# Lesson 15: Adding and Subtracting with Scientific Notation

## Goals

- Generalize (orally and in writing) a process of adding and subtracting numbers in scientific notation and interpret results in context.

## Learning Targets

- I can add and subtract numbers given in scientific notation.

## Lesson Narrative

Students add and subtract with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students must make sense and use quantitative reasoning when making comparisons, for example, when comparing whether 5 planets side by side are wider than the Sun (MP1, MP2).

### Alignments

### Addressing

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Number Talk

> **Student Learning Goals**
>
> Let’s add and subtract using scientific notation to answer questions about animals and the solar system.

# 15.1 Number Talk: Non-zero Digits

**Warm Up: 10 minutes**

The purpose of this Number Talk is to elicit strategies and understandings students have for addition, subtraction, multiplication, and division. These understandings help students develop fluency and will be helpful later in this lesson when students compute with numbers in scientific notation. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

### Addressing

- 8.EE.A.4

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 1 minute of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organization*

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### Anticipated Misconceptions

Students may write $3 \times 10^9 - 2 \times 10^7 = 1 \times 10^2$, or something similar. Ask these students to evaluate each product first before subtracting.

### Student Task Statement

Mentally decide how many non-zero digits each number will have.

$(3 \times 10^9)(2 \times 10^7)$

$(3 \times 10^9) \div (2 \times 10^7)$

$3 \times 10^9 + 2 \times 10^7$

$3 \times 10^9 - 2 \times 10^7$

### Student Response

- One non-zero digit. Multiplying 3 and 2 gives us 6, and the rest will just add a bunch of zeros.
- Two non-zero digits. Dividing 3 by 2 gives us 1.5, and the rest will just move the decimal place and add a bunch of zeros.
- Two non-zero digits. The first digit on the left will be a 3 and the third digit will be a 2.

- Three non-zero digits. The first digit on the left will be a 2, the second digit will be a 9, and the third will be an 8.

### Activity Synthesis

Ask the class how the first two questions are different than the second two questions. Note that we have to pay more attention to place value to answer the second two than the first two because when we add or subtract, we can only add or subtract digits that correspond to the same powers of 10. Consider asking:

- “Which problem was easier? Why?”
- “Of the four operations, which operations are easier to do with scientific notation? Which are harder?” (Multiplying and dividing is easier with scientific notation, because the exponent rules involve multiplication and division. Addition and subtraction are harder, because the exponent rules don’t involve those operations.)

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#### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
*Design Principle(s): Optimize output (for explanation)*

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# 15.2 Measuring the Planets

15 minutes
Students attend to precision when adding numbers in scientific notation, taking care that the numbers are first written as a decimal or with powers of 10 with the same exponent (MP6). Students critique the reasoning of Diego, Clare, and Kiran as they make sense of adding numbers in scientific notation (MP3).

### Addressing

- 8.EE.A.4

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Arrange students in groups of 2. Give 10–12 minutes to work followed by a whole-class discussion. Encourage students to share their thinking with a partner as they work.

### Access for Students with Disabilities

*Representation: Access for Perception.* Read the student task statement aloud. Students who both listen to and read the information will benefit from extra processing time.
*Supports accessibility for: Language*

## Anticipated Misconceptions

Students may make various mistakes that show a misunderstanding of exponent rules, for example multiplying the exponents like $4.7 \times 10^4 + 1.2 \times 10^5 = 5.9 \times 10^{20}$ or adding exponents like $4.7 \times 10^4 + 1.2 \times 10^5 = 1.67 \times 10^9$. For these students, emphasize that the terms are being added, not multiplied, so the factors that are 10 are not being grouped in the same way.

## Student Task Statement

Diego, Kiran, and Clare were wondering:

"If Neptune and Saturn were side by side, would they be wider than Jupiter?"

1. They try to add the diameters, $4.7 \times 10^4$ km and $1.2 \times 10^5$ km. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.
   a. Diego says, "When we add the distances, we will get $4.7 + 1.2 = 5.9$. The exponent will be 9. So the two planets are $5.9 \times 10^9$ km side by side."

   b. Kiran wrote $4.7 \times 10^4$ as 47,000 and $1.2 \times 10^5$ as 120,000 and added them:
$$\begin{array}{r} 120{,}000 \\ +47{,}000 \\ \hline 167{,}000 \end{array}$$

   c. Clare says, "I think you can't add unless they are the same power of 10." She adds $4.7 \times 10^4$ km and $12 \times 10^4$ to get $16.7 \times 10^4$.

2. Jupiter has a diameter of $1.43 \times 10^5$. Which is wider, Neptune and Saturn put side by side, or Jupiter?

## Student Response

1. a. Diego is incorrect, because their decimal place values do not match. It would be analogous to writing $47 + 1.2 = 59$.

   b. Kiran correctly added the two distances, but his answer is hard to compare to the width of Jupiter because it is not in scientific notation.

c. Clare also correctly added the two distances, and her answer is also hard to compare to the width of Jupiter because it is not in scientific notation.

2. Neptune and Saturn side by side are wider than Jupiter, because $1.67 \times 10^5 > 1.43 \times 10^5$.

### Activity Synthesis

Select students to share their reasoning about how to add $4.7 \times 10^4$ km and $1.2 \times 10^5$ km. It is important that students understand that $4.7 \times 10^4$ and $1.2 \times 10^5$ are off by roughly a factor of 10. In order to compare them, they either have to be written as decimal numbers or with the same power of 10.

Discuss some of the following questions:

- "How are Clare's and Kiran's approaches alike?" (They both reached the same sum by attending to place value.)
- "How are their approaches different?" (Kiran wrote the numbers as decimals and added them. Clare wrote the numbers with the same power of 10 and added them. Kiran's method might not work very well if the numbers are very large or very small. The decimal form of those number would be unwieldy. )
- "Why must the terms have the same power of 10 to be added?" (We can only add digits that are of the same place value. If the powers of 10 are different, the place values of the digits in the first factors of the two expressions would be different. For example, the 4 in $4.7 \times 10^4$ means 4 ten-thousands and the 1 in $1.2 \times 10^5$ means 1 hundred-thousand, so we cannot add 4.7 and 1.2.)
- "How might Clare have reasoned that $1.2 \times 10^5$ can be written as $12 \times 10^4$?" (One way is to see that changing 1.2 into 12 requires multiplying by 10. To keep the value of the expression the same, we must divide it by 10, which decreases the exponent by 1, from $10^5$ to $10^4$.)
- "How did you compare Clare and Kiran's results to the width of Jupiter?" (Converting the widths to scientific notation makes it easy to compare.)

#### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.
*Design Principle(s): Support sense-making*

# 15.3 A Celestial Dance

15 minutes

In this activity, students add quantities written in scientific notation in order to answer questions in context. To add numbers in scientific notation, students must attend to precision by aligning place value (MP6).

As students work, notice the different strategies used to align place value. One strategy would be to convert all the distances to decimal, align the place values vertically, and then add in the usual way. Another example would be to rewrite all the addends to use the same power of 10 before adding.

### Addressing

- 8.EE.A.4

### Instructional Routines

- MLR5: Co-Craft Questions

### Launch

Arrange students in groups of 2. Tell students to discuss their thinking with a partner and work to reach agreement. Give students 12 minutes to work, followed by a brief whole-class discussion.

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#### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge of working with very large numbers. Allow students to use calculators to ensure inclusive participation in the activity. *Supports accessibility for: Memory; Conceptual processing*

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#### Access for English Language Learners

*Writing, Conversing: MLR5 Co-Craft Questions.* Display only the table, and ask pairs of students to write possible questions that could be answered by the data in the table. Invite pairs to share their questions with the class. Highlight questions that involve adding or subtracting values. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in scientific notation prior to being asked to solve questions based on the values.
*Design Principle(s): Maximize meta-awareness; Support sense-making*

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### Student Task Statement

| object | diameter (km) | distance from the Sun (km) |
|---|---|---|
| Sun | $1.392 \times 10^6$ | $0 \times 10^0$ |
| Mercury | $4.878 \times 10^3$ | $5.79 \times 10^7$ |
| Venus | $1.21 \times 10^4$ | $1.08 \times 10^8$ |
| Earth | $1.28 \times 10^4$ | $1.47 \times 10^8$ |
| Mars | $6.785 \times 10^3$ | $2.28 \times 10^8$ |
| Jupiter | $1.428 \times 10^5$ | $7.79 \times 10^8$ |

1. When you add the distances of Mercury, Venus, Earth, and Mars from the Sun, would you reach as far as Jupiter?
2. Add all the diameters of all the planets except the Sun. Which is wider, all of these objects side by side, or the Sun? Draw a picture that is close to scale.

### Student Response

1. The sums of the distances is not enough to reach Jupiter because $5.79 \times 10^7 + 1.08 \times 10^8 + 1.47 \times 10^8 + 2.28 \times 10^8 = (0.579 + 1.08 + 1.47 + 2.28) \times 10^8$ $= 5.409 \times 10^8$ km, which is less than $7.79 \times 10^8$ km.
2. The first 5 planets side by side are not as wide as the Sun (about $\frac{1}{7}^{\text{th}}$ of the width) because $4.878 \times 10^3 + 1.21 \times 10^4 + 1.28 \times 10^4 + 6.785 \times 10^3 + 1.428 \times 10^5$ $= (0.4878 + 1.21 + 1.28 + 0.6785 + 14.28) \times 10^4 = 17.9363 \times 10^4 = 1.79363 \times 10^5$ km, which is less than $1.392 \times 10^6$ km. The scale drawing should show a large circle (the Sun) with 5 smaller planet circles (Jupiter much larger than the others) that reach about $\frac{1}{10}$ of the way across the large circle.

### Are You Ready for More?

The emcee at a carnival is ready to give away a cash prize! The winning contestant could win anywhere from $1 to $100. The emcee only has 7 envelopes and she wants to make sure she distributes the 100 $1 bills among the 7 envelopes so that no matter what the contestant wins, she can pay the winner with the envelopes without redistributing the bills. For example, it’s possible to divide 6 $1 bills among 3 envelopes to get any amount from $1 to $6 by putting $1 in the first envelope, $2 in the second envelope, and $3 in the third envelope (Go ahead and check. Can you make $4? $5? $6?).

How should the emcee divide up the 100 $1 bills among the 7 envelopes so that she can give away any amount of money, from $1 to $100, just by handing out the right envelopes?

### Student Response

$1 in the first envelope, $2 in the second, $4 in the third, and increasing powers of two up through $32 in the sixth envelope. At this point, there are $63 total in the six envelopes, so put the remaining $37 in the seventh envelope.

### Activity Synthesis

The main point to highlight is that values given in scientific notation can be added by carefully aligning the place values of all of the addends. Select students who show different ways of aligning the place values. Record their strategies and display them for all to see. Ask students to explain how they decided to scale the objects in their drawing.

## 15.4 Old McDonald's Massive Farm

**Optional: 15 minutes**
Consider taking the time to engage with this activity if students need more experience with negative exponents and additional practice with adding quantities expressed in scientific notation. Students work with positive and negative exponents simultaneously.

### Addressing

- 8.EE.A.4

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Arrange students in groups of 2. Tell students to explain their thinking to their partner and work to reach agreement. Give students 10-12 minutes to work followed by a whole-class discussion.

> **Student Task Statement**
>
> Use the table to answer questions about different life forms on the planet.

| creature | number | mass of one individual (kg) |
| --- | --- | --- |
| humans | $7.5 \times 10^9$ | $6.2 \times 10^1$ |
| cows | $1.3 \times 10^9$ | $4 \times 10^2$ |
| sheep | $1.75 \times 10^9$ | $6 \times 10^1$ |
| chickens | $2.4 \times 10^{10}$ | $2 \times 10^0$ |
| ants | $5 \times 10^{16}$ | $3 \times 10^{-6}$ |
| blue whales | $4.7 \times 10^3$ | $1.9 \times 10^5$ |
| antarctic krill | $7.8 \times 10^{14}$ | $4.86 \times 10^{-4}$ |
| zooplankton | $1 \times 10^{20}$ | $5 \times 10^{-8}$ |
| bacteria | $5 \times 10^{30}$ | $1 \times 10^{-12}$ |

1. On a farm there was a cow. And on the farm there were 2 sheep. There were also 3 chickens. What is the total mass of the 1 cow, the 2 sheep, the 3 chickens, and the 1 farmer on the farm?
2. Make a conjecture about how many ants might be on the farm. If you added all these ants into the previous question, how would that affect your answer for the total mass of all the animals?
3. What is the total mass of a human, a blue whale, and 6 ants all together?
4. Which is greater, the number of bacteria, or the number of all the other animals in the table put together?

### Student Response

1. 588 kg, because the sum of the masses of 1 cow, 2 sheep, 3 chickens, and 1 farmer is $4 \times 10^2 + 2(6 \times 10^1) + 3(2 \times 10^0) + 6.2 \times 10^1 = 400 + 120 + 6 + 62 = 588$ kg.
2. Answers vary. Sample response: Suppose there are roughly $10^6$ ants on the farm. The mass of these ants is $3 \times 10^{-6}$ kg per ant times $10^6$ ants, which is equal to 3 kg. The new total mass would be 591 kg.
3. 190,062.000018 kg, because the sum of the masses of 1 human, 1 blue whale, and 6 ants is $6.2 \times 10^1 + 1.9 \times 10^5 + 6(3 \times 10^{-6}) = 62 + 190,000 + 18 \times 10^{-6} = 190,062.000018$ kg.
4. The number of bacteria is greater. Strategies vary. Sample strategies:

- There are 8 entries in the list that are not bacteria, and the second-highest number of individuals is $1 \times 10^{20}$, so the sum of all the individuals must be less than $8 \times 10^{20}$, which is still much less than the number of bacteria.
- Adding all the non-bacteria individuals gives $1.0005078 \times 10^{20}$. There are roughly 50 billion times as many bacteria.

### Activity Synthesis

In a whole-class discussion, talk about what the difference between multiplying or dividing numbers in scientific notation and adding or subtracting numbers. Consider asking: "Which is easier? What do you have to be careful about?"

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#### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this to amplify mathematical uses of language to communicate about multiplying and dividing values in scientific notation. As students share their strategies for multiplying or dividing values in scientific notation, press for details by requesting that students challenge an idea, elaborate on an idea, or give an example of their process. Revoice student ideas to model mathematical language use in order to clarify, apply appropriate language, and involve more students.
*Design Principle(s): Support sense-making; Optimize output (for explanation)*

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## Lesson Synthesis

The purpose of this discussion is to check that students understand how to add and subtract numbers given in scientific notation.

Here are some questions for discussion:

- "In the first activity, which method did you prefer to make sense of adding two numbers in scientific notation?"
- "How is adding and subtracting with scientific notation different from multiplying and dividing? Which is easier? Why do you think that is?" (Multiplying and dividing with scientific notation is easier, because it is possible to use exponent rules to help do calculations.)
- "Is there anything you found surprising or interesting in the problems you did?"

# 15.5 Adding with Scientific Notation

**Cool Down: 5 minutes**
Students must attend to precision to make sure that all orders of magnitude are the same when adding numbers in scientific notation.

### Addressing

- 8.EE.A.4

#### Student Task Statement

Elena wants to add $(2.3 \times 10^5) + (3.6 \times 10^6)$ and writes
$(2.3 \times 10^5) + (3.6 \times 10^6) = 5.9 \times 10^6$.

Explain to Elena what her mistake was and what the correct solution is.

### Student Response

$3.83 \times 10^6$. Sample response: Elena added 2.3 and 3.6 without realizing $3.6 \times 10^6$ is over 10 times as large as $2.3 \times 10^5$. Instead, she should have done
$36 \times 10^5 + 2.3 \times 10^5 = (36 + 2.3) \times 10^5 = 38.3 \times 10^5 = 3.83 \times 10^6$.

## Student Lesson Summary

When we add decimal numbers, we need to pay close attention to place value. For example, when we calculate $13.25 + 6.7$, we need to make sure to add hundredths to hundredths (5 and 0), tenths to tenths (2 and 7), ones to ones (3 and 6), and tens to tens (1 and 0). The result is 19.95.

We need to take the same care when we add or subtract numbers in scientific notation. For example, suppose we want to find how much further Earth is from the Sun than Mercury. Earth is about $1.5 \times 10^8$ km from the Sun, while Mercury is about $5.8 \times 10^7$ km. In order to find

$$1.5 \times 10^8 - 5.8 \times 10^7$$

we can rewrite this as

$$1.5 \times 10^8 - 0.58 \times 10^8$$

Now that both numbers are written in terms of $10^8$, we can subtract 0.58 from 1.5 to find

$$0.92 \times 10^8$$

Rewriting this in scientific notation, Earth is

$$9.2 \times 10^7$$

km further from the Sun than Mercury.

# Lesson 15 Practice Problems

## Problem 1

## Statement

Evaluate each expression, giving the answer in scientific notation:

a. $5.3 \times 10^4 + 4.7 \times 10^4$

b. $3.7 \times 10^6 - 3.3 \times 10^6$

c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$

d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$

## Solution

a. $1 \times 10^{5}$

b. $4 \times 10^{5}$

c. $1.11 \times 10^{-2}$

d. $5 \times 10^{-6}$

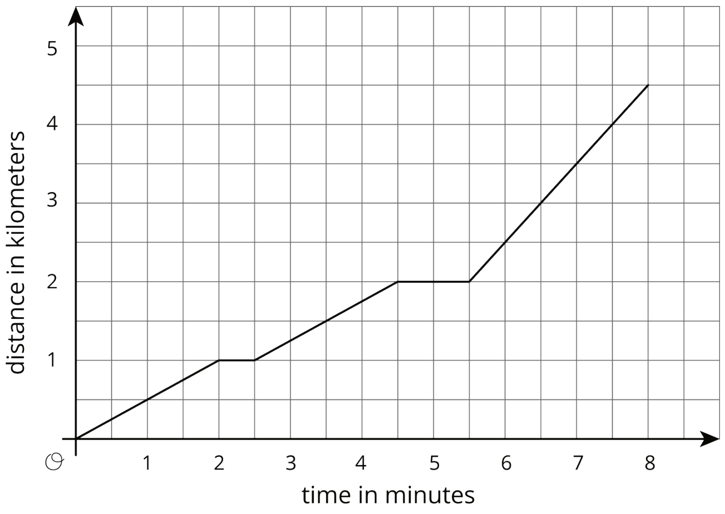
## Problem 2

### Statement

a. Write a scenario that describes what is happening in the graph.

b. What is happening at 5 minutes?

c. What does the slope of the line between 6 and 8 minutes mean?




## Solution

a. Answers vary. Sample response: A person is driving. The distance measures distance away from their house.

b. Answers vary. Sample response: The person is stopped 2 km from home.

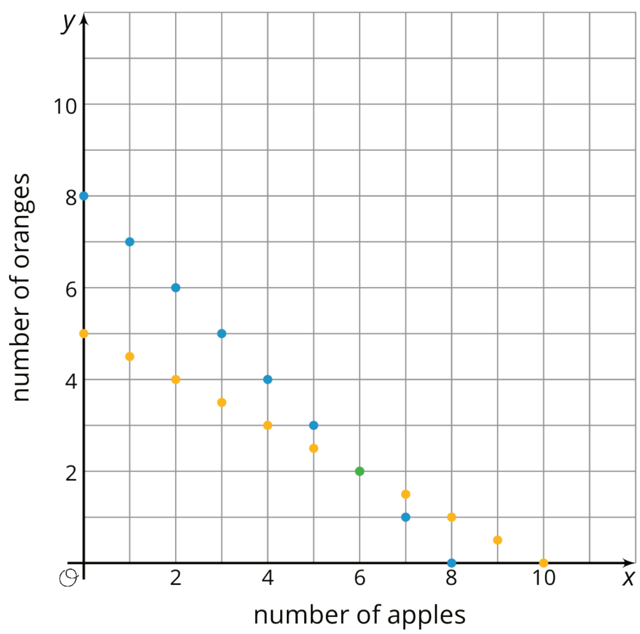
c. Answers vary. Sample response: The slope between 6 and 8 minutes indicates the speed the person is driving (1 km per minute), which is faster than any of their speeds between 0 and 6 minutes.

(From Unit 5, Lesson 10.)

## Problem 3

### Statement

Apples cost \$1 each. Oranges cost \$2 each. You have \$10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to \$10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.




a. Name one combination of 8 fruits shown on the graph that whose cost does *not* total to \$10.

b. Name one combination of fruits shown on the graph whose cost totals to \$10 that are *not* 8 fruits all together.

c. How many apples and oranges would you need to have 8 fruits that cost \$10 at the same time?

## Solution

a. Answers vary. Sample response: 4 apples, 4 oranges

b. Answers vary. Sample response: 2 apples, 4 oranges

c. 6 apples and 2 oranges

(From Unit 4, Lesson 10.)

## Problem 4

### Statement

Solve each equation and check your solution.

$-2(3x - 4) = 4(x + 3) + 6$

$\frac{1}{2}(z + 4) - 6 = -2z + 8$

$4w - 7 = 6w + 31$

## Solution

a. $x = -1$

b. $z = \frac{24}{5}$

c. $w = -19$

(From Unit 4, Lesson 5.)