different ways to write the number 437,000 using powers of 10.

Solution

Answers vary. Possible answers: $4.37 \cdot 10^5$, $43.7 \cdot 10^4$, $437 \cdot 10^3$

Problem 2

Statement

For each pair of numbers below, circle the number that is greater. Estimate how many times greater.

a. $17 \cdot 10^8$ or $4 \cdot 10^8$

b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$

c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$

Solution

a. $17 \cdot 10^8$, about 4 times larger

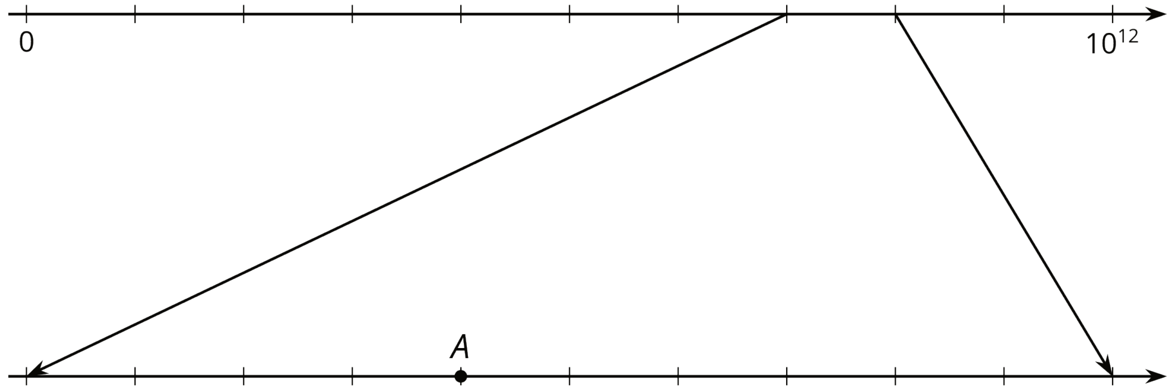
b. $7.839 \cdot 10^6$, about 4 times larger

c. $8.5 \cdot 10^8$, about 2 times larger

Problem 3

Statement

What number is represented by point A? Explain or show how you know.



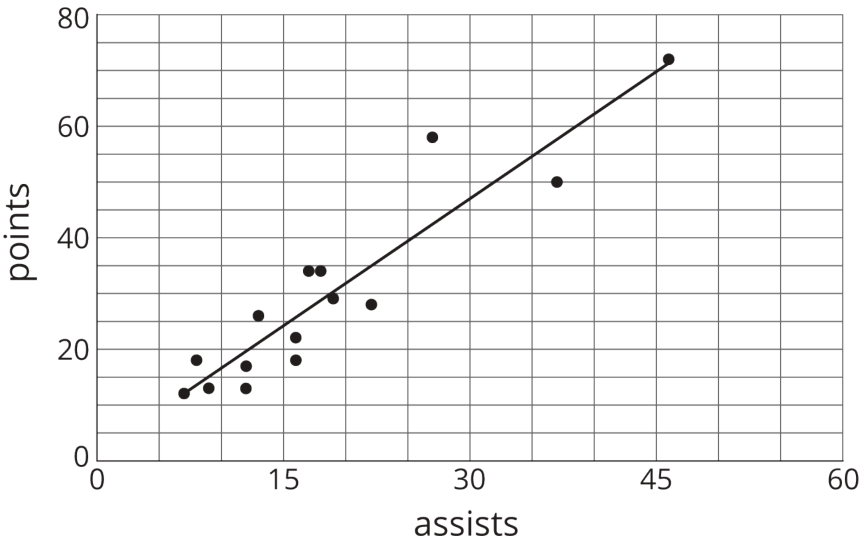
Solution

Answers vary. Sample response: $7.4 \cdot 10^{11}$. Point A lies between $7 \cdot 10^{11}$ and $8 \cdot 10^{11}$. It is $7.4 \cdot 10^{11}$ because it is four tick marks from $7.0 \cdot 10^{11}$.

Problem 4

Statement

Here is a scatter plot that shows the number of points and assists by a set of hockey players. Select **all** the following that describe the association in the scatter plot:



A. Linear association

B. Non-linear association

C. Positive association

D. Negative association

E. No association

Solution

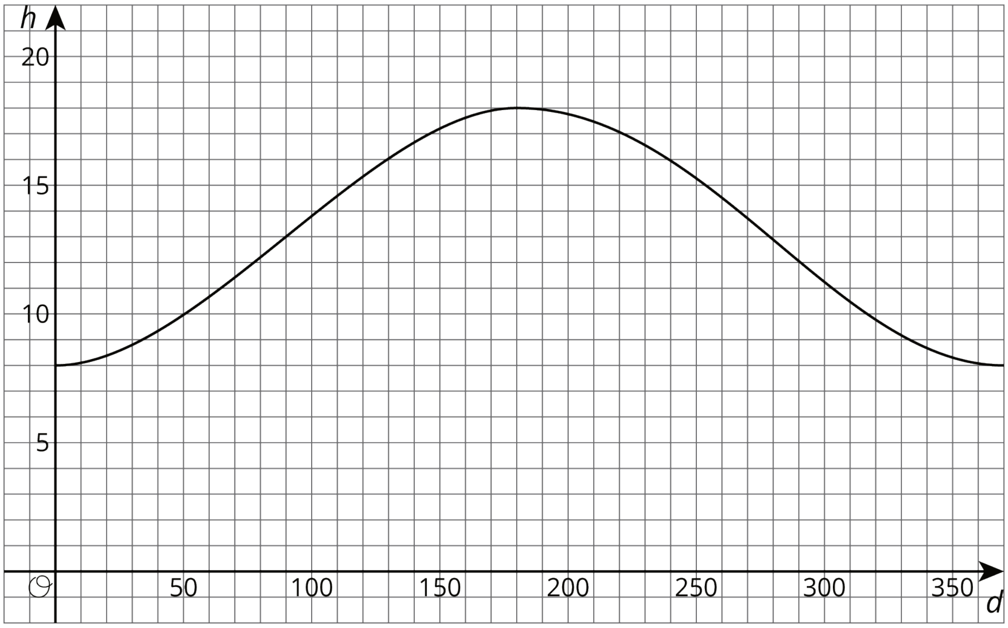
["A", "C"]

(From Unit 6, Lesson 7.)

Problem 5

Statement

Here is the graph of days and the predicted number of hours of sunlight, h, on the d-th day of the year.



a. Is hours of sunlight a function of days of the year? Explain how you know.

b. For what days of the year is the number of hours of sunlight increasing? For what days of the year is the number of hours of sunlight decreasing?

c. Which day of the year has the greatest number of hours of sunlight?

Solution

a. h is a function of d. For every d there is one and only one value of h.

b. From day 0 to day 180, the hours of sunlight are increasing. From day 180 to day 365, the hours of sunlight are decreasing.

c. The day with the greatest number of hours of sunlight is day 180.

(From Unit 5, Lesson 5.)