Lesson 12: Constructing the Coordinate Plane

Goals

- Choose and label appropriate scales for the axes of the coordinate plane, based on the coordinates to be plotted, and explain (orally and in writing) the choice.
- Compare and contrast different scales for the axes of the coordinate plane.

Learning Targets

- When given points to plot, I can construct a coordinate plane with an appropriate scale and pair of axes.

Lesson Narrative

In this lesson, students explore the idea of scaling axes appropriately to accommodate data where coordinates are rational numbers. Students attend to precision as they plan where to place axes on a grid and how to scale them to represent data clearly (MP6). In an optional activity, students practice working with coordinates in all 4 quadrants as they navigate a maze on a coordinate grid. This lesson gives students the opportunity to develop fluency with plotting coordinates in all 4 quadrants and scaling axes to fit data that is essential for the context-driven work over the next few lessons.

Alignments

Building On

- 5.G.A.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Addressing

- 6.NS.C.6.c: Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Building Towards

- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR7: Compare and Connect
- Poll the Class

> **Student Learning Goals**
>
> Let’s investigate different ways of creating a coordinate plane.

12.1 English Winter

Warm Up: 5 minutes
The purpose of this warm-up is for students to reason about the need for quadrants beyond the first quadrant in the coordinate plane when representing data within a situation's context. When choosing an appropriate set of axes, students should also notice that the scale of the axes is important for the given data. Both of these ideas will be important for students' reasoning in upcoming activities.

While option B is the preferred response, it is more important that students explain and support whatever choice they make.

Building On

- 5.G.A.1

Building Towards

- 6.NS.C.6

Instructional Routines

- Poll the Class

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion. If needed, clarify that the term "noon" refers to 12 p.m.

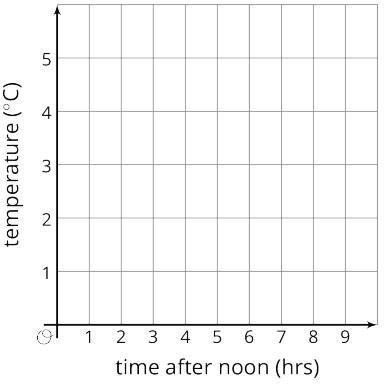
Student Task Statement

The following data were collected over one December afternoon in England.

time after noon (hours)	temperature (°C)
0	5
1	3
2	4
3	2
4	1
5	-2
6	-3
7	-4
8	-4

1. Which set of axes would you choose to represent these data? Explain your reasoning.
2. Explain why the other two sets of axes did not seem as appropriate as the one you chose.

A



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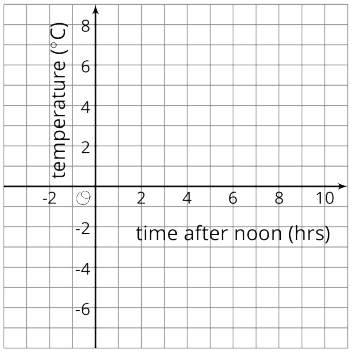

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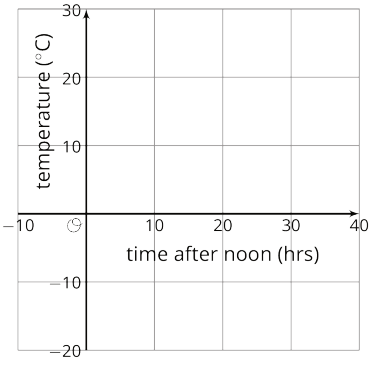


Student Response

1. Responses vary. Sample response: Option B represents the data the best because all of the data points will fit on on the graph nicely where the grid lines meet.
2. Explanations vary. Sample explanation: Option A is not the most helpful because there is nowhere to plot points with negative coordinates. Option C is not the most helpful because the grid lines are spaced 10 units apart, so it is hard to plot coordinates that are not a multiple of 10.

Activity Synthesis

Poll the class on which set of axes they chose to represent the data. Ask selected students to explain why they chose one set of axes and did not choose the other two. Record and display the responses for all to see. If possible, display and reference the three sets of axes as students explain their reasoning.

If there is time, ask students what kind of data would make the other sets of axes appropriate choices. For example, set A would be appropriate if the temperatures were all positive and set C would be appropriate if the data were collected at 10 hour intervals and happened to be close to multiples of 10.

12.2 Axes Drawing Decisions

25 minutes

The purpose of this activity is for students to draw their own axes for different sets of coordinates. They must decide which of the four quadrants they need to use and how to scale the axes. Some students may use logic such as "the largest/smallest point is this, so my axes must go at least that far." Identify these strategies for the discussion. Monitor for differences in scales and axes where the points were still able to be plotted correctly to highlight during discussion.

Addressing

- 6.NS.C.6.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Arrange students in groups of 2. Allow 10 minutes for students to construct their graphs and discuss them with their partners. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time. Invite students to share their reasoning about where they placed each axis and how they determined an appropriate scale. Some students may benefit from access to partially-created graphs with varying degrees of completion—for example, axes with or without unlabeled tick marks.
Supports accessibility for: Memory; Organization

Access for English Language Learners

Listening, Speaking: MLR2 Collect and Display. Listen for and display vocabulary and phrases students use to justify their choice of axes (e.g., "minimum/maximum x- or y-coordinate" or "appropriate units"). Continue to update collected language students used to explain their reasoning to their peers. Remind students to borrow language from the display during paired and whole-class discussions.
Design Principle(s): Maximize meta-awareness

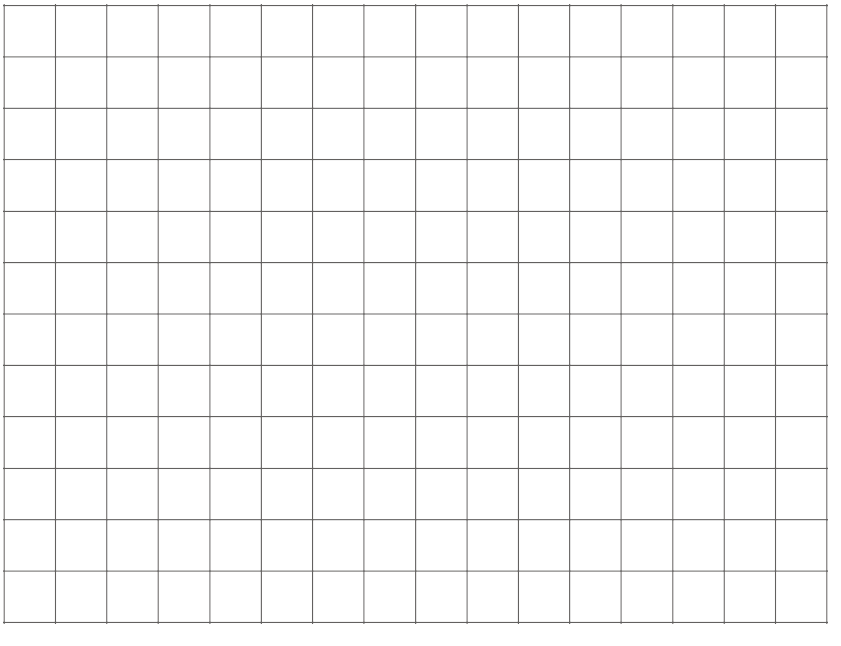
Anticipated Misconceptions

Make sure that students label distances on their axes consistently. For example, if the first tick mark after 0 is 3, then the next must be 6 in order for the spacing to be consistent.

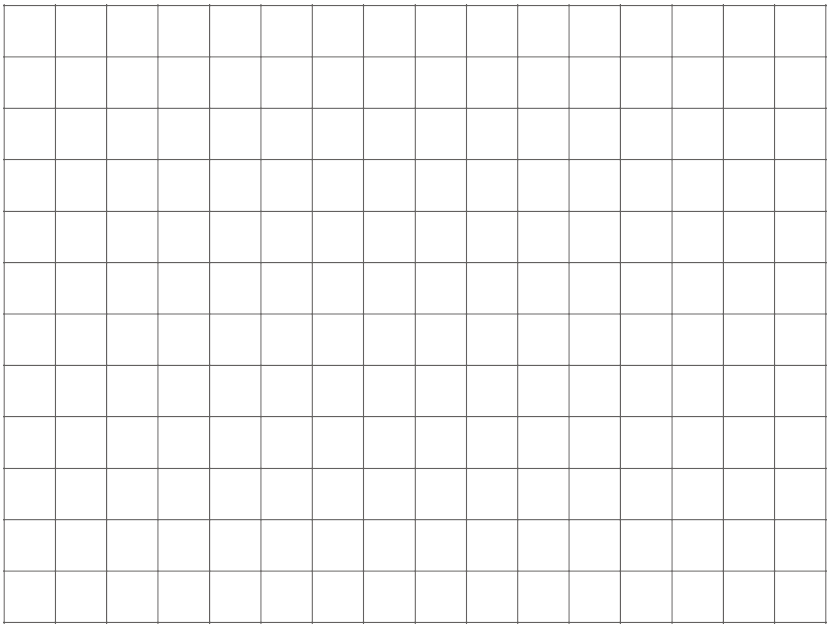
Student Task Statement

1. Here are three sets of coordinates. For each set, draw and label an appropriate pair of axes and plot the points.

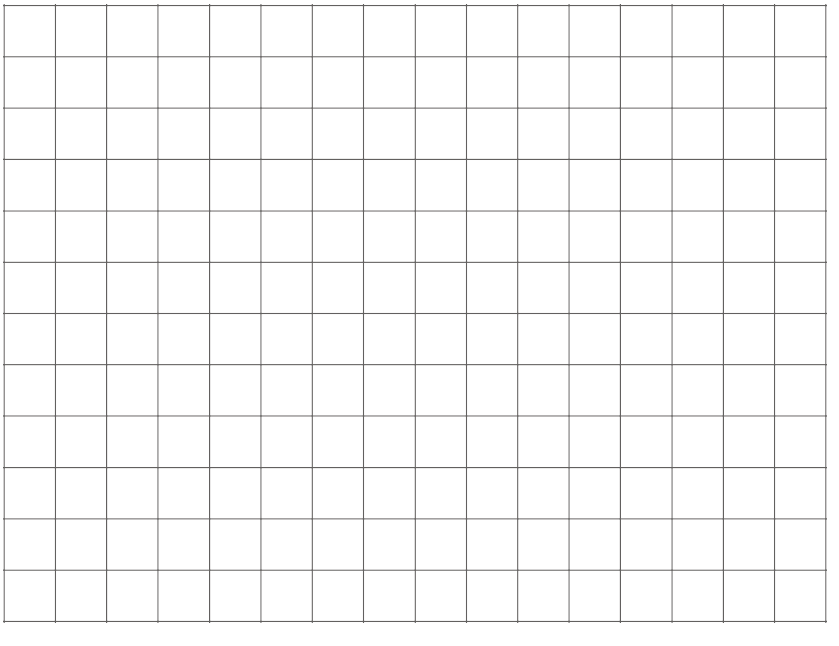
   a. $(1, 2), (3, -4), (-5, -2), (0, 2.5)$



   b. $(50, 50), (0, 0), (-10, -30), (-35, 40)$



c. $\left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{-5}{4}, \frac{1}{2}\right), \left(-1\frac{1}{4}, \frac{-3}{4}\right), \left(\frac{1}{4}, \frac{-1}{2}\right)$



2. Discuss with a partner:
    - How are the axes and labels of your three drawings different?
    - How did the coordinates affect the way you drew the axes and label the numbers?

Student Response

Check student work to ensure they made reasonable choices about axes and scale that allowed them to clearly plot all the points.

Activity Synthesis

The key takeaway from this discussion is that defining axes and scale is a process of reasoning, not an exact science. Ask students to share their strategies about how to place and scale their axes. First, display previously selected student responses that capture the same data on their axes, but with slightly different origins or scales. Ask students which axes they think better represent the data. If not mentioned by students, point out that axes with a lot of empty space probably could benefit from either a different scale or a different origin. Select previously identified students to demonstrate a reliable strategy for finding the needed maximum or minimum. Link back to the warm-up when talking about how to scale the axes: if there are larger numbers, then a bigger scale makes more sense. Also draw attention to the fractional coordinates and how using decimal equivalents might make it easier to scale.

12.3 Positively A-maze-ing

Optional: 10 minutes (there is a digital version of this activity)
The purpose of this task is for students to locate and express coordinates in all four quadrants as they navigate around a maze. Students plan their route through the maze and strategically choose coordinates to correctly execute their plans (MP1).

This activity was inspired by one created by Nathan Kraft https://teacher.desmos.com/activitybuilder/custom/563c039dccdd442e107a0ce2.

Addressing

- 6.NS.C.6.c

Instructional Routines

- MLR7: Compare and Connect

Launch

Give students 8 minutes quiet work time followed by a brief whole-class discussion.

If using digital materials, students will navigate around a maze using a digital applet.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities. Provide students with access to mazes superimposed over a set of axes demarcated by single units instead of two units.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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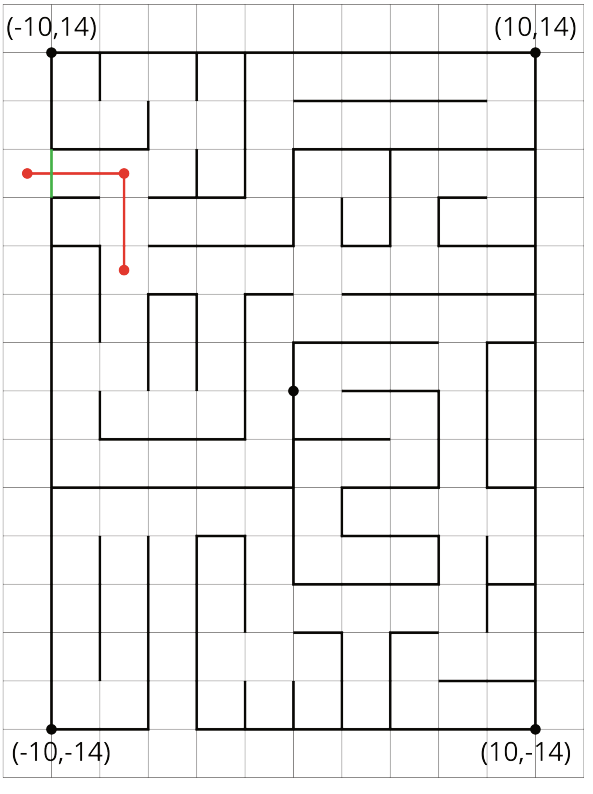
Anticipated Misconceptions

Students might disregard the fact that the side of each square grid is 2 units and just count boxes. Redirect students' attention to the relevant instruction, and ask how they will address it.

Student Task Statement

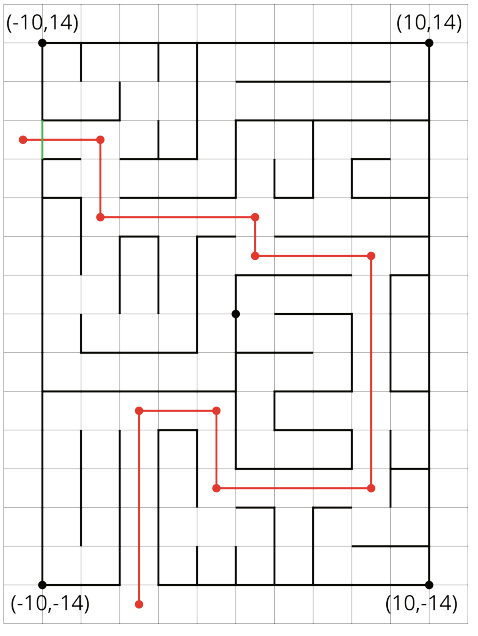
Here is a maze on a coordinate plane. The black point in the center is (0, 0). The side of each grid square is 2 units long.



1. Enter the above maze at the location marked with a green segment. Draw line segments to show your way through and out of the maze. Label each turning point with a letter. Then, list all the letters and write their coordinates.

2. Choose any 2 turning points that share the same line segment. What is the same about their coordinates? Explain why they share that feature.

Student Response

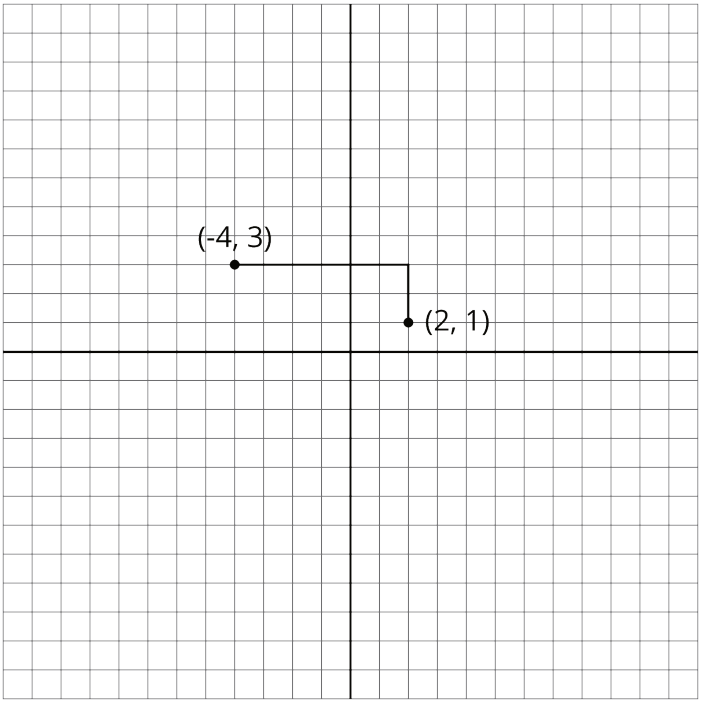


1. Start: $(-11, 9)$, A: $(-7, 9)$, B: $(-7, 5)$, C: $(1, 5)$, D: $(1, 3)$, E: $(7, 3)$, F: $(7, -9)$, G: $(-1, -9)$, H: $(-1, -5)$, I: $(-5, -5)$, J: $(-5, -15)$

2. Responses vary. Sample response: I chose points D and E. They have the same second coordinate. This is because I am moving along a straight line horizontally so my vertical position hasn't changed

Are You Ready for More?

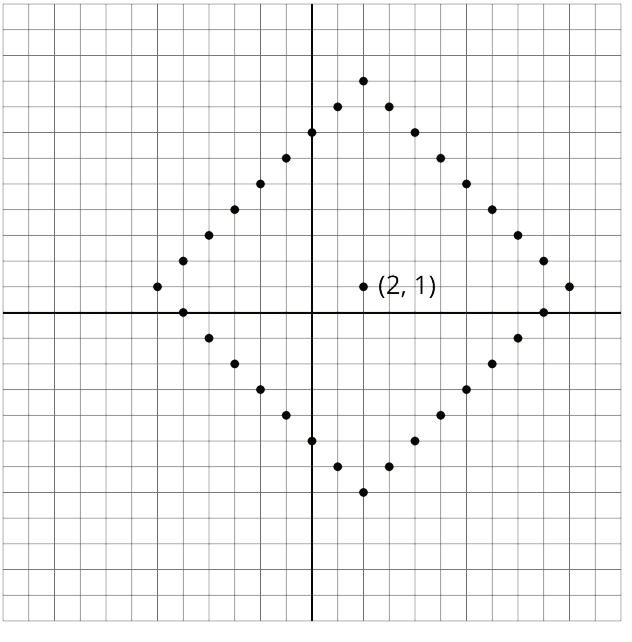
To get from the point $(2, 1)$ to $(-4, 3)$ you can go two units up and six units to the left, for a total distance of eight units. This is called the "taxicab distance," because a taxi driver would have to drive eight blocks to get between those two points on a map.



Find as many points as you can that have a taxicab distance of eight units away from $(2, 1)$. What shape do these points make?

Student Response

The points form a square with vertices $(10, 1)$, $(2, 9)$, $(-6, 1)$, and $(2, -7)$.



Activity Synthesis

Ask students to share how they planned which points to plot and how they determined the coordinates for each point with the given information. Invite students to share their responses for

question 2 to review the idea that points on a horizontal line share the same y-coordinate and points on the same vertical line share the same x-coordinate. Explain that in modern navigation, directions are precisely given in terms of coordinates. Navigation programs process coordinate data and translate it into a visual display for the driver, for example. Precise coordinates are also used to navigate through virtual space in computer simulations.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine to help students compare the various approaches used for the question, "How did you plan which points to plot and how you determined the coordinates for each point with the given information?" Prepare by looking for distinct strategies that highlight the difference between the horizontal and vertical lines and their coordinates. As students discuss their strategies, ask students to consider what information they used to decide on their coordinates and the path they chose to plot. These exchanges strengthen students' mathematical language use and reasoning about the coordinate grid.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

In this lesson, students placed and scaled axes to accommodate a variety of points with rational coordinates. To get students thinking about proper scaling, it may be helpful to start with two points plotted with axes that have been scaled improperly. Display an empty grid where both axes are labeled by 100 units from -500 to 500 and attempt to plot the points $(-2, 3)$ and $(5, 7)$. Emphasize that it is difficult to communicate information on a coordinate plane if the axes are labeled poorly. Here are some questions to consolidate what students have learned:

- When plotting $(-2, 3)$ and $(5, 7)$, how many units across would you make the x- and y-axes? How would you label the axes? (The x-axis needs to be at least 7 units across to go from -2 to 5. The y-axis needs to include values from 3 to 7, but in order to meet the x-axis, it should go at least from 0 to 7. It might look nicer to give some space in either direction, for example going from -4 to 7 in the x direction and -1 to 8 in the y direction. In this case, the grid lines could be labeled by 1 unit.)
- When plotting $(1.75, -0.5)$ and $(-2.25, 1.5)$, how many units across would you make the x- and y-axes? How would you label the axes? (The coordinates all look like multiples of 0.25, so the grid lines could be labeled by multiples of 0.25 units. The x-axis could go from -2.5 to 2 and the y-axis could go from -0.75 to 1.75.)
- When plotting $(-3, 40)$ and $(4, -60)$, how many units across would you make the x- and y-axes? How would you label the axes? (The x axis could be labeled by 1 unit and the y axis could be labeled by 10 units. The x-axis could go from -5 to 5 and the y-axis could go from -70 to 50.)

It may be helpful to display an empty grid to place and label the axes and plot the points for each example.

12.4 What Went Wrong: Graphing Edition

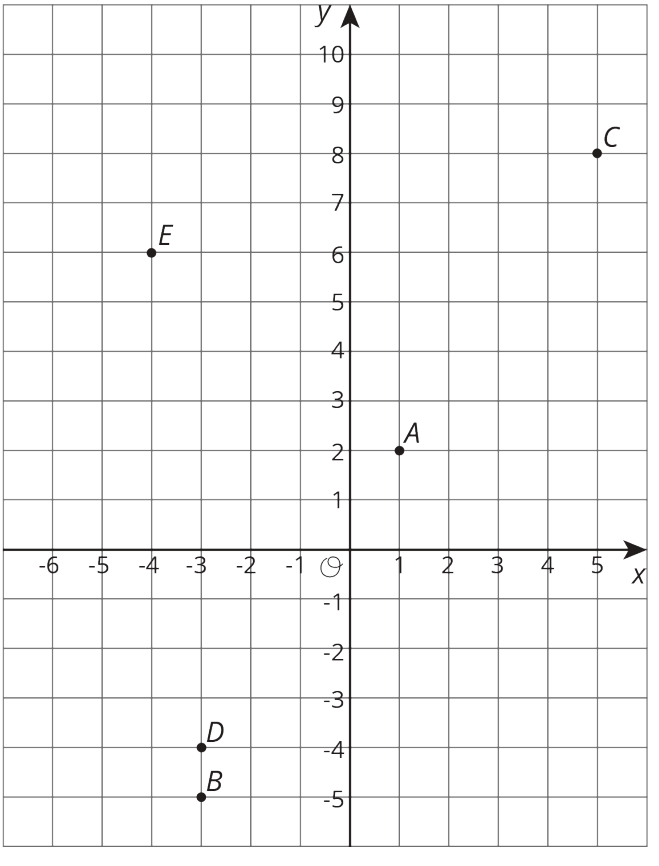
Cool Down: 5 minutes

Addressing

- 6.NS.C.6.c

Student Task Statement

Lin drew this set of axes and plotted the points $A = (1, 2)$, $B = (-3, -5)$, $C = (5, 7)$, $D = (-4, -3)$, and $E = (-4, 6)$ on them.



Identify as many mistakes as you notice in Lin's graph.

Student Response

Point C is plotted at $(5, 8)$ instead of $(5, 7)$. Point D is plotted at $(-3, -4)$ instead of $(-4, -3)$.

Student Lesson Summary

The coordinate plane can be used to show information involving pairs of numbers.

When using the coordinate plane, we should pay close attention to what each axis represents and what scale each uses.

Suppose we want to plot the following data about the temperatures in Minneapolis one evening.

time (hours from midnight)	temperature (degrees C)
-4	3
-1	-2
0	-4
3	-8

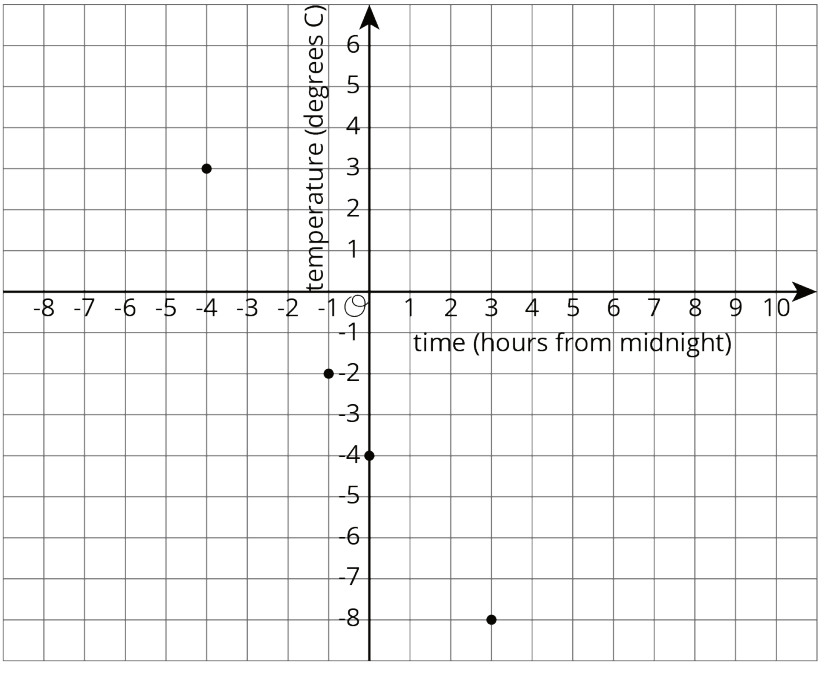
We can decide that the x-axis represents number of hours in relation to midnight and the y-axis represents temperatures in degrees Celsius.

- In this case, x-values less than 0 represent hours before midnight, and x-values greater than 0 represent hours after midnight.
- On the y-axis, the values represents temperatures above and below the freezing point of 0 degrees Celsius.

The data involve whole numbers, so it is appropriate that the each square on the grid represents a whole number.

- On the left of the origin, the x-axis needs to go as far as -4 or less (farther to the left). On the right, it needs to go to 3 or greater.
- Below the origin, the y-axis has to go as far as -8 or lower. Above the origin, it needs to go to 3 or higher.

Here is a graph of the data with the axes labeled appropriately.



On this coordinate plane, a point at $(0, 0)$ would mean a temperature of 0 degrees Celsius at midnight. The point at $(-4, 3)$ means a temperature of 3 degrees Celsius at 4 hours before midnight (or 8 p.m.).

Lesson 12 Practice Problems

Problem 1

Statement

Draw and label an appropriate pair of axes and plot the points.

$(\frac{1}{5}, \frac{4}{5})$

$(-\frac{3}{5}, \frac{2}{5})$

$(-1\frac{1}{5}, -\frac{4}{5})$

$(\frac{1}{5}, -\frac{3}{5})$

Solution

Answers vary. Check student work to ensure they made reasonable choices about axes and scale that allowed them to clearly plot all the points.

Problem 2

Statement

Diego was asked to plot these points: $(-50, 0)$, $(150, 100)$, $(200, -100)$, $(350, 50)$, $(-250, 0)$. What interval could he use for each axis? Explain your reasoning.

Solution

Answers vary. Sample response: Use an interval of 50, because all the coordinates involve points that are greater than 50 and multiples of 50.

Problem 3

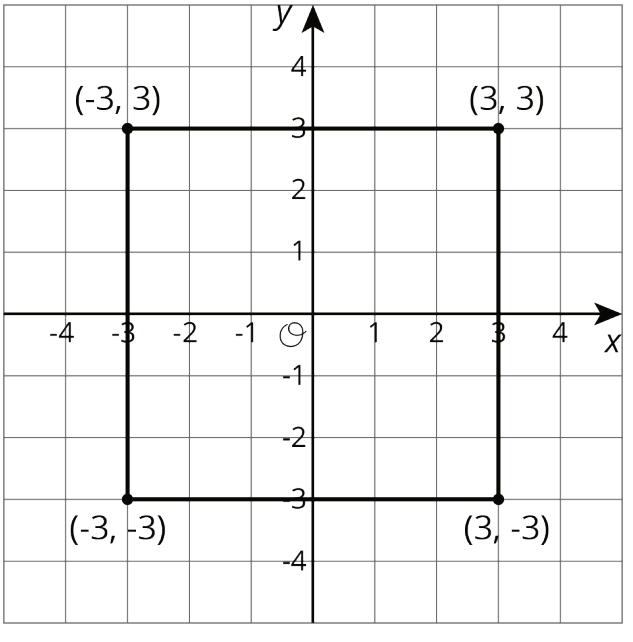
Statement

a. Name 4 points that would form a square with the origin at its center.

b. Graph these points to check if they form a square.

Solution

Answers vary. Sample response: $A = (3, 3)$, $B = (3, -3)$, $C = (-3, 3)$, $D = (-3, -3)$



Problem 4

Statement

Which of the following changes would you represent using a negative number? Explain what a positive number would represent in that situation.

a. A loss of 4 points

b. A gain of 50 yards

c. A loss of $10

d. An elevation above sea level

Solution

Answers vary. Sample response:

1 and 3 can be represented with negative numbers. A loss of 4 points is -4, but if any points are gained, the value becomes positive. A loss of $10 is -10, but if any money is earned, the value becomes positive.

(From Unit 7, Lesson 5.)

Problem 5

Statement

Jada is buying notebooks for school. The cost of each notebook is $1.75.

a. Write an equation that shows the cost of Jada's notebooks, c, in terms of the number of notebooks, n, that she buys.

b. Which of the following could be points on the graph of your equation?

$(1.75, 1)$ $(2, 3.50)$ $(5, 8.75)$ $(17.50, 10)$ $(9, 15.35)$

Solution

a. $c = 1.75n$

b. b and c

(From Unit 6, Lesson 16.)

Problem 6

Statement

A corn field has an area of 28.6 acres. It requires about 15,000,000 gallons of water. About how many gallons of water per acre is that?

A. 5,000

B. 50,000

C. 500,000

D. 5,000,000

Solution

C

(From Unit 5, Lesson 13.)