# Lesson 3: Comparing Positive and Negative Numbers

## Goals

- Compare rational numbers in the context of temperature or elevation, and express the comparisons (in writing) using the symbols > and <.>
- Comprehend the word “sign” (in spoken language) to refer to whether a number is positive or negative.
- Critique (orally and in writing) statements comparing rational numbers, including claims about relative position and claims about distance from zero.

## Learning Targets

- I can explain how to use the positions of numbers on a number line to compare them.
- I can explain what a rational number is.
- I can use inequalities to compare positive and negative numbers.

## Lesson Narrative

Returning to the temperature context, students compare rational numbers representing temperatures and learn to write inequality statements that include negative numbers. Students then consider rational numbers in all forms (fractions, decimals) and learn to compare them by plotting on a number line and considering their relative positions. Students abstract from “hotter” and “colder” to “greater” and “less,” so if a number $a$ is to the right of a number $b$, we can write the inequality statements $a > b$ and $b < a$. Students also find that the greatest number is not always the one farthest from zero, which was the case before students encountered negative numbers. For example, -100 is much farther away from zero than $-\frac{1}{100}$, but since $-\frac{1}{100}$ is to the right of -100, it is larger and we can write $-\frac{1}{100} > -100$. Students are briefly introduced to the word **sign** (i.e., algebraic sign) since it is often used to talk about whether numbers are positive or negative. Students use the structure of the number line to reason about relationships between numbers (MP7).

### Alignments

### Building On

- 4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.
- 5.NBT.A.3.b: Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

### Addressing

- 6.NS.C.7.a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a number line oriented from left to right.
- 6.NS.C.7.b: Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{\circ}C > -7^{\circ}C$ to express the fact that $-3^{\circ}C$ is warmer than $-7^{\circ}C$.

### Building Towards

- 6.NS.C.7.a: Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that $-3$ is located to the right of $-7$ on a number line oriented from left to right.
- 6.NS.C.7.d: Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than $-30$ dollars represents a debt greater than 30 dollars.

### Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Which One Doesn't Belong?

### Student Learning Goals

Let's compare numbers on the number line.

# 3.1 Which One Doesn't Belong: Inequalities

**Warm Up: 5 minutes**

This warm-up prompts students to compare four expressions that will prime them for writing inequality statements involving signed numbers in later activities. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the inequalities in comparison to one another. To allow all students to access the activity, each inequality has one obvious reason it does not belong. During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the unit.

At this time, students might explain the direction of the inequality symbol in terms of the size of the numbers. This explanation is acceptable for students to give during this warm-up, but the next activity introduces a more correct concept of ordering that includes negative numbers.

### Building On

- 4.NBT.A.2
- 5.NBT.A.3.b

### Building Towards

- 6.NS.C.7.a

### Instructional Routines

- Which One Doesn’t Belong?

### Launch

Arrange students in groups of 2–4. Display the questions for all to see. Ask students to indicate when they have noticed one question that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. In their groups, tell each student to share their reasoning why a particular question does not belong and together find at least one reason each question doesn’t belong.

### Student Task Statement

Which inequality doesn’t belong?

- $\frac{5}{4} < 2$
- $8.5 > 0.95$
- $8.5 < 7$
- $10.00 < 100$

### Student Response

Answers vary. Sample responses:

- $\frac{5}{4} < 2$ doesn’t belong because it is the only one that has a fraction.
- $8.5 > 0.95$ is the only one with a greater than symbol or doesn’t belong because it is the only one with two decimals.
- $8.5 < 7$ doesn’t belong because it is the only one that is false.
- $10.00 < 100$ doesn’t belong because it is the only one comparing two whole numbers.

### Activity Synthesis

Ask each group to share one reason why a particular image does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students’ explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as “greater than” or “less than.” Also, press students on unsubstantiated claims.

# 3.2 Comparing Temperatures

**10 minutes**

The purpose of the task is for students to compare signed numbers in a real-world context and then use inequality signs accurately with negative numbers (MP2). The context should help students understand “less than” or “greater than” language. Students evaluate and critique another's reasoning (MP3).

### Addressing

- 6.NS.C.7.a
- 6.NS.C.7.b

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Allow students 5–6 minutes quiet work time followed by whole-class discussion.

#### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Some students may benefit from access to partially completed or blank number lines. Consider preparing blank number lines with just the tick marks to get students started.
*Supports accessibility for: Visual-spatial processing; Organization*

### Anticipated Misconceptions

Some students may have difficulty comparing numbers on the negative side of the number line. Have students plot the numbers on a number line in order to sequence them from least to greatest. Make it evident that temperatures become warmer (i.e., greater) as they move from left to right on the number line.

### Student Task Statement

Here are the low temperatures, in degrees Celsius, for a week in Anchorage, Alaska.

| day | Mon | Tues | Weds | Thurs | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| temperature | 5 | -1 | -5.5 | -2 | 3 | 4 | 0 |

a. Plot the temperatures on a number line. Which day of the week had the lowest low temperature?

b. The lowest temperature ever recorded in the United States was -62 degrees Celsius, in Prospect Creek Camp, Alaska. The average temperature on Mars is about -55 degrees Celsius.

i. Which is warmer, the coldest temperature recorded in the USA, or the average temperature on Mars? Explain how you know.

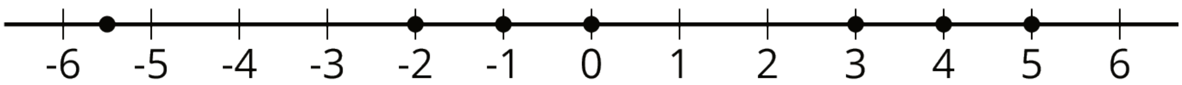
ii. Write an inequality to show your answer.

c. On a winter day the low temperature in Anchorage, Alaska, was -21 degrees Celsius and the low temperature in Minneapolis, Minnesota, was -14 degrees Celsius.

Jada said, "I know that 14 is less than 21, so -14 is also less than -21. This means that it was colder in Minneapolis than in Anchorage."

Do you agree? Explain your reasoning.

### Student Response




a. Wednesday

b. i. The average temperature on Mars is warmer. On the number line, -55 is farther to the right and closer to positive numbers than -62. This means that -55 degrees Celsius is warmer than -62 degrees Celsius.

ii. $-55 > -62$

c. Disagree. Explanations vary. Sample response: On the number line, -14 is closer to the positive numbers than -21. Positive temperatures are warmer than negative temperatures. This means that -14 degrees Celsius is warmer than -21 degrees Celsius.

### Are You Ready for More?

Another temperature scale frequently used in science is the *Kelvin scale*. In this scale, 0 is the lowest possible temperature of anything in the universe, and it is -273.15 degrees in the Celsius scale. Each $1\text{ K}$ is the same as $1^\circ\text{C}$, so $10\text{ K}$ is the same as $-263.15^\circ\text{C}$.

a. Water boils at $100^\circ\text{C}$. What is this temperature in $\text{K}$?

b. Ammonia boils at $-35.5^\circ\text{C}$. What is the boiling point of ammonia in $\text{K}$?

c. Explain why only positive numbers (and 0) are needed to record temperature in $\text{K}$.

### Student Response

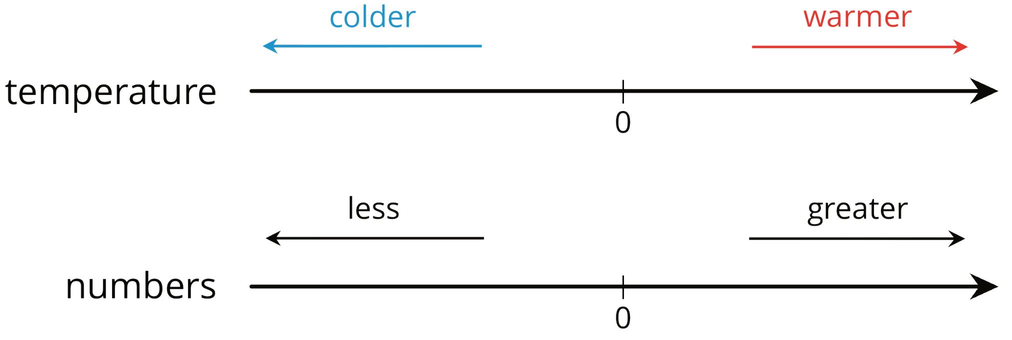
a. 373.15. Water boils at 100 degrees Celsius, and 0 Kelvin is the same as 273.15 degrees *below* zero Celsius, so these two numbers are added together to get the temperature in Kelvin.

b. 237.65. Since 0 Kelvin is 273.15 degrees Celsius below zero and the boiling point of ammonia is 35.5 degrees Celsius below zero, in Kelvin this will be $273.15 - 35.5$.

c. The temperature cannot go below absolute zero so, in Kelvin, all temperatures will be 0 or positive.

### Activity Synthesis

Ask students to explain how they could tell which of the two negative numbers was greater in problem 2. After gathering 1 or 2 responses, display the following image for all to see:




Explain to students that as we go to the right on the number line, we can think of the temperature as getting hotter, and as we go to the left, we can think of the temperature as getting colder. Numbers don't always describe temperature, though. So we use the word "greater" to describe a number that is farther to the right, and "less" to describe numbers that are farther to the left. For example, we would write $6 > -50$ and say "6 is greater than -50 because it is farther to the right on the number line." Equivalently, we could write $-50 < 6$ and say "-50 is less than 6 because -50 is farther to the left on the number line."

Ask students to make up their own list of negative numbers and plot on a number line. Then ask them to write several inequality statements with their numbers. As time allows, have students share one inequality statement with the class to see if all agree that it is true. If time allows, have students swap their list with a partner to plot and check their partner's number lines and inequality statements.

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#### Access for English Language Learners

*Speaking: MLR7 Compare and Connect.* Use this routine to support students' understanding of both the language and the symbolic notation of inequalities. Ask students to consider what is the same and what is different between the following sentences: "6 degrees is warmer than -50 degrees," "6 is greater than -50," and "$6 > -50$." This helps students make the connection between the language of the context, the written inequality statement and the symbolic inequality statement.
*Design Principle(s): Optimize output (for comparison); Maximize meta-awareness*

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# 3.3 Rational Numbers on a Number Line

**15 minutes (there is a digital version of this activity)**

The purpose of this task is for students to understand that for a given number, numbers to the left are always less than the number, and numbers to the right are always greater than the number. The precise use of the term "absolute value" is not expected at this time.

### Addressing

- 6.NS.C.7.a

### Building Towards

- 6.NS.C.7.d

### Instructional Routines

- MLR2: Collect and Display

### Launch

Allow 10 minutes quiet work time followed by whole-class discussion.

Students using the digital materials can graph the points and check them with the applet. Marks at each half, quarter, and eighth of a unit can be shown to help plot the points or to self-check for accuracy.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge about equivalent fractions. Some students may benefit from access to pre-prepared number lines to compare fractions for the final two problems.
*Supports accessibility for: Conceptual processing; Organization*

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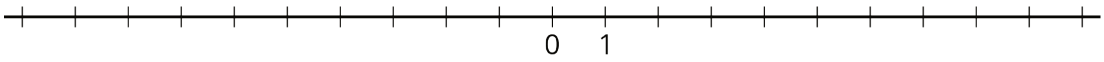
### Anticipated Misconceptions

Some students may have difficulty comparing numbers on the negative side of the number line. Have students plot the numbers on a number line from least to greatest. It may be helpful to provide an example that students can use as a visual aid while they are working independently.

Some students may have difficulty comparing fractions in question 4. Remind them that comparing fractions is easier using a common denominator. Suggest that they subdivide the interval from 0 to 1 into fourths or eighths.

### Student Task Statement

a. Plot the numbers -2, 4, -7, and 10 on the number line. Label each point with its numeric value.




b. Decide whether each inequality statement is true or false. Be prepared to explain your reasoning.

   i. $-2 < 4$

   ii. $-2 < -7$

   iii. $4 > -7$

   iv. $-7 > 10$

c. Andre says that $\frac{1}{4}$ is less than $-\frac{3}{4}$ because, of the two numbers, $\frac{1}{4}$ is closer to 0. Do you agree? Explain your reasoning.

d. Answer each question. Be prepared to explain how you know.

   i. Which number is greater: $\frac{1}{4}$ or $\frac{5}{4}$?

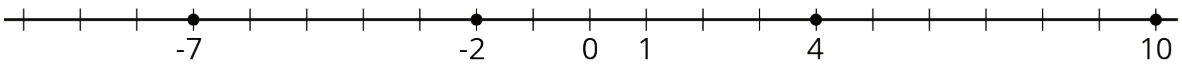
   ii. Which is farther from 0: $\frac{1}{4}$ or $\frac{5}{4}$?

   iii. Which number is greater: $-\frac{3}{4}$ or $\frac{5}{8}$?

   iv. Which is farther from 0: $-\frac{3}{4}$ or $\frac{5}{8}$?

   v. Is the number that is farther from 0 always the greater number? Explain your reasoning.

**Student Response**

a. 


b. i. $-2 < 4$ true

   ii. $-2 < -7$ false

   iii. $4 > -7$ true

   iv. $-7 > 10$ false

c. Andre is incorrect. Explanations vary. Sample response: $\frac{1}{4}$ is to the right of $-\frac{3}{4}$ so it is greater.

d. i. $\frac{5}{4}$

   ii. $\frac{5}{4}$

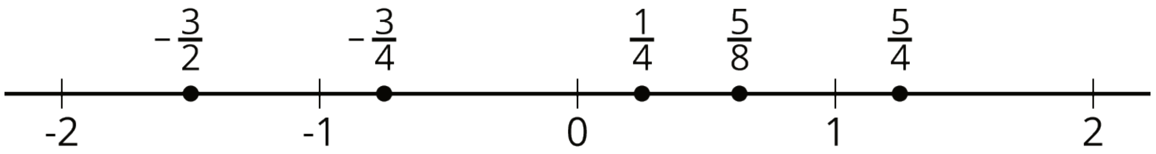
   iii. $\frac{5}{8}$

iv. $-\frac{3}{4}$

v. No. Explanations vary. Sample response: It depends on their positions relative to each other. Numbers to the right are greater than numbers to the left.

### Activity Synthesis

The key takeaway from the discussion is that now that we have numbers on both sides of 0, distance from 0 isn't enough to compare two numbers. Instead, we call numbers farther to the left on the number line "less" and numbers farther to the right "greater." Display the following number line for all to see:




One inequality at a time, ask students to indicate whether they think each of the following is true or false:

- $\frac{5}{4} > -\frac{3}{2}$ (true)
- $\frac{5}{4}$ is farther from 0 than $-\frac{3}{2}$ (false)
- $-\frac{3}{2} < -\frac{3}{4}$ (true)
- $-\frac{3}{2}$ is farther from 0 than $-\frac{3}{4}$ (true)

Invite students to share their reasoning. Here are some sentence frames that might be helpful:

- "__ is greater (less) than __ because __."
- "__ is farther from 0 than __ because __."

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#### Access for English Language Learners

*Speaking: MLR2 Collect and Display.* During the class discussion, record and display words and phrases that students use to explain why they decided certain inequality statements are true or false. Highlight phrases that include a reference to "to the right of," "to the left of," and a distance from zero. If students use gestures to support their reasoning, do your best to connect words to the gestures.
*Design Principle(s): Optimize output (for explanation); Support sense-making*

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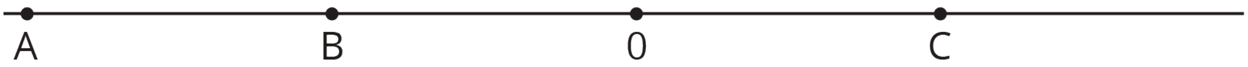
## Lesson Synthesis

Introduce the word **sign** to mean whether a number is positive or negative and give a few examples like "The sign of -3 is negative. The sign of 5 is positive." Explain that 0 has no sign because it is neither positive nor negative. Then display the number line for all to see.




- What is the sign of A? B? C? Which number is closest to 0? (negative; negative; positive; B and C both look equally close but it is hard to be sure)
- Is A greater than B? How can we write an inequality statement comparing A and B? (no; $A < B$)
- Is A less than C? How can we write an inequality statement comparing A and C? (yes; $A < C$)
- Is B equal to C? Write a statement that correctly compares B and C. (no, $B < C$ or $C > B$)
- If we plot two numbers on the number line, how can we tell which one is greater? (We call the one to the right "greater")

# 3.4 Making More Comparisons

**Cool Down: 10 minutes**

**Addressing**

- 6.NS.C.7.a
- 6.NS.C.7.b

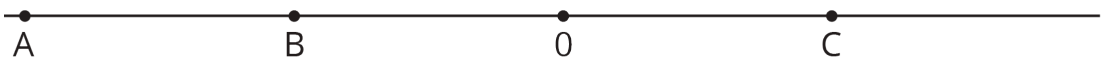
### Student Task Statement

a. The elevation of Death Valley, California, is -282 feet. The elevation of Tallahassee, Florida, is 203 feet. The elevation of Westmorland, California, is -157 feet.

   i. Compare the elevations of Death Valley and Tallahassee using $<$ or $>$.

   ii. Compare the elevations of Death Valley and Westmorland.

b. Here are the points $A$, $B$, $C$, and 0 plotted on a number line.




The points $B$ and $C$ are opposites. Decide whether each of the following statements is true.

   i. $A$ is greater than $B$.

   ii. $A$ is farther from 0 than $C$.

   iii. $A$ is less than $C$.

iv. $B$ and $C$ are equally far away from 0.

v. $B$ and $C$ are equal.

**Student Response**

a. i. -282 < 203 or 203 > -282.

ii. -157 > -282 or -282 < -157.

b. i. False. B is greater than A.

ii. True

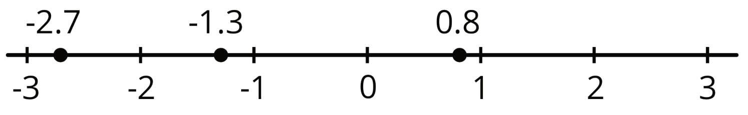
iii. True

iv. True

v. False. One is positive and one is negative.

## Student Lesson Summary

We use the words *greater than* and *less than* to compare numbers on the number line. For example, the numbers -2.7, 0.8, and -1.3, are shown on the number line.




Because -2.7 is to the left of -1.3, we say that -2.7 is less than -1.3. We write:

$$-2.7 < -1.3$$

In general, any number that is to the left of a number $n$ is less than $n$.

We can see that -1.3 is greater than -2.7 because -1.3 is to the right of -2.7. We write:

$$-1.3 > -2.7$$

In general, any number that is to the right of a number $n$ is greater than $n$

We can also see that $0.8 > -1.3$ and $0.8 > -2.7$. In general, any positive number is greater than any negative number.

## Glossary

- sign

# Lesson 3 Practice Problems

## Problem 1

### Statement

Decide whether each inequality statement is true or false. Explain your reasoning.

i. $-5 > 2$

ii. $3 > -8$

iii. $-12 > -15$

iv. $-12.5 > -12$

### Solution

i. False, -5 is to the left of 2.

ii. True, 3 is to the right of -8.

iii. True, -12 is to the right of -15.

iv. False, -12.5 is to the left of -12.

## Problem 2

### Statement

Here is a true statement: $-8.7 < -8.4$. Select **all** of the statements that are equivalent to $-8.7 < -8.4$.

A. -8.7 is further to the right on the number line than -8.4.

B. -8.7 is further to the left on the number line than -8.4.

C. -8.7 is less than -8.4.

D. -8.7 is greater than -8.4.

E. -8.4 is less than -8.7.

F. -8.4 is greater than -8.7.

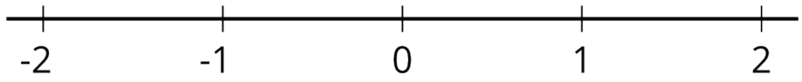
### Solution

["B", "C", "F"]

## Problem 3

### Statement

Plot each of the following numbers on the number line. Label each point with its numeric value. 0.4, -1.5, $-1\frac{7}{10}$, $-\frac{11}{10}$




### Solution

A correct solution has four points plotted in the following order from left to right: $-1\frac{7}{10}$, -1.5, $-\frac{11}{10}$ (between -2 and -1), 0.4 (between 0 and 1).

(From Unit 7, Lesson 2.)

## Problem 4

### Statement

The table shows five states and the lowest point in each state.

Put the states in order by their lowest elevation, from least to greatest.

| state | lowest elevation (feet) |
|---|---|
| California | -282 |
| Colorado | 3350 |
| Louisiana | -8 |
| New Mexico | 2842 |
| Wyoming | 3099 |

### Solution

California, Louisiana, New Mexico, Wyoming, Colorado

(From Unit 7, Lesson 4.)

## Problem 5

### Statement

Each lap around the track is 400 meters.

i. How many meters does someone run if they run:

2 laps? 5 laps? $x$ laps?

ii. If Noah ran 14 laps, how many meters did he run?

iii. If Noah ran 7,600 meters, how many laps did he run?

## Solution

i. 800 meters ($400 \cdot 2 = 800$), 2,000 meters ($400 \cdot 5 = 2,000$), $400x$ meters or equivalent

ii. 5,600 ($400 \cdot 14 = 5,600$)

iii. 19 ($7600 \div 400 = 19$)

(From Unit 6, Lesson 6.)

# Problem 6

## Statement

A stadium can seat 16,000 people at full capacity.

i. If there are 13,920 people in the stadium, what percentage of the capacity is filled? Explain or show your reasoning.

ii. What percentage of the capacity is not filled?

## Solution

i. 87% is filled, because $13{,}920 \div 16{,}000 = 0.87$.

ii. 13% remains, because $100 - 87 = 13$.

(From Unit 3, Lesson 16.)