Lesson 3: What Are Probabilities?

Goals

- Generalize (orally) the relationship between the probability of an event and the number of possible outcomes in the sample space, for an experiment in which each outcome in the sample space is equally likely.
- List (in writing) the sample space of a simple chance experiment.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.

Learning Targets

- I can use the sample space to calculate the probability of an event when all outcomes are equally likely.
- I can write out the sample space for a simple chance experiment.

Lesson Narrative

In this lesson students begin to assign probabilities to chance events. They understand that the greater the probability, the more likely the event will occur. They define an *outcome* as a possible result for a chance experiment. They learn that the **sample space** is the set of all possible outcomes, and understand that a process is called **random** when the outcome of an experiment is based on chance. They reason that if there are n equally likely outcomes for a chance experiment, they construct the argument (MP3) that the **probability** of each of these outcomes is $\frac{1}{n}$.

In future lessons students will be asked to design and use simulations. Each lesson leading up to that helps prepare students by giving them hands-on experience with different types of chance experiments they could choose to use in their simulations. In this lesson students work with drawing paper slips out of a bag.

Alignments

Addressing

- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

- 7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- 7.SP.C.7.a: Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Paper bags
Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut up slips from the What's in the Bag? blackline master. One copy is needed for every 4 students. Each set of slips should be put into a paper bag.

Student Learning Goals

Let's find out what's possible.

3.1 Which Game Would You Choose?

Warm Up: 5 minutes
The purpose of this warm-up is for students to choose the more likely event based on their intuition about the possible outcomes of two chance experiments. The activities in this lesson that follow define probability and give ways to compute numerical values for the probability of chance events such as these.

Addressing

- 7.SP.C.5
- 7.SP.C.7.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by time to share their response with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may have trouble comparing $\frac{1}{2}$ and $\frac{2}{6}$. Review how to compare fractions with these students.

Some students may struggle with the wording of the second game. Help them understand what it means for a number to be divisible by a certain number and consider providing them with a standard number cube to examine the possible values.

> ### Student Task Statement
>
> Which game would you choose to play? Explain your reasoning.
>
> Game 1: You flip a coin and win if it lands showing heads.
>
> Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

Student Response

Answers vary. Sample response: I would rather play game 1 since half of the time the coin would land so I would win. In game 2, I only have 2 out of 6 ways to win.

Activity Synthesis

Have partners share their answers and display the results for all to see. Select at least one student for each answer provided to give a reason for their choice.

If no student mentions it, explain that the number of possible outcomes that count as a win and the number of total possible outcomes are both important to determining the likelihood of an event. That is, although there are two ways to win with the standard number cube and only one way to win on the coin, the greater number of possible outcomes in the second game makes it less likely to provide a win.

3.2 What's Possible?

15 minutes

Following the warm-up discussion in which the importance of the number of possible outcomes from an experiment is discussed, students are introduced to the term sample space. Students examine experiments to determine the set of outcomes in the sample space and then use the sample spaces to think about the likelihood of the events. Following the activity, students engage in MP6 by attaching numerical values to the likelihood of events through the word probability.

In some cases, the actual sample space is unknown. For example, in the warm-up for the first lesson of this unit, all the different types of fish in the lake may not be known. In these cases, probabilities may be less precise, but can still be estimated based on the outcomes from previous experiments. Students will begin to explore this idea in the next activity and later lessons will explore the concept in more detail.

Addressing

- 7.SP.C.7.a

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2.

Define *random* as doing something so that the outcomes are based on chance. An example is putting the integers 1 through 20 on a spinner with each number in an equal sized section. Something that is not random might be answering a multiple choice question on a test for a subject you've studied.

Explain that, for a chance experiment, each possible result is called an *outcome*. The set of all possible outcomes is called the *sample space*. Spinning a spinner with equal sized sections marked 1 through 20 has a possible outcome of 8, but neither heads nor green is a possible outcome. The sample space is made up of all integers from 1 through 20.

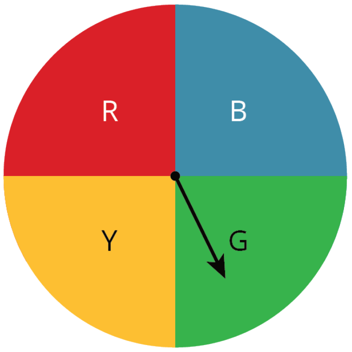
Give students 10 minutes of partner work time followed by whole-class discussion.

Student Task Statement

1. For each situation, list the **sample space** and tell how many outcomes there are.

   a. Han rolls a standard number cube once.

   b. Clare spins this spinner once.



   c. Kiran selects a letter at **random** from the word "MATH."

   d. Mai selects a letter at random from the alphabet.

   e. Noah picks a card at random from a stack that has cards numbered 5 through 20.

2. Next, compare the likelihood of these outcomes. Be prepared to explain your reasoning.

   a. Is Clare more likely to have the spinner stop on the red or blue section?

b. Is Kiran or Mai more likely to get the letter T?

c. Is Han or Noah more likely to get a number that is greater than 5?

3. Suppose you have a spinner that is evenly divided showing all the days of the week. You also have a bag of papers that list the months of the year. Are you more likely to spin the current day of the week or pull out the paper with the current month?

Student Response

1. a. Sample space: 1, 2, 3, 4, 5, 6. There are 6 outcomes.

   b. Sample space: Red, Blue, Green, Yellow. There are 4 outcomes.

   c. Sample space: M, A, T, H. There are 4 outcomes.

   d. Sample space: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. There are 26 outcomes.

   e. Sample space: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20. There are 16 outcomes.

2. a. Both are equally likely.

   b. Kiran is more likely to get a T since there are fewer possibilities in the sample space.

   c. Noah is more likely to get a number that is greater than 5, because even though he has more possibilities in the sample space, all but one of them are greater than 5.

3. It is more likely to spin the current day, since there are only 7 possible days, but 12 possible months.

Are You Ready for More?

Are there any outcomes for two people in this activity that have the same likelihood? Explain or show your reasoning.

Student Response

Answers vary. Sample response: It is equally likely that Clare will spin red and that Kiran will select an A. Since both outcomes only show up once and they both have sample spaces of 4 equally likely outcomes, these two events should be equally likely.

Activity Synthesis

Note that all of the outcomes are equally likely within each sample space. This is not always the case, but it is in these examples.

Explain that sometimes it is important to have an actual numerical value rather than a vague sense of likelihood. To answer how probable something is to happen, we assign a **probability**.

Probabilities are values between 0 and 1 and can be expressed as a fraction, decimal, or percentage. Something that has a 50% chance of happening, like a coin landing heads up, can also be described by saying, “The probability of a coin landing heads up is $\frac{1}{2}$.” or “The probability of the coin landing heads up is 0.5.”

When each outcome in the sample space is equally likely, we may calculate the probability of a desired event by dividing the number of outcomes for which the event occurs by the total number of outcomes in the sample space.

- "A standard number cube is rolled. What is the sample space?" (1, 2, 3, 4, 5, 6)
  - "How many outcomes are in the sample space?" (6)
- "What is the probability of rolling a 3? Explain your reasoning." ($\frac{1}{6}$ since there is a single 3 and 6 outcomes in the sample space.)
- "An experiment has one of each different possible outcome. The probability of getting one of the outcomes is $\frac{1}{30}$. How many outcomes are in the sample space?" (30)

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding and memory. Include the following terms and maintain the display for reference throughout the unit: random, outcome and sample space.
Supports accessibility for: Conceptual processing; Language; Memory

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Access for English Language Learners

Speaking: MLR1 Stronger and Clearer Each Time. After students decide whether it will be more likely to spin the current day of the week or the pull out the paper with the current month, ask students to write a brief explanation of their reasoning. Invite students to meet with 2–3 other partners in a row for feedback. Encourage students to ask questions such as: “How many days are there in a week?”, “How many months are there in a year?”, and “How did you determine the likelihood of spinning the current day of the week?” Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine both their reasoning and their verbal and written output.
Design Principle(s): Optimize output (for explanation)

3.3 What’s in the Bag?

15 minutes

In this activity, students are introduced to the idea that not all sample spaces are obvious before actually doing the experiment. Therefore, it is not always possible to calculate the exact probabilities for events before doing or simulating the experiment. In such situations, it is important to reason abstractly about the scenario (MP2) to gain an understanding of the situation. Students refine their guesses about the sample space by repeatedly drawing items from a bag and looking for patterns in this repetition (MP8). In later lessons, students learn how to estimate probabilities from simulations.

You will need the blackline master for this activity.

Addressing

- 7.SP.C.6
- 7.SP.C.7

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Provide each group with a paper bag containing 1 set of slips cut from the blackline master. 10 minutes partner work followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding after 3–5 minutes of work time. *Supports accessibility for: Organization; Attention*

Anticipated Misconceptions

Students may think that the phrase "equally likely" means there is a 50% chance of it happening. Tell students that, in this context, each outcome is equally likely if the probability does not change if you change the question to a different outcome in the sample space. For example, "What is the probability you get a letter A from this bag?" has the same answer as the question, "What is the probability you get a letter B from this bag?"

Student Task Statement

Your teacher will give your group a bag of paper slips with something printed on them. Repeat these steps until everyone in your group has had a turn.

- As a group, guess what is printed on the papers in the bag and record your guess in the table.

- Without looking in the bag, one person takes out one of the papers and shows it to the group.
- Everyone in the group records what is printed on the paper.
- The person who took out the paper puts it back into the bag, shakes the bag to mix up the papers, and passes the bag to the next person in the group.

	Guess the sample space.	What is printed on the paper?
person 1		
person 2		
person 3		
person 4		

1. How was guessing the sample space the fourth time different from the first?
2. What could you do to get a better guess of the sample space?
3. Look at all the papers in the bag. Were any of your guesses correct?
4. Are all of the possible outcomes equally likely? Explain.
5. Use the sample space to determine the **probability** that a fifth person would get the same outcome as person 1.

Student Response

1. Answers vary. Sample response: There was more information in round 4, so it narrowed the possibilities.
2. Answers vary. Sample response: Keep drawing out more papers.
3. Answers vary. Sample response: No, it wasn't the whole alphabet, just A through O.
4. Yes, there is one of each letter.
5. $\frac{1}{15}$ since there is one out of 15 possible things to choose from.

Activity Synthesis

The purpose of this discussion is for students to understand that often, in the real-world, we do not know the entire sample space before doing the experiment. They will learn in later lessons how to estimate the probabilities for such experiments.

Consider asking some of the following questions:

- "After the first paper is drawn, a group guesses, 'A bunch of letter Cs.' What might they have picked on their first paper that would lead to that guess? What could that group get on their second paper that would make them change their guess? Could they get something for the second paper that would make them sure their guess was right?" (They probably picked a letter C on their first draw. Any other letter on the second draw should make them change their guess. No, they might get another C that would make them think they are right, but with only two tries, there is still a good chance that other letters are in the bag.)
- "After the second paper is drawn, a group guesses, 'All of the consonants.' What might they have picked in their first two papers that would lead to that guess? What could that group get on their third paper that would make them change their guess? Could they get something that would make them more sure of their guess?" (They probably picked two consonants on their first two draws. If they picked a vowel, they would have to change their guess. If they picked another consonant, they might feel better about the guess, but should still not be certain of it.)
- "How did you refine your predictions with each round?"
- "If you had a new bag of papers and you took out papers 50 times and never got a 'Z,' would that mean there is no 'Z' in the bag?" (Not necessarily, but it might make me wonder if it's not in there.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Press students for details in their explanations by asking questions such as: “How many possible outcomes are there?”, “How do you know that all the possible outcomes are equally likely?”, and “How did you determine the probability of one of those outcomes?” Listen for and amplify the language students use to describe the sample space as one of each letter A through O for a total of 15 possible outcomes. This will a support rich and inclusive discussion about how to use the sample space to determine the probability of an outcome.
Design Principle(s): Support sense-making

Lesson Synthesis

Consider asking some of the questions:

- "If you choose one letter at random from the English alphabet, how many outcomes are in the sample space? How many outcomes are in the event that a vowel (not including Y) is chosen?" (There are 26 outcomes in the sample space. There are 5 outcomes that are vowels: A, E, I, O, and U.)
- "What is the sample space of a chance experiment? How is the number of outcomes in the sample space related to the probability of an event if the outcomes in the sample space are equally likely?" (The sample space is the list of possible outcomes for an experiment. When

there are more outcomes in the sample space, the probability of a single outcome occurring is lower.)

- "When there are 100 different outcomes in the sample space that are equally likely, what is the probability that a specific outcome will happen?" ($\frac{1}{100}$ or 1% or 0.01)

3.4 Letter of the Day

Cool Down: 5 minutes

In the cool-down, students have the opportunity to practice finding the sample space by seeing what is possible for a described experiment. They then use a sample space to write a probability for a single event. Consider having a calendar or some other resource available for students who struggle to spell the days of the week.

Addressing

- 7.SP.C.7.a

Anticipated Misconceptions

Some students may struggle to spell the days of the week. Consider having a calendar or other reference available to aid these students.

Student Task Statement

A mother decides to teach her son about a letter each day of the week. She will choose a letter from the name of the day. For example, on Saturday she might teach about the letter S or the letter U, but not the letter M.

1. What letters are possible to teach using this method? (There are 15.)
2. What are 4 letters that can't be taught using this method?
3. On TUESDAY, the mother writes the word on a piece of paper and cuts it up so that each letter is on a separate piece of paper. She mixes up the papers and picks one. What is the probability that she will choose the piece of paper with the letter Y? Explain your reasoning.

Student Response

1. A, D, E, F, H, I, M, N, O, R, S, T, U, W, Y
2. Answers vary. The complete list is: B, C, G, J, K, L, P, Q, V, X, Z
3. $\frac{1}{7}$ since there are 7 outcomes in the sample space, all outcomes are equally likely, and there is only 1 outcome that corresponds to the letter Y.

Student Lesson Summary

The **probability** of an event is a measure of the likelihood that the event will occur. Probabilities are expressed using numbers from 0 to 1.

- If the probability is 0, that means the event is impossible. For example, when you flip a coin, the probability that it will turn into a bottle of ketchup is 0. The closer the probability of some event is to 0, the less likely it is.
- If the probability is 1, that means the event is certain. For example, when you flip a coin, the probability that it will land somewhere is 1. The closer the probability of some event is to 1, the more likely it is.

If we list all of the possible outcomes for a chance experiment, we get the **sample space** for that experiment. For example, the sample space for rolling a standard number cube includes six outcomes: 1, 2, 3, 4, 5, and 6. The probability that the number cube will land showing the number 4 is $\frac{1}{6}$. In general, if all outcomes in an experiment are equally likely and there are n possible outcomes, then the probability of a single outcome is $\frac{1}{n}$.

Sometimes we have a set of possible outcomes and we want one of them to be selected at **random**. That means that we want to select an outcome in a way that each of the outcomes is *equally likely*. For example, if two people both want to read the same book, we could flip a coin to see who gets to read the book first.

Glossary

- probability
- random
- sample space

Lesson 3 Practice Problems

Problem 1

Statement

List the *sample space* for each chance experiment.

a. Flipping a coin

b. Selecting a random season of the year

c. Selecting a random day of the week

Solution

a. Heads, tails

b. Spring, summer, fall, winter

c. Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

Problem 2

Statement

A computer randomly selects a letter from the alphabet.

a. How many different outcomes are in the sample space?

b. What is the probability the computer produces the first letter of your first name?

Solution

a. 26

b. $\frac{1}{26}$

Problem 3

Statement

What is the probability of selecting a random month of the year and getting a month that starts with the letter "J?" If you get stuck, consider listing the sample space.

Solution

$\frac{3}{12}$ (or equivalent)

Problem 4

Statement

E represents an object's weight on Earth and M represents that same object's weight on the Moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

a. What does the $\frac{1}{6}$ represent in this situation?

b. Give an example of what a person might weigh on Earth and on the Moon.

Solution

a. $\frac{1}{6}$ is the constant of proportionality relating an object's weight on Earth to its weight on the Moon. Something that weighs one pound on the Earth weighs $\frac{1}{6}$ of a pound on the Moon. Or, to find weight on the Moon, multiply the weight on Earth by $\frac{1}{6}$. Or for every pound of weigh on Earth, something has $\frac{1}{6}$ of a pound of weight on the Moon.

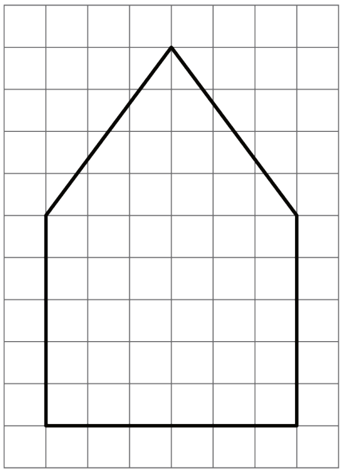
b. Answers vary. Sample response: A person who weighs 150 pounds on Earth weighs 25 pounds on the Moon.

(From Unit 2, Lesson 4.)

Problem 5

Statement

Here is a diagram of the base of a bird feeder which is in the shape of a pentagonal prism. Each small square on the grid is 1 square inch.



The distance between the two bases is 8 inches. What will be the volume of the completed bird feeder?

Solution

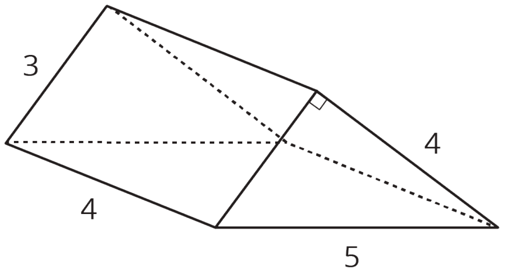
336 in^3. The area of the base is 42 in^2 because it is composed of a rectangle with area 30 in^2 and a triangle of area 12 in^2. $42 \cdot 8 = 336$.

(From Unit 7, Lesson 13.)

Problem 6

Statement

Find the surface area of the triangular prism.



Solution

60 square units

(From Unit 7, Lesson 14.)