Lesson 7: One Hundred Percent

Goals

- Critique (orally and in writing) double number line diagrams that represent situations involving percent increase or decrease.
- Generalize (orally) that the original amount corresponds to 100% and the new amount corresponds to more or less than 100%, depending on whether the situation involves an increase or decrease.
- Interpret a description of a situation to identify the original amount, the new amount, the change, and corresponding percentages. Label these on a double number line diagram.

Learning Targets

- I can use a double number line diagram to help me solve percent increase and decrease problems.
- I understand that if I know how much a quantity has grown, then the original amount represents 100%.
- When I know the new amount and the percentage of increase or decrease, I can find the original amount.

Lesson Narrative

In this second lesson about percent increase and percent decrease, students work with problems where they are given the final amount after a percent increase or decrease and must calculate the original amount, or are given the final and original amounts and must calculate the percent increase or decrease. They use double number lines to visualize such situations in order to help see clearly which of the two amounts involved, the starting amount or the final amount, is to be regarded as the whole, or 100%. They explore common misconceptions resulting from getting confused about which amount is the whole. For example, if you are given the final amount after a 10% decrease, a common error is to regard that final amount as the whole and and calculate the original amount by adding 10% of the final amount. Being clear about which quantity is the whole is a good example of attending to precision (MP6).

Alignments

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's solve more problems about percent increase and percent decrease.

7.1 Notice and Wonder: Double Number Line

Warm Up: 5 minutes
The purpose of this warm-up is to elicit the idea that percent increases and decreases can be represented with double number lines, which will be useful when students use double number lines in a later activity. While students may notice and wonder many things about these images, recognizing the original amount, represented by 100%, is an important discussion point.

Building Towards

- 7.RP.A.3

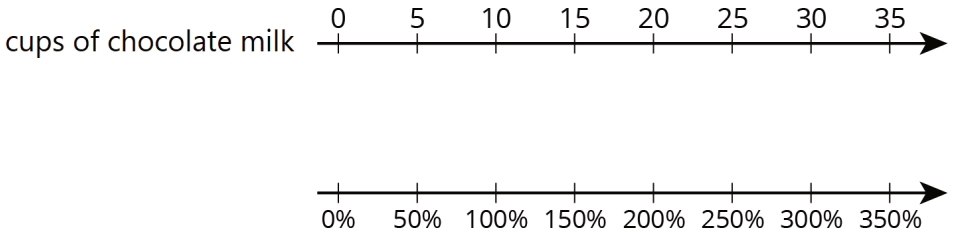
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement



What do you notice? What do you wonder?

Student Response

Things students may notice:

- If 100% is the amount needed, then 10 cups of chocolate milk is needed.
- A cup is 10% of a quantity.

Things students may wonder:

- If the number lines say how much chocolate milk is needed for a recipe, could they be used to double or triple the recipe?
- What would it mean to have negative numbers on the number lines?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the original quantity, represented by 100%, does not come up during the conversation, ask students to discuss this idea.

7.2 Double Number Lines

15 minutes

The purpose of this activity is for students to use double number line diagrams to represent situations of percent increase and decrease. Additionally, students identify the original and new amount in the double number lines to reinforce that the original amount pertains to 100%.

As students work on the task, monitor for students who created various equations for the last question.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give 5–8 minutes of quiet work time. After 5 minutes allow students to work with a partner or to continue to work alone.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

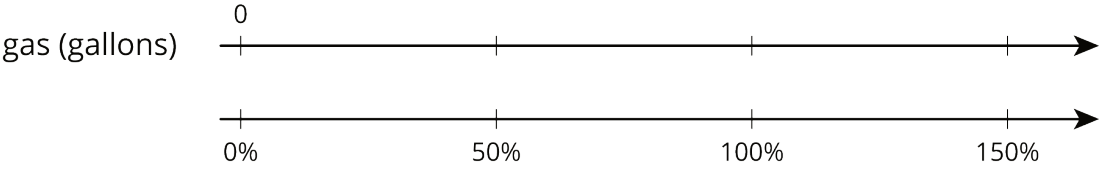
Anticipated Misconceptions

Students may continue to struggle to recognize the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

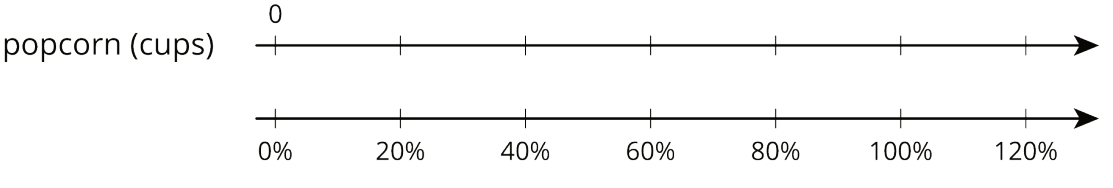
Student Task Statement

For each problem, complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.

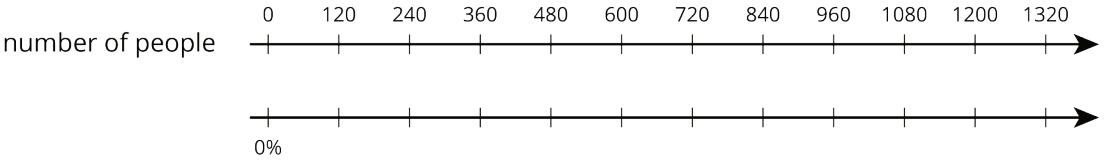
1. The gas tank in dad's car holds 12 gallons. The gas tank in mom's truck holds 50% more than that. How much gas does the truck's tank hold?



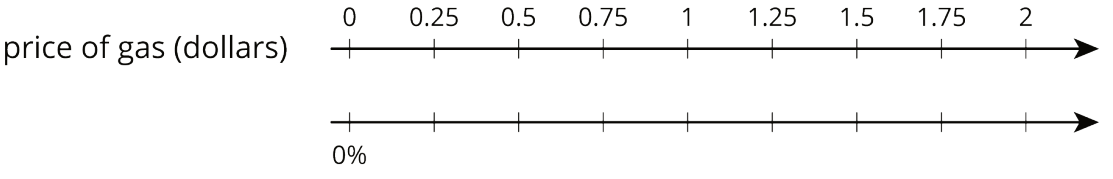
2. At a movie theater, the size of popcorn bags decreased 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?



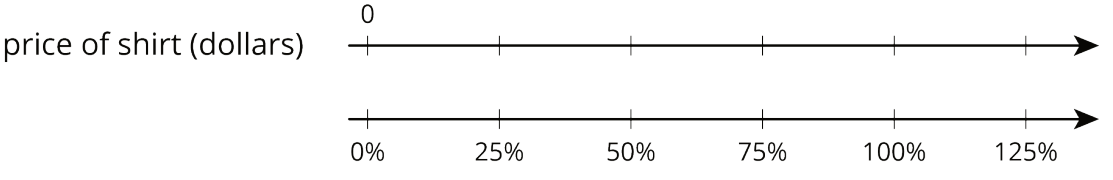
3. A school had 1,200 students last year and only 1,080 students this year. What was the percentage decrease in the number of students?



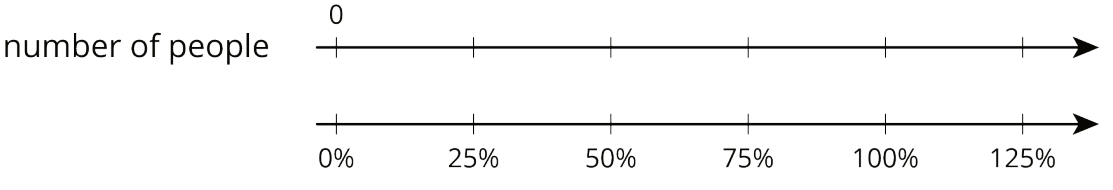
4. One week gas was $1.25 per gallon. The next week gas was $1.50 per gallon. By what percentage did the price increase?



5. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?



6. Compared to last year, the population of Boom Town has increased 25%. The population is now 6,600. What was the population last year?



Student Response

1. 18 gallons. We can put 12 on the top number line above 100% to represent the gallons of gas that dad's car tank holds. There are 2 steps on the number line to reach that number, so each step is equal to 6 gallons. It is one more step to get to the tick above 150%. Mom's truck tank holds 18 gallons because $12 + 6 = 18$.

2. 12 cups. We can write 15 on the top number line above 100% to represent the cups of popcorn the old bags held. There are 5 steps to reach 15 cups, so each step represents 3 cups. The new bags hold 80% as much popcorn, because $100 - 20 = 80$. That's one step back. So the new bags hold 12 cups of popcorn since $15 - 3 = 12$.

3. 10% decrease. We can write 100 on the bottom number line below 1,200 to represent the percentage of students the school had last year. There are 10 steps on the number line to

reach 100%, so each step represents 10%. The population of 1,080 people is one step backwards, so there was a 10% decrease.

4. 20% increase. We can write 100 on the bottom number line below 1.25 to represent the percentage of the price per gallon in the first week. There are 5 steps to reach 100%, so each step represents 20%. The price of $1.50 is one step after the price of gas the first week, so there was a 20% increase.

5. $16. After the discount, the price of the T-shirt was 75% of the price before the discount, because $100 - 25 = 75$. We can write 12 on the top number line above 75 to represent the price of the T-shirt after the discount. It takes 3 steps to reach 12, so each step represents $4. The price before the discount would be 100% of the price, which is one step later. The price before the discount was $16 because $12 + 4 = 16$.

6. 5,280 people. The population of Boom Town this year is 125% of the population last year, because $100 + 25 = 125$. We can write the number 6,600 on the top number line above the 125 to represent the population this year. It takes 5 steps to reach 6,600, so each step represents 1,320 people. The population last year would correspond to the step above 100%, which is one step before 6,600. Last year the population was 5,280 people because $6{,}600 - 1{,}320 = 5{,}280$.

Activity Synthesis

Select students to share the values they identified as original amount and the new amount for a few problems. Discuss how 100% always corresponds to the original value and when there is an increase in the value the new value corresponds to a percentage greater than the original 100%.

Select students to share the different equations they came up with. Discuss how the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.

Discuss how solving problems about percent change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how you can find the unknown number. Looking at the examples below, the first two require multiplication, but the others require division.

Using the structure A% of B is C:

- $(1.5) \cdot 12 = c$
- $(0.80) \cdot 15 = c$
- $a \cdot (1{,}200) = 1{,}080$
- $a \cdot (1.25) = 1.50$
- $(0.75) \cdot b = 12$
- $(1.25) \cdot b = 6{,}600$

Access for English Language Learners

Representing, Speaking: MLR7 Compare and Connect. Use this routine after selected students share the values they identified for the original amount and the new amount for a few problems. Ask students what is the same and what is different about the ways double number lines were used to represent percent increase and decrease for the different situations. Call students' attention to how the original amount is represented as 100% in the tape diagrams and in some of the equations. This will help strengthen students' mathematical language use and reasoning based on percent increase and decrease.
Design Principle(s): Maximize meta-awareness

7.3 Representing More Juice

5 minutes
The purpose of this activity is to help students understand that *percent increase* should be interpreted as a percent of the original or starting amount. Students are presented with two different methods to reason about and decide with which one they agree (MP3).

As students work on the activity, look for students who agree with Clare and Priya, these students should be asked to share during the whole-class discussion. The focus of the discussion should be on why Priya's number line is the correct one and how the term percent increase affects the original or starting amount (it goes over 100%).

Addressing

- 7.RP.A.3

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Give students 1 minute of quiet think time followed by partner and then whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as “I agree with _____ because _____.”
Supports accessibility for: Language; Organization

Anticipated Misconceptions

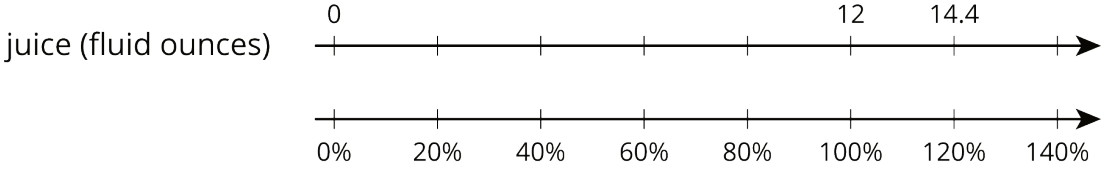
If students do not see why Clare's number line is incorrect, ask them to think about the original amount of ounces the juice box comes with. If the packaging claims to be more than what it started with should it be more or less than the original amount (100%)?

Student Task Statement

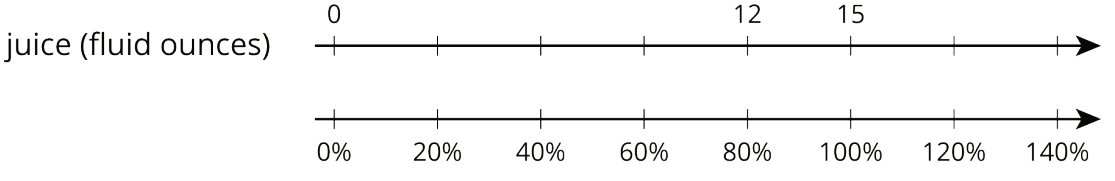
Two students are working on the same problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How much juice does the new packaging hold?

- Here is how Priya set up her double number line.



- Here is how Clare set up her double number line.



Do you agree with either of them? Explain or show your reasoning.

Student Response

Priya is correct. Explanations vary. Sample response: I agree with Priya's double number line because if the juice box is getting 20% more than what it starts with that means it will be more than the full capacity (which is 100%). So the original amount is 100% and the new amount is 120%.

Are You Ready for More?

Clare's diagram could represent a percent decrease. Describe a situation that could be represented with Clare's diagram.

Student Response

Answers vary. Sample response: A juice company modifies their packaging to ship more containers in each shipment. Their original package holds 15 fluid ounces, and their new packages holds 12 fluid ounces. What is the percent decrease made with this package change?

Activity Synthesis

Ask selected students to share their reasoning. Ask students, When the packaging says “20% more juice,” that means 20% more than *what*? (The amount of juice in the original packaging.) Explain that phrases like “percent more” or “percent less” are expressing the percent of some original amount.

So, if we use a double number line strategy, it makes sense to associate the original or starting amount with 100%.

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present an incorrect justification for why Clare's double line is correct that reflects a possible misunderstanding from the class. For example, "Clare is correct because 20% more would be 100%." Prompt discussion by asking, "Do you agree with the statement? Why or why not?" Ask students to correct the statement. This will help students understand that, in percent increase and decrease problems, original amounts are represented by 100%.
Design Principle(s): Maximize meta-awareness

7.4 Protecting the Green Sea Turtle

Optional: 10 minutes
The purpose of this activity is for students to encounter a situation where the quantity given is not the whole amount, but rather is the amount after a decrease. In this case, they are given the amount after a 10% decrease. They should make the connection from previous lessons that the amount given is 90% of the whole.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 3–5 minutes of quiet work time followed with whole-class discussion.

If time is limited pick the second problem to talk about during the discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using double number line diagrams. If students are unsure where to begin, suggest that they draw a double number line diagram to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

For the percent decrease problem, students may calculate 10% of 234, getting a change of 23.4 turtles and an original number of 257.4 turtles. Remind them that the percent decrease describes the change as a percentage of the original value, not as a percentage of the new value. If needed, prompt students to represent the situation using a double number line, placing 234 to the left by 10% of the quantity they want to find, which should be associated with 100%.

Student Task Statement

Green sea turtles live most of their lives in the ocean, but come ashore to lay their eggs. Some beaches where turtles often come ashore have been made into protected sanctuaries so the eggs will not be disturbed.

1. One sanctuary had 180 green sea turtles come ashore to lay eggs last year. This year, the number of turtles increased by 10%. How many turtles came ashore to lay eggs in the sanctuary this year?

2. At another sanctuary, the number of nesting turtles decreased by 10%. This year there were 234 nesting turtles. How many nesting turtles were at this sanctuary last year?

Student Response

1. 198 turtles. This year there was 110% of the number of turtles that came ashore to lay eggs last year, because the full amount increased by 10%. So 198 turtles came ashore to lay eggs this year, because $180 \cdot 1.1 = 198$.

2. 260 turtles. There were only 90% as many nesting turtles this year as last year, because the number decreased by 10%. If there were t nesting turtles last year, we can write the equation $0.9t = 234$. Thus, there were 260 nesting turtles last year because $t = 234 \div 0.9 = 260$.

Activity Synthesis

Ask students:

- How are the two problems the same? How are they different?
- What information is given in the first problem that is not given in the second? Second and not the first?
- In which problem were you given the quantity that represented 100%?

Connect the previous discussion about the original value being 100% by asking:

- For the percent increase question, what percentage value were you given? (students might answer 10% here but that is not actually the case)
- How did you use that to figure out the original number of green sea turtles?

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Ask 1–2 students to share how they used the given percentage value to figure out the original number of green sea turtles. Ask students to use mathematical language to restate and/or revoice one of the shared explanations. Consider providing students time to first share with a partner, before selecting one or two students to share with the class. This will support student understanding of situations where the quantity given is not the whole amount.
Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

When solving problems about percent increase and percent decrease, it is very important to start by asking yourself, "What is 100% in this situation?" Then you can find a percentage of that amount.

- "How can you find a 30% increase to 50?" (Find 30% of 50 and add it to 50. Or multiply 50 by 1.3.)
- "If you know that a 30% increase in a quantity is 50, how can you find the original quantity?" (Use a double number line that has 50 aligned with 130.)
- "How can you find a 30% decrease to 50?" (Find 30% of 50 and subtract it from 50. Or multiply 50 by 0.7.)
- "If you know that a 30% decrease in a quantity is 50, how can you find the original quantity?" (Use a double number line that has 50 aligned with 70.)

7.5 More Laundry Soap

Cool Down: 5 minutes

Addressing

- 7.RP.A.3

Student Task Statement

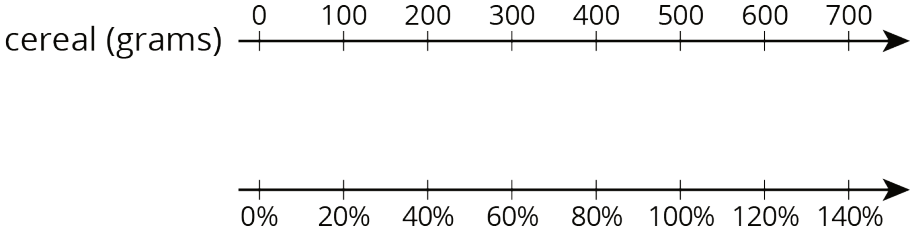
A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?

Student Response

66.25 fluid ounces. The new container holds 125% as much soap as the original container, because it holds 25% more. The new container holds 66.25 fluid ounces of laundry soap, because $53 \cdot 1.25 = 66.25$.

Student Lesson Summary

We can use a double number line diagram to show information about percent increase and percent decrease:



The initial amount of cereal is 500 grams, which is lined up with 100% in the diagram. We can find a 20% *increase* to 500 by adding 20% of 500:

$$500 + (0.2) \cdot 500 = (1.20) \cdot 500$$
$$= 600$$

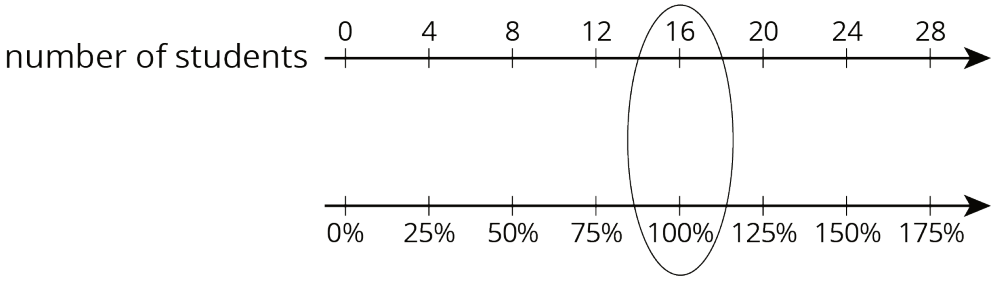
In the diagram, we can see that 600 corresponds to 120%.

If the initial amount of 500 grams is *decreased* by 40%, we can find how much cereal there is by subtracting 40% of the 500 grams:

$$500 - (0.4) \cdot 500 = (0.6) \cdot 500$$
$$= 300$$

So a 40% decrease is the same as 60% of the initial amount. In the diagram, we can see that 300 is lined up with 60%.

To solve percentage problems, we need to be clear about what corresponds to 100%. For example, suppose there are 20 students in a class, and we know this is an increase of 25% from last year. In this case, the number of students in the class *last* year corresponds to 100%. So the initial amount (100%) is unknown and the final amount (125%) is 20 students.



Looking at the double number line, if 20 students is a 25% increase from the previous year, then there were 16 students in the class last year.

Lesson 7 Practice Problems

Problem 1

Statement

A bakery used 25% more butter this month than last month. If the bakery used 240 kilograms of butter last month, how much did it use this month?

Solution

300 kilograms

Problem 2

Statement

Last week, the price of oranges at the farmer's market was $1.75 per pound. This week, the price has decreased by 20%. What is the price of oranges this week?

Solution

$1.40 per pound, because 20% of 1.75 is 0.35 and $1.75 - 0.35 = 1.40$

Problem 3

Statement

Noah thinks the answers to these two questions will be the same. Do you agree with him? Explain your reasoning.

- This year, a herd of bison had a 10% increase in population. If there were 550 bison in the herd last year, how many are in the herd this year?
- This year, another herd of bison had a 10% decrease in population. If there are 550 bison in the herd this year, how many bison were there last year?

Solution

No, the answers are different. Although the answer to both problems will be larger than 550, the number of bison in each 10% change is different because the original values are not the same.

Problem 4

Statement

Elena walked 12 miles. Then she walked 0.25 that distance. How far did she walk all together? Select **all** that apply.

A. $12 + 0.25 \cdot 12$

B. $12\,(1 + 0.25)$

C. $12 \cdot 1.25$

D. $12 \cdot 0.25$

E. $12 + 0.25$

Solution

["A", "B", "C"]
(From Unit 4, Lesson 5.)

Problem 5

Statement

A circle's circumference is 600 m. What is a good approximation of the circle's area?

A. 300 m^2

B. 3,000 m^2

C. 30,000 m^2

D. 300,000 m^2

Solution

C
(From Unit 3, Lesson 8.)

Problem 6

Statement

The equation $d = 3t$ represents the relationship between the distance (d) in inches that a snail is from a certain rock and the time (t) in minutes.

a. What does the number 3 represent?

b. How many minutes does it take the snail to get 9 inches from the rock?

c. How far will the snail be from the rock after 9 minutes?

Solution

a. The constant of proportionality or the speed of the snail, 3 inches per minute.

b. 3 minutes

c. 27 inches

(From Unit 2, Lesson 6.)