

# Lesson 7: Expressing Transformations of Functions Algebraically

- Let's express transformed functions algebraically.

## 7.1: Describing Translations

Let  $g(x) = \sqrt{x}$ . Complete the table. Be prepared to explain your reasoning.

words (the graph of $y = g(x)$ is...)	function notation	expression
translated left 5 units	$g(x + 5)$	
translated left 5 units and down 3 units		$\sqrt{x + 5} - 3$
	$g(-x)$	$\sqrt{-x}$
translated left 5 units, then down 3 units, then reflected across the $y$ -axis		

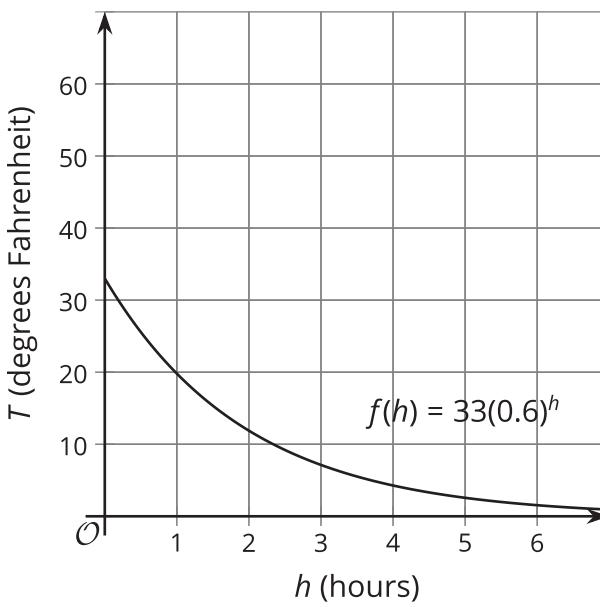
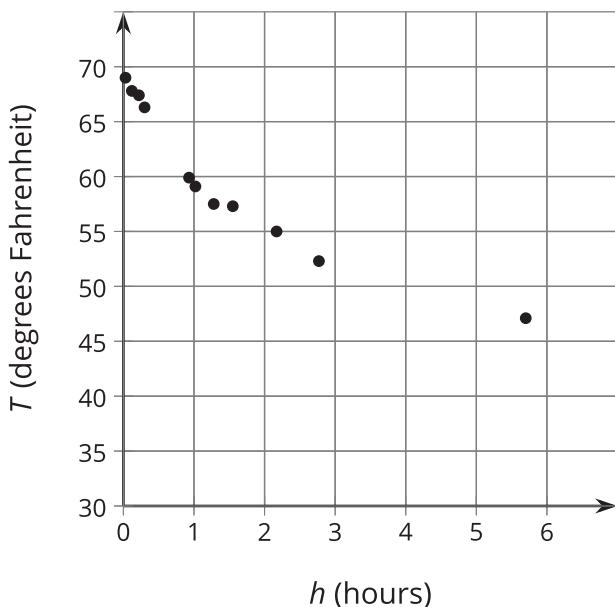
## 7.2: Translating Vertex Form

Let  $f$  be the function given by  $f(x) = x^2$ .

- Write an equation for the function  $g$  whose graph is the graph of  $f$  translated 3 units left and up 5 units.
- What is the vertex of the graph of  $g$ ? Explain how you know.
- Write an equation for a quadratic function  $h$  whose graph has a vertex at  $(1.5, 2.6)$ .
- Write an equation for a quadratic function  $k$  whose graph opens downward and has a vertex at  $(3.2, -4.7)$ .

## 7.3: An Even Better Fit

In an earlier lesson, we looked at the temperature  $T$ , in degrees Fahrenheit, of a bottle of soda water left outside for  $h$  hours. Let's model this data with a function. This time, we will start with the function  $f(h) = 33(0.6)^h$ . This graph has a shape that fits the data well.



1. Describe a translation of this graph that fits the data.
2. Write an equation defining a function  $g$  that models the data.
3. What does your function tell you about the temperature outside?

### Are you ready for more?

Han tried the following steps to model the soda water temperature. First he shifts the given graph left by one hour, then he applies a vertical shift.

1. What vertical shift does Han need to apply to model the 45 degree Fahrenheit temperature in the refrigerator?
2. How does Han's model compare to yours?

## Lesson 7 Summary

You can use the equation of a function to write an equation for its transformation. For example, let  $f(x) = x^2$ . Take the graph of  $f$ , reflect it across the  $x$ -axis, translate it up 10 units, and translate it left 3 units. What is an equation for this new function? The new function  $g$  is related to  $f$  by  $g(x) = -f(x + 3) + 10$ , since

$$g(x) = -f(x + 3) + 10$$

reflect  
across  $x$ -axis      shift  
left 3      shift  
up 10

Which means  $g(x) = -(x + 3)^2 + 10$ .

Sometimes you can recognize from the expression for a function that it is the transformation of a simpler function. For example, consider:

$$H(t) = 10 - (1.2)^{t+5}$$

One way to obtain the expression for  $H$  from  $1.2^t$  is:

- adding 5 to the input to get  $(1.2)^{t+5}$
- multiplying the output by -1 to get  $-(1.2)^{t+5}$
- adding 10 to the output to get  $10 - (1.2)^{t+5}$

So the graph of  $H$  is obtained from the graph of  $f(t) = 1.2^t$  by translating left 5 units, reflecting across the  $x$ -axis, and translating up 10 units. Consider the point  $(0, 1)$  on the graph of  $f$ . After translating, reflecting, and translating again, it becomes the point  $(-5, 9)$  on the graph of  $H$ .

