Lesson 8: Percent Increase and Decrease with Equations

Goals

- Explain (orally and in writing) how to calculate the original amount given the new amount and a percentage of increase or decrease.

- Generate algebraic expressions that represent a situation involving percent increase or decrease, and justify (orally) the reasoning.

Learning Targets

- I can solve percent increase and decrease problems by writing an equation to represent the situation and solving it.

Lesson Narrative

In this lesson, students represent situations involving percent increase and percent decrease using equations. They write equations like $y = 1.06x$ to represent growth of a bank account, and use the equation to answer questions about different starting amounts. They write equations like $t - 0.25t = 12$ or $0.75t = 12$ to represent the initial price t of a T-shirt that was $12 after a 25% discount. The focus of this unit is writing equations and understanding their connection to the context. In a later unit on solving equations the focus will be more on using equations to solve problems about percent increase and percent decrease.

When students repeatedly apply a percent increase to a quantity and see that this operation be expressed generally by an equation, they engage in MP8.

Alignments

Building On

- 5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Building Towards

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR7: Compare and Connect

- MLR8: Discussion Supports

- Number Talk

- Think Pair Share

> **Student Learning Goals**
>
> Let's use equations to represent increases and decreases.

8.1 Number Talk: From 100 to 106

Warm Up: 5 minutes
In this lesson, students will be finding percent increase and percent decrease by multiplying by an appropriate factor. For example, to find a 36% increase in a quantity, we can multiply that quantity by 1.36. This warm-up prompts them to think about the scale factor needed to multiply one number to get another.

Building On

- 5.NF.B

Building Towards

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies. Follow with a whole-class discussion.

> **Student Task Statement**
>
> How do you get from one number to the next using multiplication or division?
>
> From 100 to 106
>
> From 100 to 90
>
> From 90 to 100

> From 106 to 100

Student Response

Answers vary. Sample responses:

- Multiply by 1.06

- Multiply by 0.90

- Divide by 0.90

- Divide by 1.06

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate _____'s reasoning in a different way?"

- "Did anyone have the same strategy but would explain it differently?"

- "Did anyone solve the problem in a different way?"

- "Does anyone want to add on to _____'s strategy?"

- "Do you agree or disagree? Why?"

8.2 Interest and Depreciation

10 minutes
In this activity, students are calculating percent increase and decrease in the context of interest and depreciation. They may have encountered these contexts before this unit, but be sure that everyone understands the basic idea of interest and depreciation before students begin work.

Students have enough experience to solve these problems using various strategies and representations. Look for students writing different but correct expressions in terms of x.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR7: Compare and Connect

- Think Pair Share

Launch

Remind students what interest is, give examples of when interest is applied (savings accounts, credit cards, etc.), and discuss depreciation value of an item.

Students in groups of 2. Give students 3–5 minutes of quiet work time, followed by partner then whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Anticipated Misconceptions

If students have a hard time organizing their information, suggest that they use a table.

Students might struggle to write an expression in terms of x. Have students describe in words how they calculated the previous problems with numbers and use that to come up with an expression.

Student Task Statement

1. Money in a particular savings account increases by about 6% after a year. How much money will be in the account after one year if the initial amount is $100? $50? $200? $125? x dollars? If you get stuck, consider using diagrams or a table to organize your work.

2. The value of a new car decreases by about 15% in the first year. How much will a car be worth after one year if its initial value was $1,000? $5,000? $5,020? x dollars? If you get stuck, consider using diagrams or a table to organize your work.

Student Response

1.

$106, since $100 \cdot 1.06 = 106$.

$53. Since 6% of 100 is 6, we know 6% of 50 is 3. Increasing $50 by $3 gives $53.

$212. Since 6% of 100 is 6, we know 6% of 200 is 12. Increasing $200 by $12 gives $212.

$132.50. If we deposit $25, we would have $26.50 after one year, because it is half the amount we would have if we deposit $50. If we deposit $125, we will have $132.50 because 106+26.50=132.50.

$1.06x$ or another equivalent expression

2. $850. After one year, the car is worth 85% of its original value, because $100 - 15 = 85$. The car would be worth $ because $1,000 \cdot 0.85 = 850$.

$4,250, since $5,000 \cdot 0.85 = 4,250$.

$4,267, since $5,020 \cdot 0.85 = 4,267$.

$0.85x$ or another equivalent expression

Activity Synthesis

Select students with different ways of writing correct expressions in terms of x to share their expression. Connect both problems back to the previous work with the distributive property by asking students:

- How else can the first problem be written? ($x + 0.06x$ or $(1 + 0.06)x$ or $1.06x$)

- How else can the second problem be written? ($x - 0.15x$ or $(1 - 0.15)x$ or $0.85x$)

8.3 Matching Equations

5 minutes
In this activity, students match equations that represent a percent increase situation to the situations they represent.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. 2 minutes of quiet think time followed by 2 minutes of partner discussion.

> **Student Task Statement**
>
> Match an equation to each of these situations. Be prepared to share your reasoning.
>
> 1. The water level in a reservoir is now 52 meters. If this was a 23% increase, what was the initial depth?
>
> 2. The snow is now 52 inches deep. If this was a 77% decrease, what was the initial depth?
>
> $0.23x = 52$
>
> $0.77x = 52$
>
> $1.23x = 52$
>
> $1.77x = 52$

Student Response

1. $1.23x = 52$

2. $0.23x = 52$

Are You Ready for More?

An astronaut was exploring the moon of a distant planet, and found some glowing goo at the bottom of a very deep crater. She brought a 10-gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

1. How much goo will she have after 1 hour? After 2 hours? After 3 hours? After n hours?

2. When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for n hours to return to the original size?

Student Response

1. After 1 hour, there will be 20 grams. After 2 hours, there will be 40 grams. After 3 hours, there will be 80 grams. After n hours, there will be $10 \cdot 2^n$.

2. $\frac{n}{2}$ hours. A 75% decrease is $\frac{1}{4}$ as much, so for every hour, the amount decreases to $\frac{1}{4}$ of what was there at the beginning of the hour. For example, after 2 hours of light exposure, there will be 40 grams of goo, but after only one hour in the dark, it will be back to 10 grams.

Activity Synthesis

Ask one or more students to share which equation they matched with each situation, and resolve any discrepancies. Once the matches are agreed upon, ask students how they would solve the equation to find the amount without actually solving the equations.

8.4 Representing Percent Increase and Decrease: Equations

Optional: 15 minutes
The purpose of this activity is for students to use equations to represent situations of percent increase and decrease. Additionally, students identify the original and new amount in the double number lines to reinforce what they learned in earlier lessons (that the original amount pertains to 100%).

As students work on the task, look for students who created various equations for the last question.
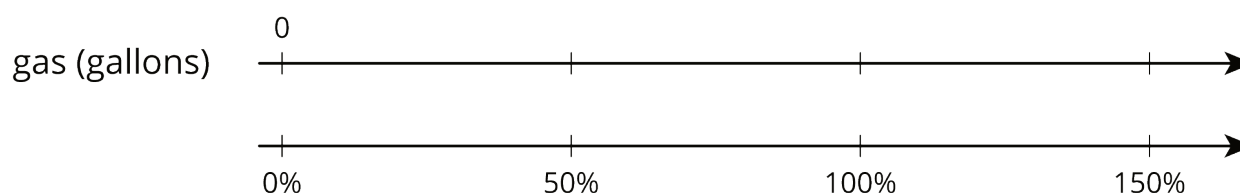
Addressing

- 7.RP.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- Think Pair Share

Launch

Show students the double number line from the activity in the previous lesson.

gas (gallons)

0

| 0% | 50% | 100% | 150% |

Help them make connections to the first problem. Tell them that they can go back to the previous lesson to see the double number lines for the others if that helps.

Arrange students in groups of 2. Give 5–8 minutes of quiet work time. After 5 minutes allow students to work with a partner or to continue to work alone.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using double number line diagrams. If students are unsure where to begin, suggest that they draw a double number line diagram to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Anticipated Misconceptions

Students may continue to struggle to recognize the original amount and new amount with the proper percentages on the double number line. Remind them that the original amount always corresponds to 100%.

Student Task Statement

1. The gas tank in dad's car holds 12 gallons. The gas tank in mom's truck holds 50% more than that. How much gas does the truck's tank hold? Explain why this situation can be represented by the equation $(1.5) \cdot 12 = t$. Make sure that you explain what t represents.

2. Write an equation to represent each of the following situations.

a. A movie theater decreased the size of its popcorn bags by 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?

b. After a 25% discount, the price of a T-shirt was $12. What was the price before the discount?

c. Compared to last year, the population of Boom Town has increased by 25%.The population is now 6,600. What was the population last year?

Student Response

1. 18 gallons. Let t represent the amount of gas that mom's truck holds. Dad's car holds 12 gallons, and mom's truck holds 50% more or $0.5 \cdot 12$ more, so it holds $12 + 0.5 \cdot 12 = 1.5 \cdot 12$. So $1.5 \cdot 12 = t$.

2. a. $(0.8) \cdot 15 = p$, where p is the amount of popcorn the new bag holds

   b. $(0.75) \cdot t = 12$, where t is the price before the discount

   c. $(1.25) \cdot p = 6,600$, where p is the population last year

Activity Synthesis

Select students to share the values they identified as original amount and the new amount for a few problems. Discuss how 100% always corresponds to the original value and when there is an increase in the value the new value corresponds to a percentage greater than the original 100%.

Select students to share the different equations they came up with. Discuss how the distributive property is useful for finding the percentage that corresponds with the new value instead of the percentage of the change.

Discuss how solving problems about percent change may require either multiplying or dividing numbers. It can be confusing, but it helps to first express the relationship as an equation and then think about how you can find the unknown number. Looking at the examples below, the first two require multiplication, but the others require division.

Using the structure A% of B is C:

- $(1.5) \cdot 12 = c$

- $(0.80) \cdot 15 = c$

- $a \cdot (1,200) = 1,080$

- $a \cdot (1.50) = 1.75$

- $(0.75) \cdot b = 12$

- $(1.25) \cdot b = 61,600$

Lesson Synthesis

Students should feel confident calculating percent increase/decrease using a method of their choice. Ask students:

- "What are some ways we have learned to solve percent increase or percent decrease problems?" (double number lines, tables, equations)

- "Which representation do you prefer to use? Why?"

8.5 Tyler's Savings Bond

Cool Down: 5 minutes
Addressing

- 7.RP.A.3

> **Student Task Statement**
>
> Tyler's mom purchased a savings bond for Tyler. The value of the savings bond increases by 4% each year. One year after it was purchased, the value of the savings bond was $156.
>
> Find the value of the bond when Tyler's mom purchased it. Explain your reasoning.

Student Response

Sample reasoning: Represent the situation using the equation $1.04x = 156$, where x represents the value of the savings bond when Tyler's mom purchased it. The solution is $x = 156 \div 1.04 = 150$, so the bond was originally worth $150.

Student Lesson Summary

We can use equations to express percent increase and percent decrease. For example, if y is 15% more than x,
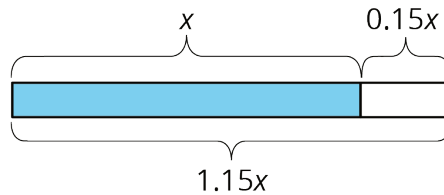
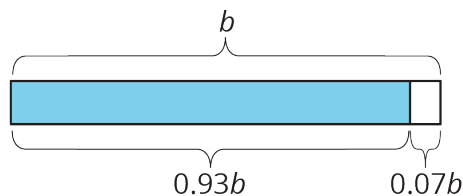

we can represent this using any of these equations:

$$y = x + 0.15x \qquad y = (1 + 0.15)x \qquad y = 1.15x$$

So if someone makes an investment of x dollars, and its value increases by 15% to $1250, then we can write and solve the equation $1.15x = 1250$ to find the value of the initial investment.

Here is another example: if a is 7% less than b,

we can represent this using any of these equations:

$$a = b - 0.07b \qquad a = (1 - 0.07)b \qquad a = 0.93b$$

So if the amount of water in a tank decreased 7% from its starting value of b to its ending value of 348 gallons, then you can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percent increase or percent decrease.

Lesson 8 Practice Problems
Problem 1
Statement

A pair of designer sneakers was purchased for $120. Since they were purchased, their price has increased by 15%. What is the new price?

Solution

$138

Problem 2

Statement

Elena's aunt bought her a $150 savings bond when she was born. When Elena is 20 years old, the bond will have earned 105% in interest. How much will the bond be worth when Elena is 20 years old?

Solution

$307.50

Problem 3

Statement

In a video game, Clare scored 50% more points than Tyler. If c is the number of points that Clare scored and t is the number of points that Tyler scored, which equations are correct? Select **all** that apply.

A. $c = 1.5t$

B. $c = t + 0.5$

C. $c = t + 0.5t$

D. $c = t + 50$

E. $c = (1 + 0.5)t$

Solution

["A", "C", "E"]

Problem 4

Statement

Draw a diagram to represent each situation:

a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

Solution

Answers vary. Sample responses:

a. A tape diagram showing 10 equal pieces labeled "number of miles driven last month" on the top with one below it that is just 7 pieces long and is labeled, "number of miles driven this month."

b. A tape diagram showing 4 equal pieces labeled "amount of paper they used last month" on the top with one below it that is 5 pieces long and is labeled, "amount of paper they used this month."

(From Unit 4, Lesson 6.)

Problem 5

Statement

Which decimal is the best estimate of the fraction $\frac{29}{40}$?

A. 0.5

B. 0.6

C. 0.7

D. 0.8

Solution

C

(From Unit 4, Lesson 5.)

Problem 6

Statement

Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

Solution

No, since $7.2 \cdot \pi \approx 22.6$.

(From Unit 3, Lesson 3.)