Lesson 1: Positive and Negative Numbers

Goals

- Comprehend the words “positive” and “negative” (in spoken and written language) and the symbol “-” (in written language). Say “negative” when reading numbers written with the “-” symbol.
- Interpret positive and negative numbers that represent temperature or elevation, and understand the convention of what “below zero” typically means in each of these contexts.
- Recognize that the number line can be extended to represent negative numbers.

Learning Targets

- I can explain what 0, positive numbers, and negative numbers mean in the context of temperature and elevation.
- I can use positive and negative numbers to describe temperature and elevation.
- I know what positive and negative numbers are.

Lesson Narrative

Students in grade 6 have spent considerable time developing their understanding and fluency with positive numbers. In this lesson, students extend their thinking to negative numbers by exploring temperature and elevation. In these two contexts, zero represents a physical situation (freezing point of water, sea level) and numbers less than zero describe a physical state in the real world. Students abstract temperatures and elevations to positive and negative numbers on a number line (MP2).

Alignments

Addressing

- 6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR6: Three Reads

- Notice and Wonder
- Think Pair Share

Required Materials

Rulers

Required Preparation

Rulers may be helpful to create number lines in the “High Places, Low Places” activity.

> **Student Learning Goals**
>
> Let’s explore how we represent temperatures and elevations.

1.1 Notice and Wonder: Memphis and Bangor

Warm Up: 5 minutes
The purpose of this task is to introduce students to temperatures measured in degrees Celsius. Many students have an intuitive understanding of temperature ranges in degrees Fahrenheit that are typical of the city or town in which they live, but many are unfamiliar with the Celsius scale.

Addressing

- 6.NS.C.5
- 6.NS.C.6

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

> **Student Task Statement**
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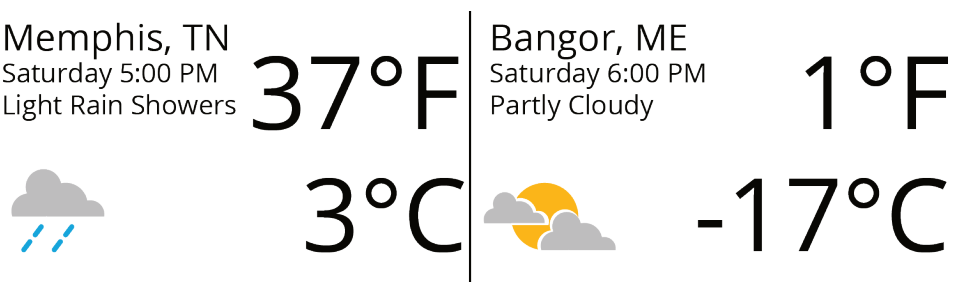
> 
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>
> What do you notice? What do you wonder?

Student Response

Things students may notice:

- The weather is different in the two cities.
- There are two temperatures for each city.
- The temperatures have different letters.
- The times are different.
- There is a minus sign on one of the temperatures.

Things students may wonder:

- What the two temperatures mean.
- Why it isn't snowing in the colder city.
- What the minus sign is doing there.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. Explain to students that temperatures are usually measured in either degrees Fahrenheit, which is what they are probably most familiar with, and degrees Celsius, which may be new for them. Tell them that many other countries measure temperature in degrees Celsius and that scientists use this temperature scale. One thing that is special about the Celsius scale is that water freezes at 0 degrees and boils at 100 degrees (at sea level).

1.2 Above and Below Zero

10 minutes (there is a digital version of this activity)
The purpose of this task is to understand that there are natural mathematical questions about certain contexts for which there are no answers if we restrict ourselves to positive numbers. The idea is to motivate the need for negative numbers and to see that there is a natural representation of them on the number line. This task is not about operations with signed numbers, but rather why we extend our number system beyond positive numbers. Students reason abstractly and quantitatively when they represent the change in temperature on a number line (MP2).

Addressing

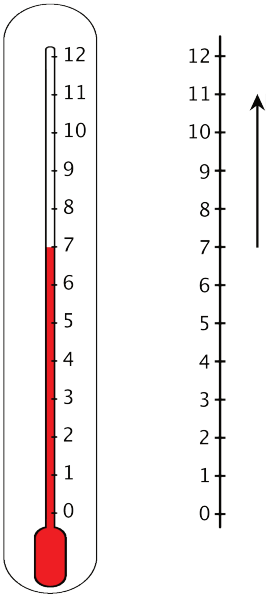
- 6.NS.C.5

Instructional Routines

- MLR6: Three Reads
- Think Pair Share

Launch

Display this image for all to see.



Tell students, “The thermometer showed a temperature of 7 degrees Celsius one morning. Later, the temperature increased 4 degrees. We can represent this change in temperature using a number line, as shown in the picture.”

Arrange students in groups of 2. Give students 2 minutes of quiet work time for question 1. Give students 2 minutes to compare their responses to their partners and to work on question 2.

Students using the digital version of the curriculum can explore the changes in temperature with the dynamic applet.

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support students' comprehension of the temperature changes for the first situation. In the first read, students read the situation with the goal of comprehending the text (e.g., the thermometer was at a certain temperature earlier in the day and then changed later in the day). Delay asking "What was the temperature late in the afternoon?". In the second read, ask students to analyze the text to understand the quantities (e.g., at noon, the temperature was 7 degrees Celsius; it increases 6 degrees by late afternoon). In the third read, direct students' attention to the question and ask students to brainstorm possible mathematical solution strategies to answer the question.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Some students may have difficulty representing change on the number line. Students sometimes count tick marks rather than counting the space between tick marks when working on a number line. For example, in the original problem image, the arrow on the number line represents a change of 4 degrees. Some students may begin at tick mark 7 and count the tick marks to yield a temperature change of 5 degrees. When reviewing that task with the whole class, be sure to make this important point and demonstrate counting on a number line by highlighting the space between the tick marks while counting out loud.

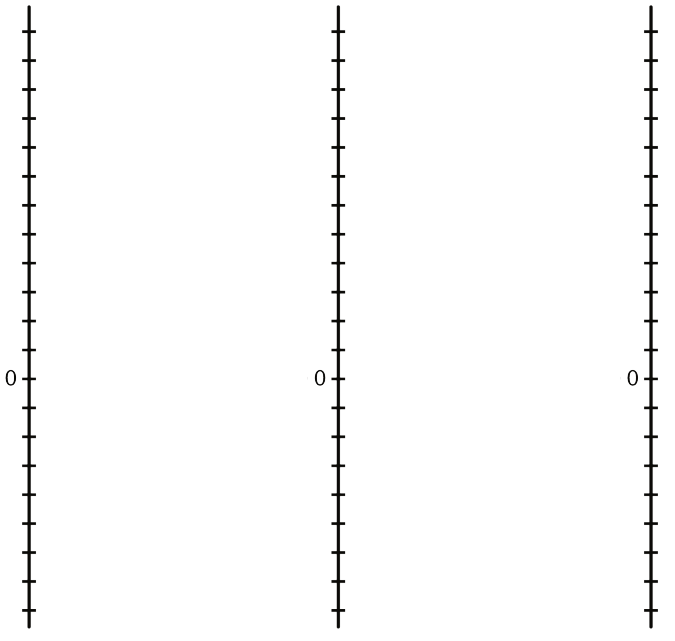
Student Task Statement

1. Here are three situations involving changes in temperature and three number lines. Represent each change on a number line. Then, answer the question.

    a. At noon, the temperature was 5 degrees Celsius. By late afternoon, it has risen 6 degrees Celsius. What was the temperature late in the afternoon?

    b. The temperature was 8 degrees Celsius at midnight. By dawn, it has dropped 12 degrees Celsius. What was the temperature at dawn?

    c. Water freezes at 0 degrees Celsius, but the freezing temperature can be lowered by adding salt to the water. A student discovered that adding half a cup of salt to a gallon of water lowers its freezing temperature by 7 degrees Celsius. What is the freezing temperature of the gallon of salt water?



2. Discuss with a partner:

   a. How did each of you name the resulting temperature in each situation?

   b. What does it mean when the temperature is above 0? Below 0?

   c. Do numbers less than 0 make sense in other contexts? Give some specific examples to show how they do or do not make sense.

Student Response

1. a. 11 degrees Celsius

   b. 4 degrees below zero, -4 degrees Celsius

   c. 7 degrees below zero, -7 degrees Celsius

2. Answers vary. Sample responses:

   a. Temperatures below 0 may be marked with a signifier labeled “below,” or equivalent.

   b. Temperatures below 0 are colder than temperatures above 0.

   c. It’s possible to go underground, so if ground level is 0, underground would be below 0. If you are counting people, on the other hand, negative numbers don’t make sense. You can’t have negative people (not mathematically, anyway).

Activity Synthesis

Some students will use the phrase “degrees below zero.” Use this activity to introduce the term **negative** as a way to represent a quantity less than zero. In contrast, ask students how they would

describe a quantity that is greater than zero on the number line. Some students will have a pre-existing understanding of positive and negative numbers. Discuss the use of + and - as symbols to denote positive and negative numbers. Notation will be important throughout the rest of this unit. Students should understand that +7 and 7 both represent positive 7. Negative 7 is represented as -7.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: negative.
Supports accessibility for: Conceptual processing; Language; Memory

1.3 High Places, Low Places

20 minutes (there is a digital version of this activity)
The purpose of this task is to present a second, natural context for negative numbers and to start comparing positive and negative numbers in preparation for ordering them. Monitor for students who make connections between elevation and temperature or come up with strategies for deciding which points are lower or higher than other points. Students may use the structure of a vertical number line in order to compare the relative location of each elevation (MP7).

Addressing

- 6.NS.C.5

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Display the table of elevations for all to see. Ask students to think of a way to explain in their own words what the numbers mean. Ask two or three students to share their ideas.

Tell students, “The term ‘elevation’ is commonly used to describe the height of a place (such as a city) or an object (such as an aircraft) compared to sea level. Denver, CO, is called ‘The Mile High City’ because its elevation is 1 mile or 5,280 feet above sea level.”

Arrange students in groups of 2 and give students 5 minutes of quiet work time to answer the first five questions. Ask students to be prepared to explain their thinking in a whole-class discussion.

Students using the digital activity are provided with an interactive map in addition to the questions about elevation. After they complete the questions in the task, they can drag each point to the elevation on the number line for the landmark it represents.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time. Check to make sure students have attended to all parts of the original figures.
Supports accessibility for: Organization; Attention

Access for English Language Learners

Reading, Writing, Speaking: MLR5 Co-Craft Questions. To help students use language related to positive and negative numbers within the context of elevation, show students the table and ask pairs to write down mathematical questions to ask about the situation. Have students share their questions with a partner and then share out with the class.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Some students may have difficulty comparing negative elevations. For example, when students are asked to find a higher elevation than Coachella, CA, they may think that -35 feet is a higher elevation than -22 feet because 35 > 22. Encourage students to create a vertical number line and plot elevations before comparing them. Alternatively, provide them with a pre-made number line to use.

Student Task Statement

1. Here is a table that shows elevations of various cities.

city	elevation (feet)
Harrisburg, PA	320
Bethell, IN	1,211
Denver, CO	5,280
Coachella, CA	-22
Death Valley, CA	-282
New York City, NY	33
Miami, FL	0

   a. On the list of cities, which city has the second highest elevation?

b. How would you describe the elevation of Coachella, CA in relation to sea level?

c. How would you describe the elevation of Death Valley, CA in relation to sea level?

d. If you are standing on a beach right next to the ocean, what is your elevation?

e. How would you describe the elevation of Miami, FL?

f. A city has a higher elevation than Coachella, CA. Select all numbers that could represent the city's elevation. Be prepared to explain your reasoning.

- -11 feet
- -35 feet
- 4 feet
- -8 feet
- 0 feet

2. Here are two tables that show the elevations of highest points on land and lowest points in the ocean. Distances are measured from sea level.

mountain	continent	elevation (meters)
Everest	Asia	8,848
Kilimanjaro	Africa	5,895
Denali	North America	6,168
Pikchu Pikchu	South America	5,664

trench	ocean	elevation (meters)
Mariana Trench	Pacific	-11,033
Puerto Rico Trench	Atlantic	-8,600
Tonga Trench	Pacific	-10,882
Sunda Trench	Indian	-7,725

a. Which point in the ocean is the lowest in the world? What is its elevation?

b. Which mountain is the highest in the world? What is its elevation?

c. If you plot the elevations of the mountains and trenches on a vertical number line, what would 0 represent? What would points above 0 represent? What about points below 0?

d. Which is farther from sea level: the deepest point in the ocean, or the top of the highest mountain in the world? Explain.

Student Response

1. a. Bethell, IN: 1,211 feet has the second highest elevation.

   b. 22 feet below sea level

   c. 282 feet below sea level

   d. My elevation would be 0 at the beach next to the sea

   e. Miami, FL, is at sea level

   f. A, C, D, and E because A is 11 feet below sea level, so it is higher than 22 feet below. C is above sea level, so it is higher. D is 8 feet below sea level, so it is higher than 22 feet below. E because it is 0 feet and that is sea level, so that is higher than 22 feet below.

2. a. Mariana Trench in the Pacific Ocean; -11,033 meters

   b. Mt. Everest in Asia; 8,848 meters

   c. 0 would represent sea level; the points above 0 would represent elevations above sea level; the points below 0 would represent elevations below sea level.

   d. The deepest point in the ocean because -11,033 meters is farther from zero on a number line than 8,848 meters.

Are You Ready for More?

A spider spins a web in the following way:

- It starts at sea level.
- It moves up one inch in the first minute.
- It moves down two inches in the second minute.
- It moves up three inches in the third minute.
- It moves down four inches in the fourth minute.

Assuming that the pattern continues, what will the spider's elevation be after an hour has passed?

Student Response

30 inches below sea level. (The pattern after each minute is +1, -1, +2, -2, etc.)

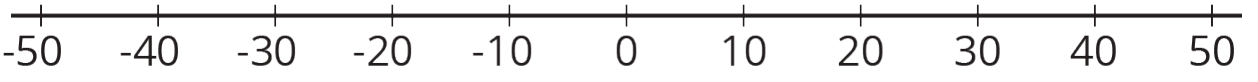
Activity Synthesis

The important concept is that elevation measures how far below or above sea level something is. Positive elevation tells us that something is above sea level, whereas negative elevation tells us that something is below sea level. In the same way, positive numbers are greater than zero and negative numbers are less than zero. Zero is neither greater than or less than zero; therefore, it is neither positive or negative. Invite selected students to share their thinking about how they compared different elevations and any similarities they may have noticed between elevation and temperature.

Lesson Synthesis

In this lesson, students considered two contexts that motivate the need for numbers less than zero. Focus their attention on what zero represents in each situation, since that choice affects the interpretation of positive and negative numbers in the context.

- What does zero represent in each situation? (freezing point of water, sea level)
- What does a positive number represent in each context? (temperatures above freezing, elevations above sea level)
- What does a negative number represent in each context? (temperatures below freezing, elevations below sea level)



- Is -30 degrees warmer or colder than -40 degrees?
- Is an elevation of -20 feet higher or lower than an elevation of -10 feet?
- In general, what is a positive number? Where are they located on a number line? (a number that is greater than zero; on the same side of 0 as 1, which is usually to the right of zero or above zero)
- In general, what is a negative number? Where are they located on a number line? (a number that is less than zero; on the opposite side of 0 as 1, which is usually to the left of zero or below zero)

1.4 Agree or Disagree?

Cool Down: 5 minutes

The purpose of this cool-down is to review the previous contexts used for working with positive and negative numbers. Students will consider distances from zero as a preview for an upcoming lesson. Students are asked to make comparisons in relation to these contexts and consider relative positions on the number line.

Addressing

- 6.NS.C.5

Student Task Statement

State whether you agree with each of the following statements. Explain your reasoning.

1. A temperature of 35 degrees Fahrenheit is as cold as a temperature of -35 degrees Fahrenheit.
2. A city that has an elevation of 15 meters is closer to sea level than a city that has an elevation of -10 meters.
3. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.

Student Response

1. Disagree. 35 degrees Fahrenheit is above 0 degrees Fahrenheit and -35 degrees Fahrenheit is below 0 degrees Fahrenheit. -35 degrees is 70 degrees colder than 35 degrees.
2. Disagree. -10 meters is 10 meters from sea level. 15 meters is 15 meters from sea level. -10 meters is closer to sea level.
3. Agree. -17 meters is 17 meters from sea level. -40 meters is 40 meters from sea level. -17 meters is closer to sea level.

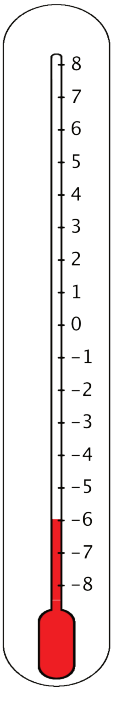
Student Lesson Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than zero. The meaning of a negative number in a context depends on the meaning of zero in that context.

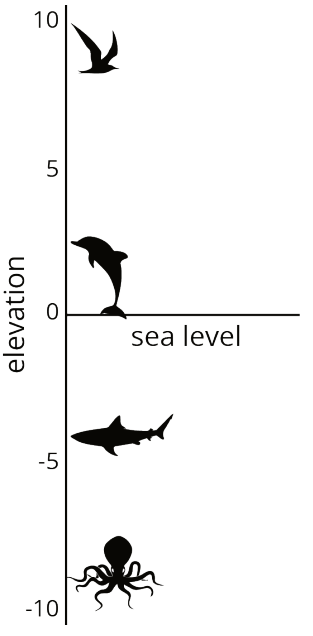
For example, if we measure temperatures in degrees Celsius, then 0 degrees Celsius corresponds to the temperature at which water freezes.

In this context, positive temperatures are warmer than the freezing point and negative temperatures are colder than the freezing point. A temperature of -6 degrees Celsius means that it is 6 degrees away from 0 and it is less than 0. This thermometer shows a temperature of -6 degrees Celsius.

If the temperature rises a few degrees and gets very close to 0 degrees without reaching it, the temperature is still a negative number.



Another example is elevation, which is a distance above or below sea level. An elevation of 0 refers to the sea level. Positive elevations are higher than sea level, and negative elevations are lower than sea level.



Glossary

- negative number
- positive number

Lesson 1 Practice Problems

Problem 1

Statement

a. Is a temperature of -11 degrees warmer or colder than a temperature of -15 degrees?

b. Is an elevation of -10 feet closer or farther from the surface of the ocean than an elevation of -8 feet?

c. It was 8 degrees at nightfall. The temperature dropped 10 degrees by midnight. What was the temperature at midnight?

d. A diver is 25 feet below sea level. After he swims up 15 feet toward the surface, what is his elevation?

Solution

a. Warmer

b. Farther

c. -2 degrees

d. -10 feet or 10 feet below sea level

Problem 2

Statement

a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?

b. The whale swims down 300 feet to feed. What is the whale's elevation now?

c. The whale swims down 150 more feet more. What is the whale's elevation now?

d. Plot each of the three elevations as a point on a vertical number line. Label each point with its numeric value.

Solution

a. 0. (Sea level is 0 feet above or below sea level.)

b. -300 feet. (The whale is 300 feet *below* sea level.)

c. -450 feet. (The whale was 300 feet *below* sea level, and now it is an additional 150 feet below sea level.)

d. A number line with 0, -300, and -450 marked.

Problem 3

Statement

Explain how to calculate a number that is equal to $\frac{2.1}{1.5}$.

Solution

Answers vary. Sample response: $\frac{2.1}{1.5}$ means $2.1 \div 1.5$. This can be done by long division. (The question doesn't require it, but the quotient is 1.4.)

(From Unit 6, Lesson 5.)

Problem 4

Statement

Write an equation to represent each situation and then solve the equation.

a. Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x for the number of ounces of water the bottle holds.

b. A bottle holds 15 ounces of water. Jada drank 8.5 ounces of water. How many ounces of water are left in the bottle? Use y for the number of ounces of water left in the bottle.

c. A bottle holds z ounces of water. A second bottle holds 16 ounces, which is $\frac{8}{5}$ times as much water. How much does the first bottle hold?

Solution

a. $\frac{3}{5}x = 15$. Solution: 25.

b. $y + 8.5 = 15$. Solution: 6.5.

c. $\frac{8}{5}z = 16$ Solution: 10. Equations equivalent to these are also acceptable.

(From Unit 6, Lesson 4.)

Problem 5

Statement

A rectangle has an area of 24 square units and a side length of $2\frac{3}{4}$ units. Find the other side length of the rectangle. Show your reasoning.

Solution

$8\frac{8}{11}$. Sample reasoning: $24 \div \frac{11}{4} = 24 \cdot \frac{4}{11} = \frac{96}{11} = 8\frac{8}{11}$.

(From Unit 4, Lesson 13.)