# Lesson 13: Measurement Error

## Goals

- Compare and contrast (orally) multiple measurements of the same length that result from using rulers with different levels of precision.
- Describe (orally) possible sources of “measurement error” when measuring lengths.
- Generalize a process for calculating measurement error and expressing it as a percentage of the actual length.

## Learning Targets

- I can represent measurement error as a percentage of the correct measurement.
- I understand that all measurements include some error.

## Lesson Narrative

This is the first of three lessons where students encounter the idea of percent error. Unlike situations involving percent increase and percent decrease, where there is an initial amount and a final amount, situations expressed with percent error involve a measured amount and a correct amount. The measurement error is the positive difference between the measured amount and the correct amount, and the percent error is the measurement error expressed as a percentage of the correct amount. In this first lesson students see how measurement error can arise in two different ways: from the level of precision in the measurement device, and from human error. In this lesson students encounter one of the important aspects of mathematical modeling, namely that mathematical representations are usually an approximation of the real situation (MP4).

### Alignments

### Building On

- 2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

### Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

### Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

### Required Preparation

Print the Measuring to the Nearest blackline master. Prepare 1 copy for every 2 students. The blackline master contains two versions of a centimeter ruler that students will cut out (or you may cut out ahead of time) and use to measure things, so card stock would be preferable if available. In the instructions, students are told to cut out the rulers they will use from the blackline master, but to save class time you may want to do this for them ahead of time. These same rulers are also used in the Measuring Your Classroom activity in this lesson, so they should be used carefully during the warm-up.

Measure the height or length of several objects in your classroom to the nearest tenth of a centimeter. If possible, have at least one object per student in the class so that students don't have to wait too long to measure things. Most of the items should be greater than 20 cm in length, but some can be less than or equal to 20 cm in length. Examples of such objects might be the width of the door, the length of the stick that holds a flag, the length of an eraser, or a side of a table or desk top.

### Student Learning Goals

Let's use percentages to describe how accurately we can measure.

# 13.1 Measuring to the Nearest

**Warm Up: 10 minutes**

The purpose of this task is to notice how differences in recorded measurements can result from the level of precision of your measuring device.  Students use rulers that have varying levels of accuracy to measure the same lines. This warm-up gets the conversation started around measurement error that will continue in the follow activities.

Students will need to use the rulers again later in this lesson, so make sure they keep track of them.

### Building On

- 2.MD.A.2

- 5.NBT.B.7

### Building Towards

- 7.RP.A.3

### Instructional Routines

- Think Pair Share

### Launch

Arrange students in groups of 2. Give each group one copy of the blackline master and access to scissors or cut out the rulers provided ahead of time.  Remind students that they are to use the two different rulers to measure the line segments.

Give students 3–5 minutes of quiet work time to complete the task with their partner. Follow with whole-class discussion.

### Anticipated Misconceptions

Students might not line up the edge of the ruler with the end of the line. Remind students that we need to line up the 0 mark on the ruler (in this case, the edge of the ruler) with the beginning edge of the line being measured.

### Student Task Statement

Your teacher will give you two rulers and three line segments labeled A, B, and C.

1. Use the centimeter ruler to measure each line segment to the nearest centimeter. Record these lengths in the first column of the table.
2. Use the millimeter ruler to measure each line segment to the nearest tenth of a centimeter. Record these lengths in the second column of the table.

| line segment | length (cm) as measured with the first ruler | length (cm) as measured with the second ruler |
|---|---|---|
| A | | |
| B | | |
| C | | |

### Student Response

Answers vary. Sample response:

| line segment | length (cm) as measured with the first ruler | length (cm) as measured with the second ruler |
|---|---|---|
| A | 7 | 6.7 |
| B | 7 | 6.9 |
| C | 7 | 7.3 |

### Activity Synthesis

Ask students if they noticed anything between the lengths they got using the two different measuring devices (resulted in different recorded measurements). Explain to students that measurement error can result from the precision level of your measuring device. Ask students, "Assuming the measurements to the nearest tenth are exact, by how much was each measurement in error when you used the centimeter scaled ruler?" (7 cm was 0.3 cm too long, 0.1 cm too long, and 0.3 cm too short respectively.)

# 13.2 Measuring a Soccer Field

**10 minutes**

In the warm-up, students learned that measurement error can result from the level of precision in a measuring device. In this activity, students learn about how real-world limitations on humans using measuring devices can introduce measurement errors. They discuss possible sources of error and express the error both as an amount and as a percentage. This is their first introduction into the concept of measurement error and how we use that to calculate percent error.

### Addressing

- 7.RP.A.3

### Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

### Launch

Keep students in groups of 2. Tell students that a soccer field is 120 yards long and ask them how they can measure that length using a 30-foot-long tape measure. (Note the use of two different units of measure, here.) If not mentioned by students, suggest measuring off 30 feet, making a mark, measuring off another 30 feet, and so on. Ask the class if they would all get exactly the same answer by this method. Tell students they are going to think more deeply about a specific measurement made by a person. Give the students 1–2 minutes of quiet work time to calculate the amount of the error and the percent error followed by partner and whole-group discussions.

### Anticipated Misconceptions

If students fail to see the need for converting units of measure, ask them how many feet are in 120 yards? How many inches?

### Student Task Statement

A soccer field is 120 yards long. Han measures the length of the field using a 30-foot-long tape measure and gets a measurement of 358 feet, 10 inches.

1. What is the amount of the error?
2. Express the error as a percentage of the actual length of the field.

### Student Response

1. 1 foot, 2 inches or 14 inches. 120 yards is 360 feet. The amount of error is the difference between 360 feet and 358 feet, 10 inches.
2. The percent error is 0.32%. The soccer field is 120 yards long, which is 4,320 inches, because $120 \cdot 3 \cdot 12 = 4{,}320$. Han's measurement error is 14 inches, and $14 \div 4{,}320 \approx 0.00324$.

### Activity Synthesis

Ask students, "What is a possible cause of the error?" Possible reasons include:

- He did not position the tape measure precisely every time he measured another 30 feet.
- He didn't go in a completely straight line. (Although this would result in a longer measurement.)
- Han did not correctly use the measuring tape.

Ask a few students to share their solutions for the last problem.

Explain to students, **measurement error** is the positive difference between the measurement and the actual value. The **percent error** is the error expressed as a percentage of the actual value. We always use a positive number to express percent error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value. In this case, we might say that the measured length was less than the actual length with a percent error of 0.32%.

Ask, "When might percent error be more useful than measurement error?" If needed, give an example of measuring a student's height and being incorrect by an inch versus measuring the height of a skyscraper and being incorrect by an inch. Although the measurement error is the same (1 inch), the percent error is very different since the 1 inch difference is significantly more important when measuring shorter distances.

### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: measurement error and percent error.
*Supports accessibility for: Memory; Language*

### Access for English Language Learners

*Speaking: MLR7 Compare and Connect.* Use this routine when students present their strategies for calculating the percent error. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the associations between the measurement, measurement error, and percent error. As students share what they noticed between the strategies, revoice their statements using the terms “measurement error” and “percent error.” These exchanges can strengthen students' mathematical language use and reasoning based on measurement errors.
*Design Principle(s): Maximize meta-awareness*

# 13.3 Measuring Your Classroom

**20 minutes**
This activity has students measuring things around the classroom to connect to the previous activity about measurement error. Students will work with their partner to measure 3 different things found in the classroom that the teacher has measured ahead of time (to obtain an “actual” measurement). They will use two different rulers (one with mm markings and one without).

After both students in a group have measured their three objects, provide them with the actual measurements of those items. They will calculate the measurement and percent errors after given the actual measurements.

Monitor for students who develop different procedures for computing percent error. For example: describing an algorithm in words (verbally or in writing), describing an algorithm more symbolically like $\frac{|\text{actual} - \text{measured}|}{\text{actual}}$, or constructing a table of values.

Also monitor for a group whose two tables clearly shows how using the less-precise ruler results in greater percent error.

### Addressing

- 7.RP.A.3

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

### Launch

Keep students in the same groups of 2. Students will need the two rulers from the blackline master from the warm-up of this lesson.

Assign each group 3 objects to measure (that you have measured ahead of time). If possible, select some items that are longer than the rulers provided so that students may encounter the issue raised in the previous activity.

Tell students that they will fill in only the first 2 columns of each table using their rulers to measure. After groups have completed measuring their 3 objects, tell students the measurements you made of these same objects for them to fill in the third column of the tables. Students should then work to fill in the last 2 columns of the table.

Give students 3–5 minutes of quiet work time and then allow time for them to discuss with a partner, followed by whole-class discussion.

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#### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
*Supports accessibility for: Organization; Attention*

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### Student Task Statement

Your teacher will tell you which three items to measure. Keep using the paper rulers from the earlier activity.

1. Between you and your partner, decide who will use which ruler.
2. Measure the three items assigned by your teacher and record your measurements in the first column of the appropriate table.

   Using the cm ruler:

| item | measured length (cm) | actual length (cm) | difference | percentage |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |

Using the mm ruler:

| item | measured length (cm) | actual length (cm) | difference | percentage |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |

3. After you finish measuring the items, share your data with your partner. Next, ask your teacher for the actual lengths.
4. Calculate the difference between your measurements and the actual lengths in both tables.
5. For each difference, what percentage of the actual length is this amount? Record your answers in the last column of the tables.

### Student Response

Answers vary. Sample response:

| item | measured length (cm) | actual length (cm) | difference | percentage |
|---|---|---|---|---|
| **eraser length** | 19 | 19 | 0 | 0% |
| **table width** | 106 | 110.4 | 4.4 | 4% |
| **book height** | 28 | 27.5 | 0.5 | 1.8% |

| item | measured length (cm) | actual length (cm) | difference | percentage |
|---|---|---|---|---|
| **eraser length** | 19.1 | 19 | 0.1 | 0.5% |
| **table width** | 106 | 110.4 | 4.4 | 4% |
| **book height** | 27.7 | 27.5 | 0.2 | 0.7% |

### Are You Ready for More?

Before there were standard units of measurement, people often measured things using their hands or feet.

1. Measure the length of your foot to the nearest centimeter with your shoe on.
2. How many foot-lengths long is your classroom? Try to determine this as precisely as possible by carefully placing your heel next to your toe as you pace off the room.
3. Use this information to estimate the length of your classroom in centimeters.
4. Use a tape measure to measure the length of your classroom. What is the difference between the two measurements? Which one do you think is more accurate?

### Student Response

Answers vary. The tape measure should be more accurate.

### Activity Synthesis

There are two desired outcomes of this activity: to develop a procedure that makes sense to students for computing percent error, and to reinforce that less-precise measuring devices result in greater percent error.

Select a few students who came up with different, correct procedures for computing percent error to explain their reasoning. A procedure might be described like, “Find the difference between the measured length and the actual length. Divide this difference by the actual length. Express the result as a percentage.” Other procedures may also be appropriate. For example, students might construct a table of values or a double number line to help them reason about expressing the error as a percentage of the actual length.

To highlight the effects of using a less-precise measuring device, select a pair of students whose tables clearly show that the measurements taken with the centimeter ruler had greater percent error than the measurements taken with the millimeter ruler. Display their work and ask them to explain why the measurements taken with the centimeter ruler had greater percent error.

**Access for English Language Learners**

*Conversing: MLR8 Discussion Supports.* Before selecting students to display their work, allow students think time to write and reflect on their understanding of measurement and percent error and describe how each is calculated. Then have a whole-class discussion pressing for detail about how less-precise measuring devices result in greater percent error. This will help students produce and make sense of the language needed to communicate their own ideas. *Design Principle(s): Support sense-making; Cultivate conversation*

## Lesson Synthesis

Students should have a basic understanding of what measurement error is and how to use it to calculate percent error. Ask students:

- "What is measurement error? What causes measurement error?" (The difference between a measurement of an object and it's actual measure. It may exist due to human error in using a measurement tool or because the tool itself is not precise.)
- "How can we minimize the amount of error?" (Use precision tools and care when using them.)
- "What is the relationship between measurement error and percent error?" (Percent error is the measurement error divided by the actual quantity.)

# 13.4 Off by a Little Bit?

**Cool Down: 5 minutes**
In this cool-down, students are assessed on their ability to compute measurement error and percent error from two measurements, one estimate and another actual measurement.

### Addressing

- 7.RP.A.3

### Anticipated Misconceptions

If students do not think to put all the measurements in the same units of measure, they may divide 2 inches by 4 feet and come up with a percent error of 50%. Ask students to explain why an error of 50% is not reasonable in this situation.

### Student Task Statement

Clare estimates that her brother is 4 feet tall. When they get measured at the doctor's office, her brother's height is 4 feet, 2 inches.

1. Should Clare's or the doctor's measurement be considered the actual height? Explain your reasoning.

2. What was the error, expressed in inches?

3. What was the error, expressed as a percentage of the actual height?

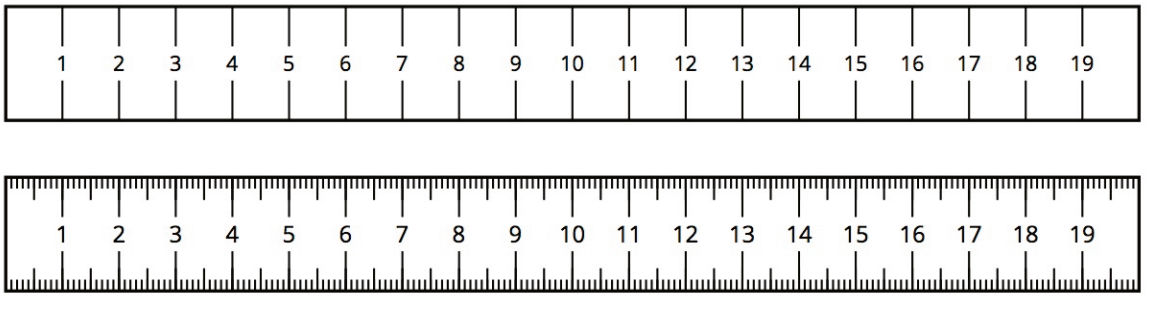
**Student Response**

1. The doctor's measurement, since Clare's is only an estimate, the doctor's is more precise, and the doctor is probably more skilled at measuring heights than Clare.
2. 2 inches
3. 4%, because 4 feet 2 inches is equivalent to 50 inches, and $2 \div 50 = 0.04$.

## Student Lesson Summary

When we are measuring a length using a ruler or measuring tape, we can get a measurement that is different from the actual length. This could be because we positioned the ruler incorrectly, or it could be because the ruler is not very precise. There is always at least a small difference between the actual length and a measured length, even if it is a microscopic difference!

Here are two rulers with different markings.




The second ruler is marked in millimeters, so it is easier to get a measurement to the nearest tenth of a centimeter with this ruler than with the first. For example, a line that is actually 6.2 cm long might be measured to be 6 cm long by the first ruler, because we measure to the nearest centimeter.

The **measurement error** is the positive difference between the measurement and the actual value. Measurement error is often expressed as a percentage of the actual value. We always use a positive number to express measurement error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value.

For example, if we get 6 cm when we measure a line that is actually 6.2 cm long, then the measurement error is 0.2 cm, or about 3.2%, because $0.2 \div 6.2 \approx 0.032$.

## Glossary

- measurement error

# Lesson 13 Practice Problems

## Problem 1

### Statement

The depth of a lake is 15.8 m.

a. Jada accurately measured the depth of the lake to the nearest meter. What measurement did Jada get?

b. By how many meters does the measured depth differ from the actual depth?

c. Express the measurement error as a percentage of the actual depth.

## Solution

a. 16 m

b. 0.2 m

c. 1.27%, because $0.2 \div 15.8 \approx 0.01265$.

## Problem 2

### Statement

A watermelon weighs 8,475 grams. A scale measured the weight with an error of 12% under the actual weight. What was the measured weight?

## Solution

7,458 grams, $8{,}475 \times 0.88 = 7{,}458$

## Problem 3

### Statement

Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.

a. If the actual temperature is $325^\circ\text{F}$, what will the thermometer reading be?

b. If the thermometer reading is $76^\circ\text{F}$, what is the actual temperature?

## Solution

a. 331.5 degrees Fahrenheit, $325 \times 1.02 = 331.5$

b. Approximately 74.5 degrees Fahrenheit, $76 \div 1.02 \approx 74.5$

## Problem 4

### Statement

At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Now, there is $\frac{1}{3}$ less than that. How many ounces of peanut butter are in the pantry now?

A. $\frac{2}{3} \cdot 80$

B. $\frac{1}{3} \cdot 80$

C. $80 - \frac{1}{3}$

D. $\left(1 + \frac{1}{3}\right) \cdot 80$

## Solution

A

(From Unit 4, Lesson 4.)

## Problem 5

### Statement

a. Fill in the table for side length and area of different squares.

| side length (cm) | area ($cm^2$) |
|---|---|
| 3 | |
| 100 | |
| 25 | |
| $s$ | |

b. Is the relationship between the side length of a square and the area of a square proportional?

## Solution

a.

| side length (cm) | area ($cm^2$) |
|---|---|
| 3 | 9 |
| 100 | 10,000 |
| 25 | 625 |
| $s$ | $s^2$ |

b. No. There is no number the numbers in the first column of the table can be multipled by to get the numbers in the second column.

(From Unit 3, Lesson 7.)