Lesson 7: Comparing Numbers and Distance from Zero

Goals

- Critique comparisons (expressed using words or symbols) of rational numbers and their absolute values.
- Generate values that meet given conditions for their relative position and absolute value, and justify the comparisons (using words and symbols).
- Recognize that the value of -a can be positive or negative, depending on the value of a.

Learning Targets

- I can explain what absolute value means in situations involving elevation.
- I can use absolute values to describe elevations.
- I can use inequalities to compare rational numbers and the absolute values of rational numbers.

Lesson Narrative

In this lesson, students use precise language to distinguish between order and absolute value of rational numbers (MP6). It is a common mistake for students to mix up "greater" or "less" with absolute value. A confused student might say that -18 is greater than 4 because they see 18 as being the "bigger" number. What this student means to express is $|\text{-}18| > 4$. The *absolute value* of -18 is greater than 4 because -18 is more than 4 units away from 0. In the "Submarine" activity, students visualize possible elevations of characters with sticky notes on a vertical number line. The freedom to move a sticky note within a specified range anticipates the concept of a solution to an inequality in the next section.

Alignments

Addressing

- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.C.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- 6.NS.C.7: Understand ordering and absolute value of rational numbers.

- 6.NS.C.7.d: Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR4: Information Gap Cards
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Sticky notes

Required Preparation

For every 4 students, create a set of 5 sticky notes that read Clare, Andre, Han, Lin, and Priya. These are for the launch of the "Submarine" activity.

Student Learning Goals

Let's use absolute value and negative numbers to think about elevation.

7.1 Opposites

Warm Up: 10 minutes

The purpose of this warm-up is to use opposites to get students to think about distance from 0. Problem 3 also reminds students that the opposite of a negative number is positive.

Notice students who choose 0 or a negative number for a and how they reason about -a.

Addressing

- 6.NS.C.6.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Think Pair Share

Launch

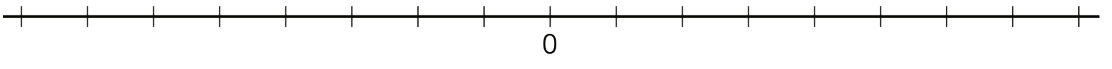
Arrange students in groups of 2. Give students 5 minutes of quiet think time, then 2 minutes of partner discussion. Follow with whole-class discussion.

Anticipated Misconceptions

For problem 3, students might assume that $-a$ is always a negative number. Ask these students to start with a negative number and find its opposite. For example, starting with $a = -3$, we can find its opposite, $-(-3)$, to be equal to 3.

Student Task Statement

1. a is a rational number. Choose a value for a and plot it on the number line.



2. a. Based on where you plotted a, plot $-a$ on the same number line.

   b. What is the value of $-a$ that you plotted?

3. Noah said, "If a is a rational number, $-a$ will always be a negative number." Do you agree with Noah? Explain your reasoning.

Student Response

1. Responses vary.
2. a. The point $-a$ will be plotted the same distance from 0 as a, but on the opposite side of 0.

   b. Responses vary. The value of $-a$ has the opposite sign as the value for a.

3. Noah is incorrect. Sample response: If a itself is negative, then $-a$ will be its opposite, which will be positive. If a is 0, then $-a$ is also 0 and neither is positive or negative.

Activity Synthesis

The main idea of discussion is that opposites have the same distance to 0 (i.e., same absolute value) and that the opposite of a negative number is positive. Ask students to discuss their reasoning with a partner. In a whole-class discussion, ask a student who chose a to be positive to share their reasoning about how to plot $-a$ and whether they agreed with Noah in problem 3. Then, select previously identified students who chose a to be negative to share their thinking. If not mentioned by students, emphasize both symbolic and geometric statements of the fact that the opposite of a negative number is positive. For example, if $a = -3$, write $-(-3) = 3$ and show that 3 is the opposite of -3 on the number line because they are the same distance to 0. If time allows, select a student who chose a to be 0 and compare to cases where a is negative or positive. The number 0 is its own opposite because no other number is 0 units away from 0. Sequencing the discussion to look at positive, negative, and 0 values of a helps students to visualize and generalize the concept of opposites for rational numbers.

7.2 Submarine

15 minutes

Students distinguish between absolute value and order in the context of elevation. Students express their ideas carefully using symbols, verbally, and using a vertical number line. Placing

possible elevations on the number line serves as a transition to thinking about solutions to inequalities. Look for students who choose positive and negative elevations for Han and Lin to compare in the discussion.

Addressing

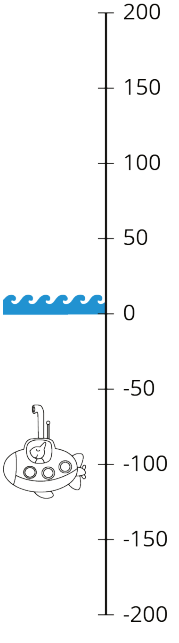
- 6.NS.C.7

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Distribute one set of sticky notes to each group, where each note contains one name: Clare, Andre, Han, Lin, and Priya. Display the image for all to see throughout the activity.



Ask students to read the instructions for the task and the description of each person's elevation. Give them a few minutes to use their sticky notes, as a group, to decide where each person (except Priya) could be located.

Place Clare's sticky note on the number line according to the completed first row of the table. Explain the completed first row of the table to students as it pertains to Clare's description. Use precise language when explaining the symbols in the table:

- One possible elevation for Clare is 150 feet because 150 is greater than -100, and it is also farther from sea level.
- 150 is greater than -100.
- The absolute value of 150 is greater than the absolute value of -100.

Ask groups to complete the rest of the table for the other people (except Priya), and then answer the question about Priya. Note that it is possible to come up with different, correct responses that fit the descriptions. Give students 10 minutes to work followed by whole-class discussion.

Access for Students with Disabilities

Representation: Access for Perception. Activate or supply background knowledge. Give students 1–2 minutes to review the first row of the table that shows a possible elevation for Clare. Invite 1–2 students to think aloud and share connections they make between the display with the sticky notes, and the values in the table. Record their thinking on a display of the table and keep the work visible as students work.
Supports accessibility for: Organization; Attention

Student Task Statement

A submarine is at an elevation of -100 feet (100 feet below sea level). Let's compare the elevations of these four people to that of the submarine:

- Clare's elevation is greater than the elevation of the submarine. Clare is farther from sea level than the submarine.
- Andre's elevation is less than the elevation of the submarine. Andre is farther away from sea level than the submarine.
- Han's elevation is greater than the elevation of the submarine. Han is closer to sea level than is the submarine.
- Lin's elevation is the same distance away from sea level as the submarine's.

1. Complete the table as follows.
    a. Write a possible elevation for each person.
    b. Use $<$, $>$, or $=$ to compare the elevation of that person to that of the submarine.
    c. Use absolute value to tell how far away the person is from sea level (elevation 0).

As an example, the first row has been filled with a possible elevation for Clare.

	possible elevation	compare to submarine	distance from sea level
Clare	150 feet	$150 > -100$	$\lvert 150 \rvert$ or 150 feet
Andre			
Han			
Lin			

2. Priya says her elevation is less than the submarine's and she is closer to sea level. Is this possible? Explain your reasoning.

Student Response

1. Responses vary. Andre could have any elevation less than -100 feet. Han could have any elevation between -100 and 100 feet. Lin could be at either -100 feet or 100 feet. Sample response:

	possible elevation	compare to submarine	distance from sea level
Clare	150 feet	$150 > -100$	$\lvert 150 \rvert$ or 150 feet
Andre	-250 feet	$-250 < -100$	$\lvert -250 \rvert$ or 250 feet
Han	20	$20 > -100$	$\lvert 20 \rvert$ or 20 feet
Lin	100	$100 > -100$	$\lvert 100 \rvert$ or 100 feet

   Sample explanations:
   - Andre could have an elevation of -250 feet because $-250 < -100$ and $\lvert -250 \rvert > \lvert -100 \rvert$. We can write $\lvert -250 \rvert = 250$ because Andre is 250 feet away from sea level.
   - Han could have an elevation of 20 feet because $20 > -100$ and $\lvert 20 \rvert < \lvert -100 \rvert$. We can write $\lvert 20 \rvert = 20$ because Han is 20 feet away from sea level.
   - Lin could have an elevation of 100 feet because $\lvert 100 \rvert = \lvert -100 \rvert$. We can write $\lvert 100 \rvert = 100$ because Lin is 100 feet away from sea level.

2. It is not possible. Sample response: Priya's description is impossible because any elevation that is less than the elevation of the submarine (below the submarine) must also be farther away from sea level.

Activity Synthesis

The purpose of the discussion is to let students practice using proper vocabulary to express ideas that distinguish order from absolute value with positive and negative numbers. Select previously identified students to share different elevations for Han and for Lin that show both positive and negative possibilities. Encourage students to explain why the elevation they chose satisfies the description in the problem. As students speak, record their statements using $<, >, =$ and $|\cdot|$. Allow students to rearrange sticky notes on the vertical number line display. If time allows, use the sticky notes to show the range of possible solutions for each character; this will help to further prepare students for the concept of graphing solutions of an inequality on the number line.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support students' use of vocabulary related to absolute value and positive and negative numbers, provide sentence frames related to each column heading. Some examples include: "_____ could have an elevation of _____ because _____," "Comparing _____'s elevation to the submarine's, I notice _____," or "_____'s distance from sea level is _____ because _____."
Design Principle(s): Cultivate conversation

7.3 Info Gap: Points on the Number Line

Optional: 15 minutes
In this info gap activity, students use comparisons of order and absolute value of rational numbers to determine the location of unknown points on the number line. In doing so students reinforce their understanding that a number and its absolute value are different properties. Students will also begin to understand that the distance between two numbers, while being positive, could be in either direction between the numbers. This concept is expanded on further when students study arithmetic with rational numbers in grade 7.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Info Gap: Points on the Number Line

Problem Card 1

The points A, B, C, and D are located on the number line. What is the location of point A?

Info Gap: Points on the Number Line

Data Card 1

- Point A has the same absolute value as B, but a different sign.
- B is less than D.
- Point C is located at -2.
- D is the opposite of C.
- The distance between B and D is $1\frac{1}{2}$.

Info Gap: Points on the Number Line

Problem Card 2

The points X, Y, and Z are located on the number line. What is the location of point Z?

Info Gap: Points on the Number Line

Data Card 2

- The absolute value of X is 2.
- Y is greater than X.
- Point Y is closer to zero than point X is.
- Z is positive.
- The distance between X and Y is 1.
- The distance between Y and Z is 4.

Addressing

- 6.NS.C.6
- 6.NS.C.7

Instructional Routines

- MLR4: Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute the first problem card to one student and a data card to the other student. After debriefing on the first problem, distribute the cards for the second problem, in which students switch roles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Access for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to determine the location of unknown points on the number line. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Students may struggle to make sense of the abstract information they are given if they don't choose to draw a number line. Rather than specifically instructing them to use this strategy, consider asking them a question like "How could you keep track of the information you've learned about the points so far?"

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Student Response

1. Point A is at $-\frac{1}{2}$.
2. Point Z is at 3.

Activity Synthesis

Select students with different strategies to share their approaches. Invite them to share which of the clues they thought were more helpful and which were least helpful. Ask students to explain how drawing a number line helped them and how they decided on the appropriate order for the unknown numbers.

7.4 Inequality Mix and Match

Optional: 15 minutes
The goal of this activity is for students to practice comparing rational numbers.

Notice students who compare fractions to decimals, fractions to integers, or who compare absolute values to negative numbers.

Addressing

- 6.NS.C.7.d

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Give students 10 minutes to work before whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Create alternatives for physically interacting with materials. Consider creating a set of cards for each of the numbers and inequality symbols that students can select from and sequence to create true comparison statements. Invite students to talk about their statements before writing them down.
Supports accessibility for: Visual-spatial processing; Conceptual processing

---

Access for English Language Learners

Speaking: MLR5 Co-Craft Questions. To create space for students to produce the language of mathematical questions themselves, display only the array of numbers that the students will be using in this activity. Ask students to think about the values of the numbers and write a mathematical question using two or more numbers from the array. Students may generate questions such as "How many values are greater than zero?" or "Which numbers are opposites?" Notice students that have questions about comparing and ordering the numbers and ask them to share their questions. This will help students use conversation skills to generate, choose, and improve their questions as well as develop meta-awareness of the language used in mathematical questions.
Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Here are some numbers and inequality symbols. Work with your partner to write true comparison statements.

-0.7	$-\frac{3}{5}$	1	4	$\lvert -8 \rvert$	$<$
$-\frac{6}{3}$	-2.5	2.5	8	$\lvert 0.7 \rvert$	$=$
-4	0	$\frac{7}{2}$	$\lvert 3 \rvert$	$\lvert -\frac{5}{2} \rvert$	$>$

One partner should select two numbers and one comparison symbol and use them to write a true statement using symbols. The other partner should write a sentence in words with the same meaning, using the following phrases:

- is equal to
- is the absolute value of
- is greater than
- is less than

For example, one partner could write $4 < 8$ and the other would write, "4 is less than 8." Switch roles until each partner has three true mathematical statements and three sentences written down.

Student Response

Responses vary. Sample mathematical responses:

- $-0.7 > -\frac{6}{3}$
- $\frac{7}{2} < 4$

- $|-8| = 8$

Sample sentence responses:

- -0.7 is greater than $-\frac{6}{3}$.
- $\frac{7}{2}$ is less than 4.
- The absolute value of -8 is 8.

Are You Ready for More?

For each question, choose a value for each variable to make the whole statement true. (When the word *and* is used in math, both parts have to be true for the whole statement to be true.) Can you do it if one variable is negative and one is positive? Can you do it if both values are negative?

1. $x < y$ and $|x| < y$.
2. $a < b$ and $|a| < |b|$.
3. $c < d$ and $|c| > d$.
4. $t < u$ and $|t| > |u|$.

Student Response

Answers vary. Sample responses:

1. $-1 < 5$ and $|-1| < 5$.
2. $-2 < 3$ and $|-2| < |3|$.
3. $-12 < -8$ and $|-12| > -8$.
4. $-10 < -1$ and $|-10| > |-1|$.

Activity Synthesis

The goal of discussion is to allow students to use precise language when comparing rational numbers and absolute values verbally. Select previously identified students to share their responses that compare fractions to decimals, fractions to integers, or absolute values to negative numbers. Display their responses using absolute value and $>, <, =$ symbols for all to see. Ask students to indicate whether they agree that each response is true, and ask students to share their reasoning about whether they agree or disagree.

Lesson Synthesis

During this lesson, students have used precise language to distinguish absolute value from order of rational numbers. Display $|-8|$ and 3 questions for all to see:

- “How do you say this?” (The absolute value of -8.)
- “What does it mean in an elevation situation?” (It’s the distance from 8 feet below sea level to sea level.)
- “What does it mean on a number line?” (It’s the distance from -8 to 0 on the number line.)
- “What is its value?” (8.)

Next, display $|-8| < 5$ and two questions for all to see:

- “How do you say this?” (The absolute value of -8 is less than 5.)
- “What does it mean on a number line?” (-8 is less than 5 units away from 0.)
- “Is it true?” (No, -8 is more than 5 units away from 0.)

7.5 True or False?

Cool Down: 5 minutes
This cool-down asks students to think about the differences between absolute value and order. The numbers -5 and 3 illustrate this difference because while -5 is greater in absolute value than 3, it is also less than 3. Encourage students who struggle with this cool-down to review absolute value, “greater than,” “less than,” and to use a number line to reason about the statements.

Addressing

- 6.NS.C.7

Student Task Statement

Mark each of the following as true or false and explain how you know.

1. $-5 < 3$
2. $-5 > 3$
3. $|-5| < 3$
4. $|-5| > 3$

Student Response

1. True, because -5 is farther to the left on the number line than 3.
2. False, because -5 is farther to the left on the number line than 3.
3. False, because $|-5| = 5$ and $5 > 3$.
4. True, because $|-5| = 5$ and $5 > 3$.

Student Lesson Summary

We can use elevation to help us compare two rational numbers or two absolute values.

- Suppose an anchor has an elevation of -10 meters and a house has an elevation of 12 meters. To describe the anchor having a lower elevation than the house, we can write $-10 < 12$ and say "-10 is less than 12."
- The anchor is closer to sea level than the house is to sea level (or elevation of 0). To describe this, we can write $|-10| < |12|$ and say "the distance between -10 and 0 is less than the distance between 12 and 0."

We can use similar descriptions to compare rational numbers and their absolute values outside of the context of elevation.

- To compare the distance of -47.5 and 5.2 from 0, we can say: $|-47.5|$ is 47.5 units away from 0, and $|5.2|$ is 5.2 units away from 0, so $|-47.5| > |5.2|$.
- $|-18| > 4$ means that the absolute value of -18 is greater than 4. This is true because 18 is greater than 4.

Lesson 7 Practice Problems

Problem 1

Statement

In the context of elevation, what would $|-7|$ feet mean?

Solution

The vertical distance between the point at -7 feet and sea level (0 feet).

Problem 2

Statement

Match the the statements written in English with the mathematical statements.

A. The number -4 is a distance of 4 units away from 0 on the number line.

B. The number -63 is more than 4 units away from 0 on the number line.

C. The number 4 is greater than the number -4.

D. The numbers 4 and -4 are the same distance away from 0 on the number line.

E. The number -63 is less than the number 4.

F. The number -63 is further away from 0 than the number 4 on the number line.

1. $|-63| > 4$
2. $-63 < 4$
3. $|-63| > |4|$
4. $|-4| = 4$
5. $4 > -4$
6. $|4| = |-4|$

Solution

- A: 4
- B: 1
- C: 5
- D: 6
- E: 2
- F: 3

Problem 3

Statement

Compare each pair of expressions using >, <, or =.

- -32 ____ 15
- $|-32|$ ____ $|15|$
- 5 ____ -5
- $|5|$ ____ $|-5|$
- 2 ____ -17
- 2 ____ $|-17|$
- $|-27|$ ____ $|-45|$
- $|-27|$ ____ -45

Solution

a. $-32 < 15$ because 15 is further right on the number line.

b. $|-32| > |15|$ because -32 is further from zero than 15.

c. $5 > -5$ because 5 is further right on the number line than -5.

d. $|5| = |-5|$, because 5 and -5 are the same distance away from zero.

e. $2 > -17$ because 2 is further right on the number line than -17.

f. $2 < |-17|$ because -17 is more than 2 units away from zero.

g. $|-27| < |-45|$ because -45 is further from zero than -27.

h. $|-27| > -45$ because $27 > -45$.

Problem 4

Statement

Mai received and spent money in the following ways last month. For each example, write a signed number to represent the change in money from her perspective.

a. Her grandmother gave her $25 in a birthday card.

b. She earned $14 dollars babysitting.

c. She spent $10 on a ticket to the concert.

d. She donated $3 to a local charity

e. She got $2 interest on money that was in her savings account.

Solution

a. +25 or 25

b. +14 or 14

c. -10

d. -3

e. + 2 or 2

(From Unit 7, Lesson 5.)

Problem 5

Statement

Here are the lowest temperatures recorded in the last 2 centuries for some US cities.

- Death Valley, CA was -45°F in January of 1937.
- Danbury, CT was -37°F in February of 1943.
- Monticello, FL was -2°F in February of 1899.
- East Saint Louis, IL was -36°F in January of 1999.

- Greenville, GA was $-17^\circ \mathrm{F}$ in January of 1940.

a. Which of these states has the lowest record temperature?

b. Which state has a lower record temperature, FL or GA?

c. Which state has a lower record temperature, CT or IL?

d. How many more degrees colder is the record temperature for GA than for FL?

Solution

a. CA

b. GA

c. CT

d. 15 degrees

(From Unit 7, Lesson 1.)

Problem 6

Statement

Find the quotients.

a. $0.024 \div 0.015$

b. $0.24 \div 0.015$

c. $0.024 \div 0.15$

d. $24 \div 15$

Solution

a. 1.4

b. 14

c. 0.14

d. 1.4

(From Unit 5, Lesson 13.)