Lesson 4: Ordering Rational Numbers

Goals

- Compare rational numbers without a context and express the comparisons using the terms "greater than," "less than," and "opposite" (orally and in writing).
- Comprehend that all negative numbers are less than all positive numbers.
- Order rational numbers from least to greatest, and explain (orally and through other representations) the reasoning.

Learning Targets

- I can compare and order rational numbers.
- I can use phrases like "greater than," "less than," and "opposite" to compare rational numbers.

Lesson Narrative

This lesson solidifies what students have learned in the past several lessons about the ordering of rational numbers on the number line. Students practice ordering rational numbers and use precise language to describe the relationships between numbers plotted on a number line (MP6). These phrases include "greater than," "less than," "negative," and "opposite."

Alignments

Building On

- 4.NBT.A.2: Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.
- 5.NBT.A.3.b: Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, =, and $<$ symbols to record the results of comparisons.

Addressing

- 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.
- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.C.6.a: Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- 6.NS.C.7: Understand ordering and absolute value of rational numbers.

Building Towards

- 6.NS.C.7: Understand ordering and absolute value of rational numbers.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Pre-printed cards, cut from copies of the blackline master

Required Preparation

Make 1 copy of the "Ordering Rational Number Cards" activity blackline master for every 2 students, and cut them up ahead of time. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let's order rational numbers.

4.1 How Do They Compare?

Warm Up: 10 minutes

The purpose of this warm-up is for students to review strategies for comparing whole numbers, decimal numbers, and fractions as well as the use of inequality symbols. The numbers in each pair have been purposefully chosen based on misunderstandings students typically have when comparing. Since there are many pairs of numbers to compare, it may not be possible to share all of the students' strategies for each pair. Consider sharing only one strategy for each pair if all of the students agree and more than one if there is a disagreement among the students.

Building On

- 4.NBT.A.2
- 5.NBT.A.3.b

Building Towards

- 6.NS.C.7

Launch

Give students 3 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

Some students may not remember the inequality symbols that represent the phrases: greater than, less than, and equal to. Show these students each of the inequality symbols in an example that they can refer back to as they work.

Student Task Statement

Use the symbols >, <, or = to compare each pair of numbers. Be prepared to explain your reasoning.

- 12 ____ 19
- 15 ____ 1.5
- 6.050 ____ 6.05
- $\frac{19}{24}$ ____ $\frac{19}{21}$
- 212 ____ 190
- 9.02 ____ 9.2
- 0.4 ____ $\frac{9}{40}$
- $\frac{16}{17}$ ____ $\frac{11}{12}$

Student Response

- 12 < 19 because 12 is farther left on the number line than 19.
- 212 > 190 because 212 is farther right on the number line than 190.
- 15 > 1.5 because 15 is 10 times farther to the right than 1.5 on the number line.
- 9.02 < 9.2 because both numbers have 9 wholes and 9.2 has 2 tenths and 9.02 doesn't have any.
- 6.050 = 6.05 because both numbers have the same number of ones, tenths, hundredths, and thousandths.
- 0.4 > $\frac{9}{40}$ because 0.4 is greater than $\frac{1}{4}$ and $\frac{9}{40}$ is less than $\frac{1}{4}$.
- $\frac{19}{24} < \frac{19}{21}$ because both fractions are the same number of pieces and $\frac{1}{21}$ is greater than $\frac{1}{24}$.
- $\frac{16}{17} > \frac{11}{12}$ because both fractions are 1 unit from a whole and $\frac{1}{17}$ is less than $\frac{1}{12}$.

Activity Synthesis

For each pair of numbers, ask one or two students to share their reasoning. Record and display their reasoning for all to see. If the whole class agrees, move on to the next question, but if there is a disagreement, ask students to explain their thinking until an agreement is reached. If possible, spend more time on the questions with numbers expressed with decimals and fractions.

4.2 Ordering Rational Number Cards

15 minutes

In this activity, students order rational numbers from least to greatest in 2 steps. They first order positive rational numbers, and then negative ones. The numbers are written as fractions, decimals, and integers. By manipulating physical cards, students get a tangible sense of how rational numbers relate to each other.

Notice conversations students have when deciding how to place fractions and decimals, especially on the negative side of the number line. Pay attention to proper use and understanding of "less" or "greater," and improper use of "bigger" or "smaller." One strategy to look for is fitting new numbers between known numbers (e.g., $-\frac{9}{8}$ is between -1 and -2).

Addressing

- 6.NS.C
- 6.NS.C.7

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- Take Turns

Launch

Arrange students in groups of 2. Distribute the first set of cards to each group. Give students 5 minutes to order the first set of cards, taking turns to place each number. When a group finishes ordering, check their ordering before giving them the second set of cards. To speed up the checking process, consider referring groups finishing the first set to compare their ordering to a group you have already checked. Give students 5 minutes to order the second set of cards followed by whole-class discussion. When collecting the cards, ask groups to separate the negative set of numbers and randomize each set for the next class.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have placed their initial cards in the correct order.
Supports accessibility for: Conceptual processing; Organization

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Anticipated Misconceptions

Some students may place negative numbers in order of increasing absolute value on the left side of 0. Ask these students to draw a number line that goes 5 units right and 5 units left of 0. Point out that negative numbers progress as -1, -2, -3, -4, -5 as they move outward from 0.

Student Task Statement

Your teacher will give you a set of number cards. Order them from least to greatest.

Your teacher will give you a second set of number cards. Add these to the correct places in the ordered set.

Student Response

-23, $-22\frac{1}{2}$, -22, -10, -9, -8, -7.5, -7, $-5\frac{1}{2}$, -5, -3, -2.5, -2, $-\frac{9}{8}$, -1, $-\frac{1}{4}$, 0, $\frac{1}{4}$, 1, $\frac{9}{8}$, 2, 2.5, $\frac{8}{3}$, 3, 4, 5, $5\frac{1}{2}$, 6, 7, 7.5, 8, 9, 10, 11, 14, 15, 16, 17, $22\frac{1}{2}$, 25, 29, 30, 53, 62, 78, 87, 99, 100.

Activity Synthesis

The purpose of the discussion is to solidify students' understanding of the order of rational numbers. Select previously identified students to share how they decided how to place numbers like $-\frac{9}{8}$, $\frac{9}{8}$, $\frac{8}{3}$, and $-22\frac{1}{2}$. Here are some questions to consider:

- Which numbers were hardest to place and which were the least difficult?
- How does placing negative numbers compare to placing positive numbers?
- How did you use numbers you had already placed to reason about where to place new numbers?

Access for English Language Learners

Speaking: MLR2 Collect and Display. During the class discussion record words and phrases that students use to explain how to order negative numbers. Highlight phrases that include a reference to "greater than," "less than," "to the right of" and "to the left of." As a class, discuss how to order -23, -22 and one half, and -22. Record student words on a visual display of a number line. This will help students read and use mathematical language during future coursework.
Design Principle(s): Support sense-making

4.3 Comparing Points on A Line

15 minutes
Students practice using relational language "greater than" and "less than" to describe order and position on number line.

Addressing

- 6.NS.C.6
- 6.NS.C.6.a

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 7 minutes of quiet work time for both problems before 3–5 minutes for partner discussion, followed by whole-class discussion.

Access for Students with Disabilities

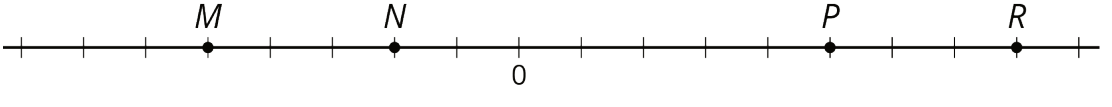
Representation: Internalize Comprehension. Activate or supply background knowledge. Some students may benefit from access to blank or partially completed number lines for the final question.
Supports accessibility for: Visual-spatial processing; Organization

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. To support students to produce statements with justification during the conversation between partners, provide student sentence frames such as "____ is greater than ____ because ____." ; "____ is the opposite of ____ because ____." ; "I know that ____ is a negative number because _____."
Design Principle(s): Optimize output (for justification)

Student Task Statement

1.



Use each of the following terms at least once to describe or compare the values of points M, N, P, R.

- greater than
- less than
- opposite of (or opposites)
- negative number

2. Tell what the value of each point would be if:

   a. P is $2\frac{1}{2}$

b. N is -0.4

c. R is 200

d. M is -15

Student Response

1. Responses vary. Sample response: R is greater than N. M is less than P. M and P are opposites. N is a negative number.

2. a. $M = -2\frac{1}{2}$, $N = -1$, $P = 2\frac{1}{2}$, $R = 4$.

   b. $M = -1$, $N = -0.4$, $P = 1$, $R = 1.6$.

   c. $M = -125$, $N = -50$, $P = 125$, $R = 200$.

   d. $M = -15$, $N = -6$, $P = 15$, $R = 24$.

Are You Ready for More?

The list of fractions between 0 and 1 with denominators between 1 and 3 looks like this:

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}$$

We can put them in order like this: $\frac{0}{1} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{1}{1}$

Now let's expand the list to include fractions with denominators of 4. We won't include $\frac{2}{4}$, because $\frac{1}{2}$ is already on the list.

$$\frac{0}{1} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{1}{1}$$

1. Expand the list again to include fractions that have denominators of 5.
2. Expand the list you made to include fractions have have denominators of 6.
3. When you add a new fraction to the list, you put it in between two "neighbors." Go back and look at your work. Do you see a relationship between a new fraction and its two neighbors?

Student Response

1. $\frac{0}{1} < \frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{1}{1}$
2. $\frac{0}{1} < \frac{1}{6} < \frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{2}{5} < \frac{1}{2} < \frac{3}{5} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5} < \frac{5}{6} < \frac{1}{1}$
3. The numerator of a new fraction is always the sum of the two numerators of its neighbors. The denominator of a new fraction is always the sum of the two denominators of its neighbors.

Activity Synthesis

The purpose for discussion is to give students the opportunity to use precise language as they compare the relative positions of rational numbers. Give students 3–5 minutes to discuss their responses with a partner before whole-class discussion. Ask students to share their partner's reasoning, especially if it was different than their own. Here are some questions to consider for whole-class discussion:

- "Did you ever have a different answer than your partner? If so, were you both correct? If not, how did you work to reach agreement?"
- "How did your partner decide the value of each unit on the number line in problem 2? Did you think of it a different way?"
- "How can we tell if two numbers are opposites?" (They are the same distance from 0.)
- "How can we tell if one number is greater or less than another number?" (Numbers toward the right are considered greater, and numbers toward the left are considered less.)

Lesson Synthesis

Ask students to summarize the ideas they have developed in the last few lessons about plotting and comparing rational numbers. Here are some questions to consider:

- "What are some situations where negative numbers make sense? What do the words 'positive,' 'negative,' and 0 mean in those situations?" (Elevation: 0 represents sea level, negative represents below sea level, and positive represents above sea level. Temperature: 0°C represents the standard freezing point of water, positive represents temperatures warmer than freezing, and negative represents temperatures below freezing.)
- "What about on the number line? What do 'positive' and 'negative' mean on the number line? Is 0 positive or negative?" (Negative numbers are numbers left of 0 on the number line, and positive numbers are to the right of 0. The number 0 is neither positive nor negative.)
- "What are some ideas you have about 'opposites?' What is the opposite of 0?" (Opposites are numbers that are the same distance from 0. They come in pairs—one positive, one negative—except for 0, which is its own opposite.)
- "How can you tell if one number is greater than or less than another? How do you write it?" (Given two rational numbers, the number toward the right on the number line is considered "greater," and the number toward the left is considered "less." We use the $<$ and $>$ symbols to indicate "less than" and "greater than," respectively.)

4.4 Getting Them in Order

Cool Down: 5 minutes
Addressing

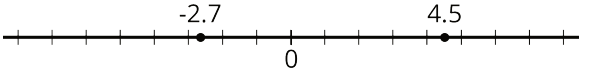
- 6.NS.C.6

Student Task Statement

1. Place these numbers in order from least to greatest:

$\frac{16}{5}$ -3 6 3.1 -2.5 $\frac{1}{4}$ $-\frac{3}{4}$ $-\frac{3}{8}$

2. Write a sentence to compare the two points shown on the number line.



Student Response

1. -3, -2.5, $-\frac{3}{4}$, $-\frac{3}{8}$, $\frac{1}{4}$, 3.1, $\frac{16}{5}$, 6

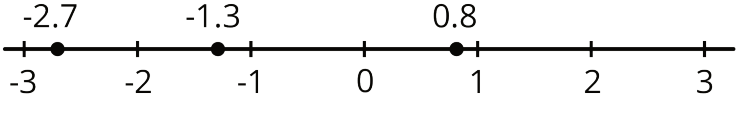
2. -2.7 is less than 4.5, or 4.5 is greater than -2.7.

Student Lesson Summary

To order rational numbers from least to greatest, we list them in the order they appear on the number line from left to right. For example, we can see that the numbers

-2.7, -1.3, 0.8

are listed from least to greatest because of the order they appear on the number line.



Lesson 4 Practice Problems

Problem 1

Statement

Select **all** of the numbers that are *greater than* -5.

A. 1.3

B. -6

C. -12

D. $\frac{1}{7}$

E. -1

F. -4

Solution

["A", "D", "E", "F"]

Problem 2

Statement

Order these numbers from least to greatest: $\frac{1}{2}$, 0, 1, $-1\frac{1}{2}$, $-\frac{1}{2}$, -1

Solution

$-1\frac{1}{2}$, -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$, 1

Problem 3

Statement

Here are the boiling points of certain elements in degrees Celsius:

- Argon: -185.8
- Chlorine: -34
- Fluorine: -188.1
- Hydrogen: -252.87
- Krypton: -153.2

List the elements from least to greatest boiling points.

Solution

Hydrogen, fluorine, argon, krypton, chlorine

Problem 4

Statement

Explain why zero is considered its own opposite.

Solution

Answer vary. Sample response: Opposites are equally distant from 0. Since 0 is the only number that is 0 units from 0, it has to be its own opposite. $0 + 0 = 0$.

(From Unit 7, Lesson 2.)

Problem 5

Statement

Explain how to make these calculations mentally.

a. $99 + 54$

b. $244 - 99$

c. $99 \cdot 6$

d. $99 \cdot 15$

Solution

Answers vary. Sample responses:

a. 153; this is one less than $100 + 54 = 154$.

b. 145; this is one more than $244 - 100 = 144$.

c. 594; this is one 6 short of 100 sixes or 600.

d. 1485; this is one 15 short of 100 fifteens or 1500.

(From Unit 6, Lesson 9.)

Problem 6

Statement

Find the quotients.

a. $\frac{1}{2} \div 2$

b. $2 \div 2$

c. $\frac{1}{2} \div \frac{1}{2}$

d. $\frac{38}{79} \div \frac{38}{79}$

Solution

a. $\frac{1}{4}$

b. 1

c. 1

d. 1

(From Unit 4, Lesson 11.)

Problem 7

Statement

Over several months, the weight of a baby measured in pounds doubles. Does its weight measured in kilograms also double? Explain.

Solution

Yes. Explanations vary. Sample explanation: The weight itself doubles, so any measurement of the weight using the same units will also double. We can also see that by saying if the weight is x pounds, then double that weight would be $2x$ pounds. The weight in kilograms will be $x \div 2.2$, and the double weight will be $(2x) \div 2.2$ or $2(x \div 2.2)$, which is also double.

(From Unit 3, Lesson 4.)