Lesson 13: Interpreting Points on a Coordinate Plane

Goals

- Compare points on a graph, including statements about relative position and the vertical distance between points.
- Describe (using words and inequality symbols) and interpret the range of coordinates on a graph, including the meaning of y-values that are negative.
- Identify and interpret points on a graph to answer questions about situations involving temperature or money.

Learning Targets

- I can explain how rational numbers represent balances in a money context.
- I can explain what points in a four-quadrant coordinate plane represent in a situation.
- I can plot points in a four-quadrant coordinate plane to represent situations and solve problems.

Lesson Narrative

This lesson pays particular attention to choices about what axes represent and the scale used on each axis. Graphs need to present information clearly and legibly to be useful for visualizing relationships between quantities. Students learn to make these choices purposefully when plotting points and to consider the decisions that have been made when reading and interpreting the coordinates of points from a graph. They interpret and label axes appropriately to clearly communicate their correspondence with the quantities in a problem. They reason abstractly and quantitatively as they interpret vertical distance in a coordinate plane in context (MP2).

Alignments

Building On

- 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

Addressing

- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- 6.NS.C.7.c: Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.
- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Building Towards

- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

> **Student Learning Goals**
>
> Let's examine what points on the coordinate plane can tell us.

13.1 Unlabeled Points

Warm Up: 5 minutes
In this warm-up, students practice skills that they have developed for plotting points in all 4 quadrants of the coordinate plane. This warm-up also gives students the opportunity to describe points that do not fall nicely on the intersection of grid lines. In the next few activities, students apply these skills to answer questions in context.

Addressing

- 6.NS.C.6.c

Building Towards

- 6.NS.C.8

Launch

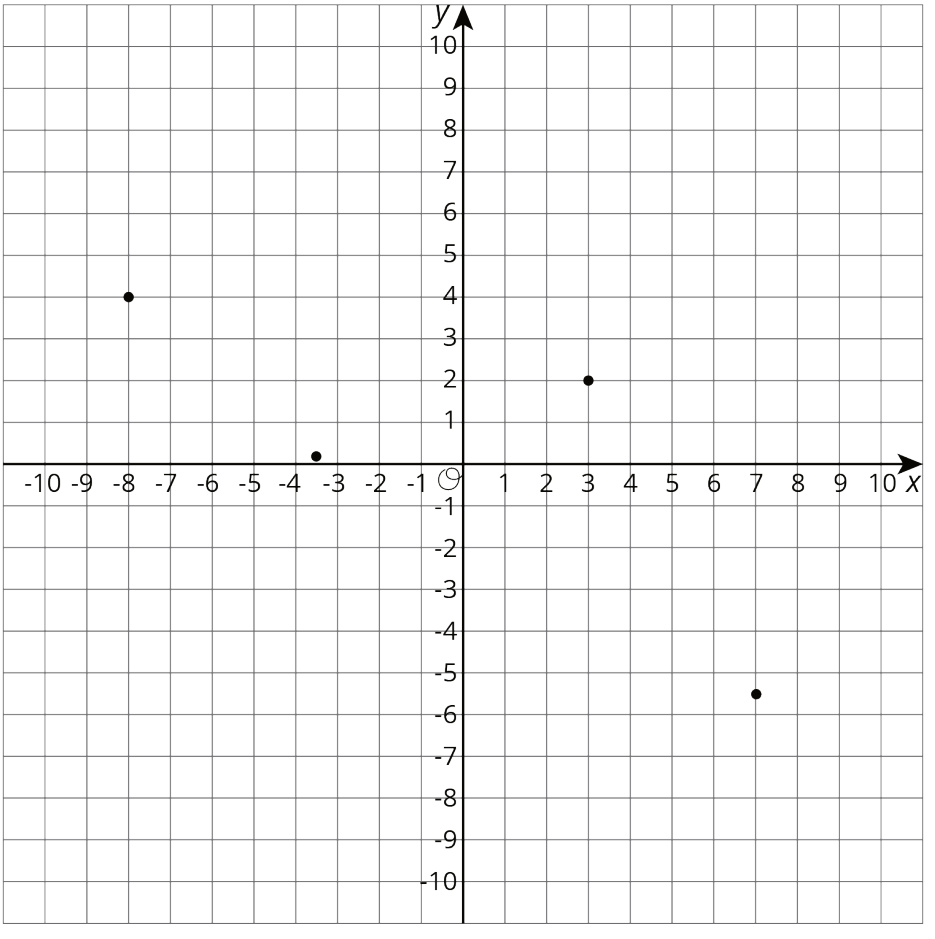
Give students 3 minutes of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may have trouble locating decimal values on a coordinate plane. (They have placed decimals on horizontal number lines before, but up to this point have mostly seen coordinates that are integers or 0.5's.) Demonstrate how point B is 4 units above the x-axis by tracing a pencil 4 units vertically to land on point B. Then ask, "What if the point was only 0.2 units above the x-axis? Where would it go?"

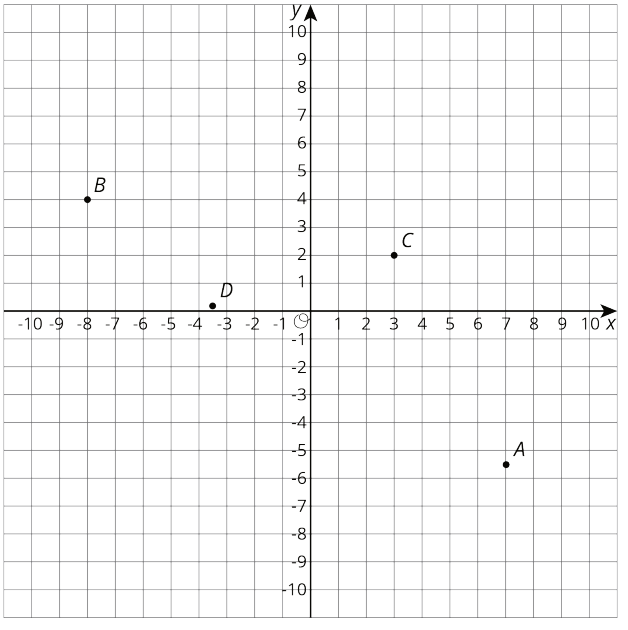
Student Task Statement

Label each point on the coordinate plane with the appropriate letter and ordered pair.



$A = (7, -5.5)$ $B = (-8, 4)$ $C = (3, 2)$ $D = (-3.5, 0.2)$

Student Response



Activity Synthesis

The main goal of discussion is to review the order of ordered pairs and make sense of points that don't fall on the intersection of grid lines. Invite students to explain how they knew which points matched with which coordinates. Ask students how they would make sense of point D, since it doesn't fall nicely where grid lines cross.

13.2 Account Balance

15 minutes

In this activity, students interpret points in the coordinate plane that correspond to the balance in a bank account (MP2). Since bank accounts are not likely to be familiar to students in grade 6, they will need to be oriented to the context.

Building On

- 6.NS.C.5

Addressing

- 6.NS.C.6.c
- 6.NS.C.7.c

Building Towards

- 6.NS.C.8

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Tell students that when someone opens a bank account, they have to put money into the account. The "account balance" is the amount of money in the account at any given time. For example, they might put $350 into the account when they open it, and then the account balance will be 350. However, sometimes they have to borrow money from the bank and then their account balance is a negative value. For instance, if they have no money in the account and borrow $200, then the account balance is -200. The graph they see shows the account balance for a person's account at the start of each day for two weeks.

Give students 10 minutes of work time. Encourage students to check in with their partner after each problem and work to reach agreement if they disagree. Follow with whole-class discussion.

Access for Students with Disabilities

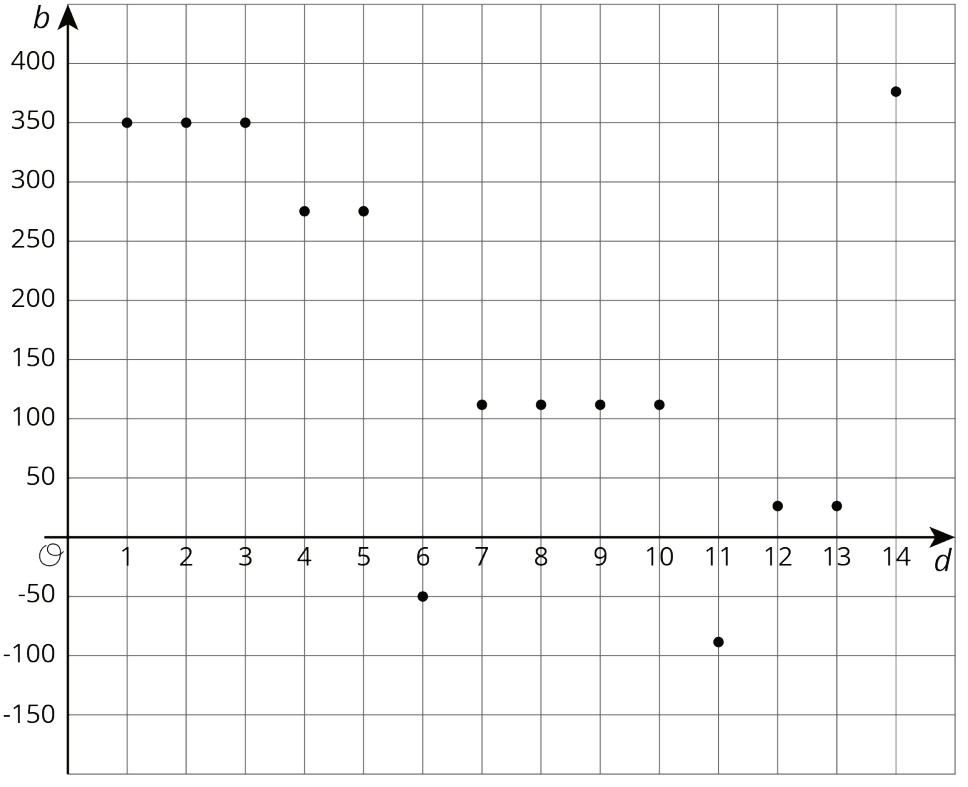
Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “I noticed _____ so I….”, “If _____ then _____ because….”
Supports accessibility for: Language; Organization

Access for English Language Learners

Conversing: MLR5 Co-craft Questions. In order to help students read and interpret the graph in this activity, first display the opening paragraph and graph without showing the questions that follow. Ask students to consider the graph and generate mathematical questions that could be answered by using the graph, such as “How much money was spent on Day 5?” or “What happened on Day 13?” Then, invite pairs to share their questions with the class. This helps students produce the language of mathematical questions and talk about the meaning behind the positive and negative coordinates in this task (e.g., owing money vs. putting money into the bank) prior to being asked to analyze the situation.
Design Principle(s): Cultivate conversation

Student Task Statement

The graph shows the balance in a bank account over a period of 14 days. The axis labeled b represents account balance in dollars. The axis labeled d represents the day.



1. Estimate the greatest account balance. On which day did it occur?
2. Estimate the least account balance. On which day did it occur?
3. What does the point $(6, -50)$ tell you about the account balance?
4. How can we interpret $|-50|$ in the context?

Student Response

1. The greatest balance was about $375 and it occurred on the 14th day.
2. The least balance was about $-90 and it occurred on the 11th day.
3. The point $(6, -50)$ tells us that the account balance was -$50 on the 6th day.
4. $|-50| = 50$ is the amount of money the person owes the bank on day 6.

Activity Synthesis

The purpose of the discussion is to check how comfortable students are with the concept of an account balance that included negative numbers and how to interpret the coordinate plane in this context. Ask for students to explain their responses to each question. To include more students in the discussion, consider asking:

- "Do you agree or disagree? Why?"
- "Who can restate ___'s reasoning in a different way?"

- "Does anyone want to add on to _____'s reasoning?"

Bring attention to the days when the account balance changed. Ask students to come up with a story of what might have happened on those days.

13.3 High and Low Temperatures

15 minutes
Students reason abstractly and quantitatively about temperatures over time graphed on coordinate axes (MP2). The goal of this activity is for students to use inequalities to describe the location of points on a coordinate grid in one direction. This activity also introduces the idea of vertical difference on the coordinate plane using a familiar context. Students may use previous strategies such as counting squares, but are not expected to explicitly add or subtract using negative numbers.

Building On

- 6.NS.C.5

Addressing

- 6.NS.C.6.c
- 6.NS.C.8

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Allow students 3–4 minutes of quiet work time and 1–2 minutes to check results with their partner. Follow with a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students about the sea levels and elevation number line representations.
Supports accessibility for: Social-emotional skills; Conceptual processing

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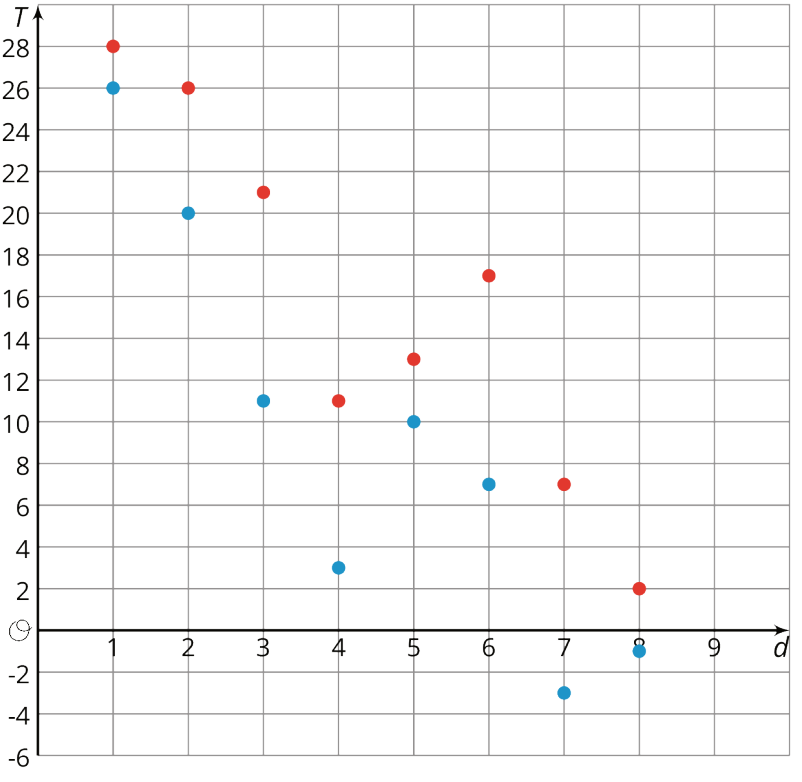
Anticipated Misconceptions

Because the high temperature on day 7 was positive and the low temperature was negative, some students may not notice that the difference was 10 degrees on this day as well. Consider prompting them to use tracing paper to compare the temperature differences on days 6 and 7.

Student Task Statement

The coordinate plane shows the high and low temperatures in Nome, Alaska over a period of 8 days. The axis labeled T represents temperatures in degrees Fahrenheit. The axis labeled d represents the day.



1. a. What was the warmest high temperature?

   b. Write an inequality to describe the high temperatures, H, over the 8-day period.

2. a. What was the coldest low temperature?

   b. Write an inequality to describe the low temperatures, L, over the 8-day period.

3. a. On which day(s) did the *largest* difference between the high and low temperatures occur? Write down this difference.

   b. On which day(s) did the *smallest* difference between the high and low temperatures occur? Write down this difference.

Student Response

1. a. The warmest high temperature was 28°F.

   b. $H < 28$ or $H = 28$. Additionally, students may write $H > 2$ or $H = 2$.

2. a. The coldest low temperature was -3°F.

   b. $L > -3$ or $L = -3$. Additionally, students may write $L < 26$ or $L = 26$.

3. a. Days 3, 6, and 7. The difference was 10°F.

   b. Day 1. The difference was 2°F.

Are You Ready for More?

Before doing this problem, do the problem about taxicab distance in an earlier lesson.

The point $(0, 3)$ is 4 taxicab units away from $(-4, 3)$ and 4 taxicab units away from $(2, 1)$.

1. Find as many other points as you can that are 4 taxicab units away from *both* $(-4, 3)$ and $(2, 1)$.
2. Are there any points that are 3 taxicab units away from both points?

Student Response

1. $(-2, 1)$ and $(-1, 2)$
2. There are no such points. This is because $(-4, 3)$ and $(2, 1)$ are 8 taxicab units apart, so their combined distance to any other point must be at least 8.

Activity Synthesis

This discussion should lead to two key takeaways. First, students express the range of values for the low and high temperatures using inequalities. Second, students share strategies for finding a difference between two values on the coordinate plane.

Ask students to share their inequalities for H and L. It is expected that students have inequalities that describe the maximum high temperature for H and the minimum low temperature for L, but the discussion should bring out that each variable has 4 statements that capture its possible values: $L > -3$, $L = -3$, $L < 26$, and $L = 26$ for the variable L and $H > 2$, $H = 2$, $H < 28$, and $H = 28$ for the variable H.

Ask students to share their strategies for finding the vertical distance between points. Push them to explain how they took the scale of the vertical axis into account. Invite students to explain how they used the context to make sense of their answers.

Access for English Language Learners

Listening, Speaking: MLR8 Discussion Supports. To support students in producing statements about details in the inequalities, provide sentence frames for students to use when they explain their reasoning to their peers, such as: “This inequality should/should not also be equal to because _____”, “A situation uses an equal sign when _____”, “I know that the inequality _____ is correct because _____.”
Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, students graphed temperature and account balance over time on coordinate axes and interpreted questions involving vertical distance. Consider asking some of the following questions:

- "Sketch a graph of $(4, 450)$ and $(4, -47)$. Which of the situations we looked at today would make these points make the most sense? What would the x- and y-axes represent?" (The account balance over the course of several days makes most sense in this situation. Then x would represent the number of days and y would represent the account balance. A temperature of 450 degrees Celsius doesn't make as much sense if we are talking about Earth.)
- "Suppose two people open their own bank accounts on the same day. Graphing their account balances over several days, one person's situation is represented by $(4, 450)$ and the other person's is represented by $(4, -47)$. What does this mean in the situation? How do their account balances compare?" (This means the first person has $450 on day 4 and the other person owes $47 by that same day. The first person's balance is $497 higher than the second person's.)
- "The high temperature on day 6 of a 10 day period is 30 degrees Celsius and the low temperature on that same day is 12 degrees Celsius. Sketch a graph, label the axes, and plot the high and low temperatures on day 6. How much warmer is the high temperature than the low temperature?"

Invite students to display their sketches for all to see. Highlight the vertical distance between points and compare students' strategies for finding that distance.

13.4 Time and Temperature

Cool Down: 5 minutes

This cool-down checks whether students can represent data on a coordinate plane and interpret points in context. Students who label time as the vertical axis and temperature as the horizontal axis need to be reminded of the convention that the first column in a table represents values of the first (horizontal) coordinate and the second column represents values of the second (vertical) coordinate. Given the way the axes are scaled, students need to estimate the location of points that don't fall precisely on the intersection of grid lines.

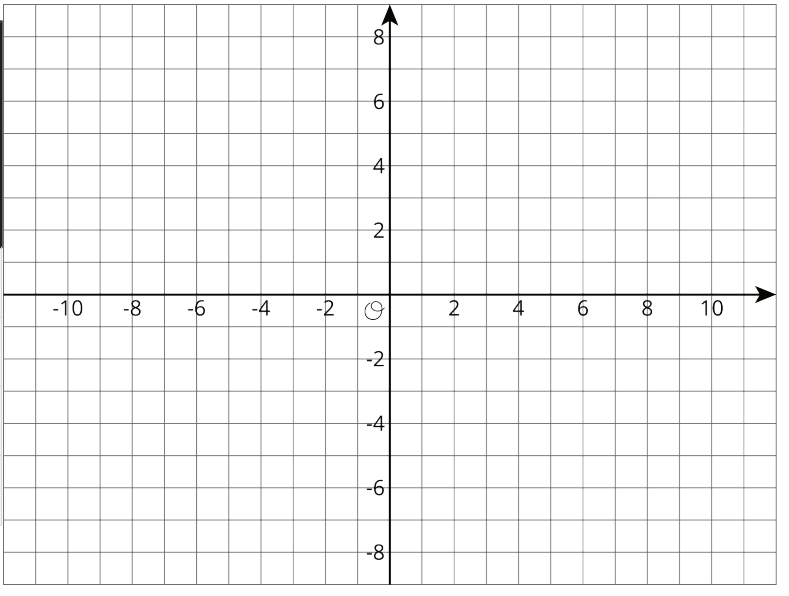
Addressing

- 6.NS.C.8

Student Task Statement

The temperature in Princeton was recorded at various times during the day. The times and temperatures are shown in the table.

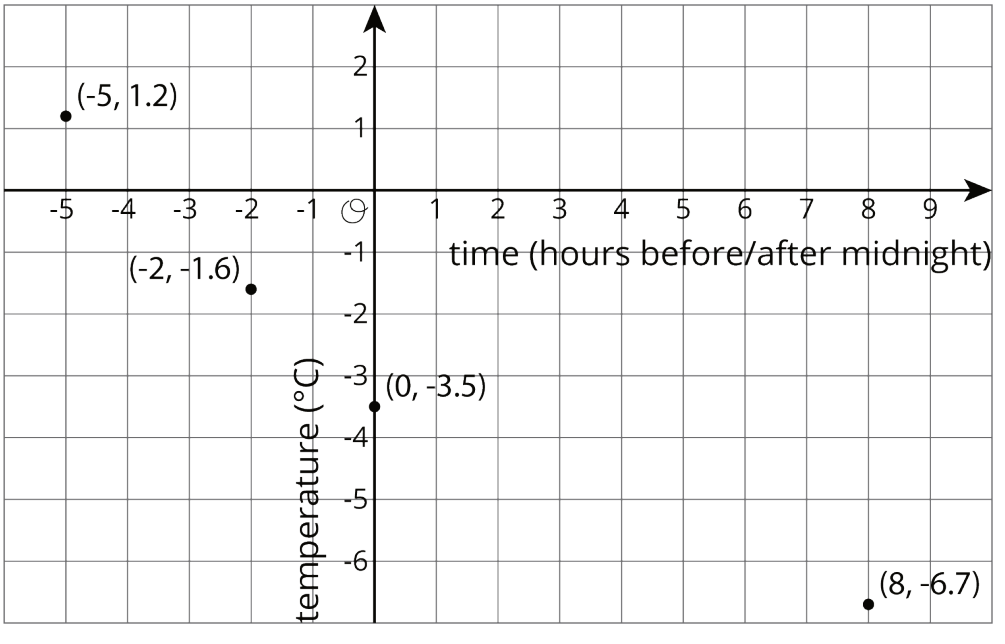
time (hours before or after midnight)	temperature (degrees C)
-5	1.2
-2	-1.6
0	-3.5
8	-6.7



1. Plot points that represent the data. Be sure to label the axes.
2. In the town of New Haven, the temperature at midnight was $1.2^{\circ}\mathrm{C}$. Plot and label this point. Which town was warmer at midnight, Princeton or New Haven? How many degrees warmer was it?
3. If the point $(3, -2.5)$ were also plotted on the diagram, what would it mean?

Student Response

1.



2. The point $(0, 1.2)$ should be added to the plot from the previous question. New Haven is warmer by 4.7 degrees Celsius.

3. $(3, -2.5)$ means 3 hours after midnight the temperature was -2.5 degrees.

Student Lesson Summary

Points on the coordinate plane can give us information about a context or a situation. One of those contexts is about money.

To open a bank account, we have to put money into the account. The account balance is the amount of money in the account at any given time. If we put in $350 when opening the account, then the account balance will be 350.

Sometimes we may have no money in the account and need to borrow money from the bank. In that situation, the account balance would have a negative value. If we borrow $200, then the account balance is -200.

A coordinate grid can be used to display both the balance and the day or time for any balance. This allows to see how the balance changes over time or to compare the balances of different days.

Similarly, if we plot on the coordinate plane data such as temperature over time, we can see how temperature changes over time or compare temperatures of different times.

Lesson 13 Practice Problems

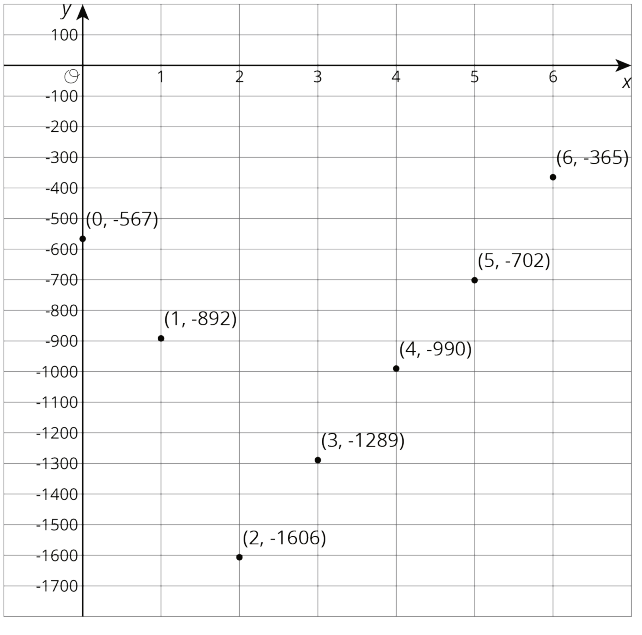
Problem 1

Statement

The elevation of a submarine is shown in the table. Draw and label coordinate axes with an appropriate scale and plot the points.

time after noon (hours)	elevation (meters)
0	-567
1	-892
2	-1,606
3	-1,289
4	-990
5	-702
6	-365

Solution



Problem 2

Statement

The inequalities $h > 42$ and $h < 60$ represent the height requirements for an amusement park ride, where h represents a person's height in inches.

Write a sentence or draw a sign that describes these rules as clearly as possible.

Solution

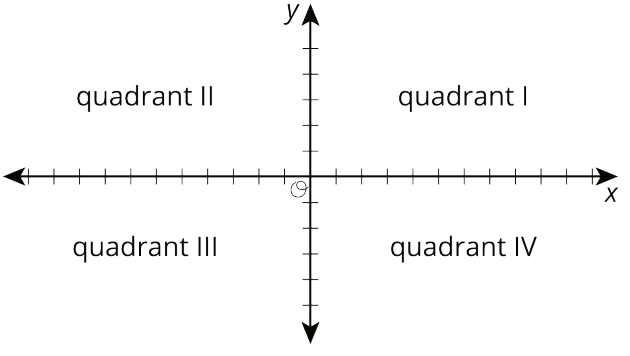
Answers vary. Sample response: To ride, a person must be more than 3 feet 6 inches tall, and no taller than 5 feet.

(From Unit 7, Lesson 8.)

Problem 3

Statement

The x-axis represents the number of hours before or after noon, and the y-axis represents the temperature in degrees Celsius.



a. At 9 a.m., it was below freezing. In what quadrant would this point be plotted?

b. At 11 a.m., it was 10°C. In what quadrant would this point be plotted?

c. Choose another time and temperature. Then tell the quadrant where the point should be plotted.

d. What does the point $(0, 0)$ represent in this context?

Solution

a. Quadrant III

b. Quadrant II

c. Answers vary. Sample response: At 11 p.m., the temperature was -5°C. This point would be plotted in Quadrant IV.

d. A point at $(0, 0)$ would represent a freezing temperature (0°C) at noon.

Problem 4

Statement

Solve each equation.

$3a = 12$

$b + 3.3 = 8.9$

$1 = \frac{1}{4}c$

$5\frac{1}{2} = d + \frac{1}{4}$

$2e = 6.4$

Solution

a. $a = 4$

b. $b = 5.6$

c. $c = 4$

d. $d = 5\frac{1}{4}$

e. $e = 3.2$

(From Unit 6, Lesson 4.)