## Lesson 16: Solving Quadratics

* Let’s solve quadratic equations.

### 16.1: Find the Perfect Squares

The expression $x^{2}+8x+16$ is equivalent to $(x+4)^{2}$. Which expressions are equivalent to $(x+n)^{2}$ for some number $n$?

1. $x^{2}+10x+25$
2. $x^{2}+10x+29$
3. $x^{2}−6x+8$
4. $x^{2}−6x+9$

### 16.2: Different Ways to Solve It

Elena and Han solved the equation $x^{2}−6x+7=0$ in different ways.

Elena said, “First I added 2 to each side:

$x^{2}−6x+7+2=2$

So that tells me:

$(x−3)^{2}=2$

I can find the square roots of both sides:

$x−3=\pm \sqrt{2}$

Which is the same as:

$x=3\pm \sqrt{2}$

So the two solutions are $x=3+\sqrt{2}$ and $x=3−\sqrt{2}$.”

Han said, “I used the quadratic formula:

$x=\frac{-b\pm \sqrt{b^{2}−4⋅a⋅c}}{2⋅a}$

Since $x^{2}−6x+7=0$, that means $a=1$, $b=-6$, and $c=7$. I know:

$x=\frac{6\pm \sqrt{36−4⋅1⋅7}}{2⋅1}$

or

$x=\frac{6\pm \sqrt{8}}{2}$

So:

$x=3\pm \frac{\sqrt{8}}{2}$

I think the solutions are $x=3+\frac{\sqrt{8}}{2}$ and $x=3−\frac{\sqrt{8}}{2}$.”

Do you agree with either of them? Explain your reasoning.

#### Are you ready for more?

Under what circumstances would solving an equation of the form $x^{2}+bx+c=0$ lead to a solution that doesn’t involve fractions?

### 16.3: Solve These Ones

Solve each quadratic equation with the method of your choice. Be prepared to compare your approach with a partner‘s.

1. $x^{2}=100$
2. $x^{2}=38$
3. $x^{2}−10x+25=0$
4. $x^{2}+14x+40=0$
5. $x^{2}+14x+39=0$
6. $3x^{2}−5x−11=0$

### Lesson 16 Summary

Consider the quadratic equation:

$x^{2}−5x=25$

It is often most efficient to solve equations like this by completing the square. To complete the square, note that the perfect square $(x+n)^{2}$ is equal to $x^{2}+(2n)x+n^{2}$. Compare the coefficients of $x$ in $x^{2}+(2n)x+n^{2}$ to our expression $x^{2}−5x$ to see that we want $2n=-5$, or just $n=-\frac{5}{2}$. This means the perfect square $\left(x−\frac{5}{2}\right)^{2}$ is equal to $x^{2}−5x+\frac{25}{4}$, so adding $\frac{25}{4}$ to each side of our equation will give us a perfect square.

$\begin{matrix}x^{2}−5x&=25\\x^{2}−5x+\frac{25}{4}&=25+\frac{25}{4}\\\left(x−\frac{5}{2}\right)^{2}&=\frac{100}{4}+\frac{25}{4}\\\left(x−\frac{5}{2}\right)^{2}&=\frac{125}{4}\end{matrix}$

The two numbers that square to make $\frac{125}{4}$ are $\frac{\sqrt{125}}{2}$ and $-\frac{\sqrt{125}}{2}$, so:

$x−\frac{5}{2}=\pm \frac{\sqrt{125}}{2}$

which means the two solutions are:

$x=\frac{5}{2}\pm \frac{\sqrt{125}}{2}$

Other times, it is most efficient to use the quadratic formula. Look at the quadratic equation:

$3x^{2}−2x=0.8$

We could divide each side by 3 and then complete the square like before, but the equation would get even messier and the chance of making a mistake might be higher. With messier equations like this, it is often most efficient to use the quadratic formula:

$x=\frac{-b\pm \sqrt{b^{2}−4ac}}{2a}$

To use this formula, we first need to put the equation in standard form and identify $a$, $b$, and $c$. Rearranging, we get:

$3x^{2}−2x−0.8=0$

so $a=3$, $b=-2$, and $c=-0.8$. We have to be careful to pay attention to the negative signs. Using the quadratic formula, we get:

$x=\frac{-(-2)\pm \sqrt{(-2)^{2}−4(3)(-0.8)}}{2(3)}$

$x=\frac{2\pm \sqrt{4+(12)(0.8)}}{6}$

Evaluating these solutions with a calculator gives decimal approximations -0.281 and 0.948.



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