Lesson 8: Writing and Graphing Inequalities

Goals

- Coordinate verbal, algebraic, and number line representations of inequalities.
- Critique (orally and in writing) possible values given for a situation with a constraint, including determining whether the boundary value is included and making sense of situations with discrete quantities.
- Interpret phrases that describe a quantity constrained by a maximum or minimum acceptable value, e.g. “at least,” “at most,” “up to,” “more than,” “less than”, etc., and write an inequality statement to represent the constraint.

Learning Targets

- I can graph inequalities on a number line.
- I can write an inequality to represent a situation.

Lesson Narrative

In extending their concept of numbers to all rational numbers, students began writing inequality statements that compared two numbers. In this lesson, students extend their work with inequality statements by considering comparisons with an unknown quantity. These quantities, represented by variables, often describe real-world situations, and their value is usually constrained by minimum or maximum allowable values. Students represent these situations with inequality statements and reason about possible values that make them true (MP2). As there are often many, even infinite, possibilities for the value of the variable that satisfy the constraint, students use the number line as a helpful tool to show all the possible values.

The activities in this lesson present students with two types of scenarios. When the variable represents a measurement, the possible values can usually be any number within the range satisfied by the constraint. When the variable represents a count of people or objects, the possible values are restricted to whole numbers within the range. Students also consider whether the constraint itself is included or excluded in the set of possible values, and learn how to indicate this result on the number line representation.

After writing inequality statements to represent situations, students test values to see if they make the statement true.

Alignments

Addressing

- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- 6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- 6.NS.C.7.b: Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{\circ}C > -7^{\circ}C$ to express the fact that $-3^{\circ}C$ is warmer than $-7^{\circ}C$.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Take Turns
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut up cards from the blackline master. Prepare 1 set of 16 cards for each group of 2 students.

> **Student Learning Goals**
>
> Let’s write inequalities.

8.1 Estimate Heights of People

Warm Up: 5 minutes
The purpose of this warm-up is for students to estimate heights and discuss their estimates using the language of inequalities.As students discuss their estimates with a partner, monitor the discussions and identify students who use different estimation strategies to share during the whole-class discussion.

Addressing

- 6.NS.C.7.b

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time to complete the first question. Pause after the first question and ask students to share their responses. Record and display them for all to see. Tell students the height of the man in the pictures is 5 feet 10 inches and give them 1 minute to discuss the second question with their partner. Follow with a whole-class discussion focused on the second question.

Student Task Statement

1. Here is a picture of a man.

   a. Name a number, in feet, that is clearly too high for this man's height.

   b. Name a number, in feet, that is clearly too low for his height.

   c. Make an estimate of his height.

   Pause here for a class discussion.

2. Here is a picture of the same man standing next to a child.

   If the man's actual height is 5 feet 10 inches, what can you say about the height of the child in this picture?

   Be prepared to explain your reasoning.

Student Response

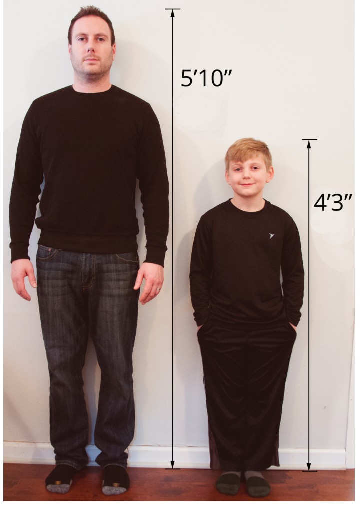
Answers vary. Sample responses:

1. a. 12 feet

   b. 4 feet

   c. 6 feet (or another number between the responses for the first two questions).

2. The child is shorter than the man, so the child’s height is less than 5 feet 10 inches.



Activity Synthesis

During the whole-class discussion, encourage students to use language such as “his height is more than . . . ” and “her height is less than . . . ” instead of “taller than . . .” and “shorter than . . . ” so they become familiar with the more precise language of inequalities.

Ask students to share their responses to the second question. As students share, ask them how to write their explanation as a sentence and inequality. For example, if students say, “The child is shorter than 5'10",” ask them how we could write that statement using “greater” or “less” and using an inequality. Record and display their responses for all to see. If students do not say, “The child’s height is less than 5'10" and $h < 5'10"$,” where h represents the child’s height, then make these statements explicit.

If there is time, ask students for the difference between their estimates and the actual heights.

8.2 Stories about 9

15 minutes (there is a digital version of this activity)
In this activity, students extend their understanding of inequality statements by considering an unknown quantity with one or more constraints in a real-world situation. Students represent situations with inequality statements and reason about possible values that make them true, showing the possibilities on a number line. They realize that these situations often have many values that make the inequality true, and often even an infinite number, making the number line representation of possible values very useful (MP7). Students examine the difference between solutions that are continuous (can take on any value, usually involving measurement of an amount) and those that are discrete (require whole number solutions because of the context, usually involving a count of people or objects) (MP2). They also consider when to include or exclude the endpoints. They learn how to represent this on the number line with a closed (include) or open (exclude) circle at the boundary of the constraint. Note that these materials don’t introduce $\geq$ or $\leq$ symbols until grade 7, but some individual teachers prefer to introduce them earlier.

Addressing

- 6.EE.B.8

Instructional Routines

- MLR7: Compare and Connect
- Take Turns

Launch

Copy and cut up the blackline master. Make enough for each group of 2 students to have a set. Arrange students in groups of 2. Give each group 16 pre-cut slips and 6–8 minutes to complete the first question, which is to match each story and question to three ways to represent the solutions: a list or description, a number line, or an inequality statement. Ask each group to compare their matching decisions with another group and come to an agreement before recording their sorted representations.

Classes using the digital materials have an applet to match the story and question to two ways to represent the solutions: a list or description and a number line. Representation with an inequality is left to class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

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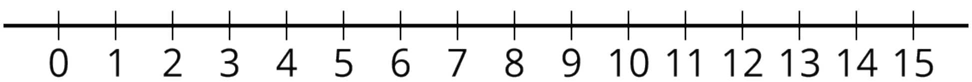
Student Task Statement

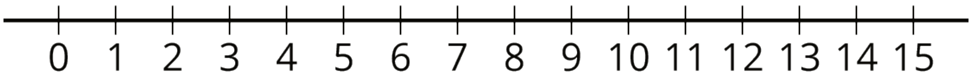
1. Your teacher will give you a set of paper slips with four stories and questions involving the number 9. Match each question to three representations of the solution: a description or a list, a number line, or an inequality statement.
2. Compare your matching decisions with another group's. If there are disagreements, discuss until both groups come to an agreement. Then, record your final matching decisions here.
   a. A fishing boat can hold fewer than 9 people. How many people (x) can it hold?
      - Description or list:
      - Number line:



- Inequality:

b. Lin needs more than 9 ounces of butter to make cookies for her party. How many ounces of butter (x) would be enough?

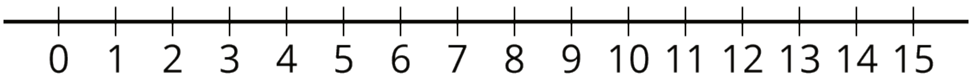
- Description or list:
- Number line:



- Inequality:

c. A magician will perform her magic tricks only if there are at least 9 people in the audience. For how many people (x) will she perform her magic tricks?

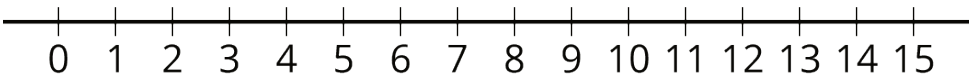
- Description or list:
- Number line:



- Inequality:

d. A food scale can measure up to 9 kilograms of weight. What weights (x) can the scale measure?

- Description or list:
- Number line:



- Inequality:

Student Response

See blackline master.

Activity Synthesis

Here are some topics for discussion:

- Differences between solutions to the people problem and the weight problem (continuous v. discrete).
- Inclusion and exclusion of 9 and how to distinguish them on the number line (including open and closed circle).
- The idea that there are many values that make these inequalities true. At times, there are an infinite number of values that make the inequality true.

- Advantages of using inequality statements or graphs instead of listing or describing the solutions.

There are some additional restrictions in some of the scenarios. For example, when counting people in a boat or measuring weight, zero is the least value that makes sense. For the people at the magic show and the amount of butter, very large numbers satisfy the inequalities; however, they don't necessarily make sense in the contexts presented. These restrictions are not a focus for this activity, but they are likely to come up during the discussion.

Toward the end of the discussion, display the statement "Jada built a robot that pushes small boxes from one place to another. The robot is able to push up to 3 pounds. For what box weights (w) can the robot push the box?" Ask students to represent all possible answers in three ways:

1. By listing or describing all numbers that answer the question. (Any weight that is equal to or less than 3 pounds.)
2. By plotting or graphing the values that work on a number line. (Closed circle at 3 and shaded to the left.)
3. By writing an inequality. (Using the given variable) ($w < 3$ and $w = 3$.)

Access for English Language Learners

Representing, Conversing, Reading: MLR7 Compare and Connect. To foster students' meta-awareness of language as they compare and contrast different mathematical situations, display these two sentences: "A fishing boat can hold fewer than 9 people" and "A magician will perform her magic tricks only if there are at least 9 people in the audience." Ask students what is the same and what is different about these two situations. Invite students to work with a partner to find values can be true for each statement. This discussion makes explicit the language of inequalities (e.g., fewer, at least), and offers an opportunity to connect verbal descriptions to solution sets of numbers.
Design Principle(s): Maximize meta-awareness

8.3 How High and How Low Can It Be?

15 minutes
The purpose of this task is to extend the use of inequalities to describe maximum and minimum possible values, using symbols and determining whether or not a particular value makes an inequality true. Though students are thinking about whether or not a particular value makes an inequality true, the term "solution" will not be formally introduced until a future lesson.

Students will estimate the maximum and minimum height of a basketball hoop in a given picture. They will then explore how inequalities with these 2 values, the maximum and minimum, can describe the restrictions and possible heights for the hoop. The inequalities are represented as symbolic statements as well as on number lines.

Once they have created inequalities, students will decide if a given value could be a possible height for the hoop. For the last question, students work with a partner so each will have a chance to check a possible solution to an inequality that they didn't estimate and write themselves.

Addressing

- 6.EE.B.8

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Give students 8 minutes of quiet work time to complete questions 1 through 4 followed by 2 minutes partner discussion for question 5. Follow with whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students to check for understanding within the first 2–3 minutes of work time. Look for students who are making reasonable estimates, filling in blanks and beginning to plot estimates on number lines.
Supports accessibility for: Memory; Organization

Access for English Language Learners

Writing, Speaking: MLR5 Co-Craft Questions. Before students begin work, use this routine to help students interpret the situation before being asked to make statements about the basketball hoop. Display just the photograph for all to see, and arrange students in groups of 2. Invite groups to discuss what they notice and to write mathematical questions they could ask about the situation depicted in the photograph. Once 2–3 pairs share their questions with the class, reveal the rest of the activity.
Design Principle(s): Supporting sense-making

Anticipated Misconceptions

Students may understand that maximum means the largest value but may have trouble with the concept that all other values are therefore less than the maximum. A similar misunderstanding may occur with the concept of minimum value. For these students, have a short conversation about their maximum and minimum height estimates and how those values restrict the possible heights of the hoop.

Student Task Statement

Here is a picture of a person and a basketball hoop. Based on the picture, what do you think are reasonable estimates for the maximum and minimum heights of the basketball hoop?

1. Complete the first blank in each sentence with an estimate, and the second blank with "taller" or "shorter."

   a. I estimate the *minimum* height of the basketball hoop to be ________ feet; this means the hoop cannot be ____________ than this height.

   b. I estimate the *maximum* height of the basketball hoop to be ________ feet; this means the hoop cannot be ____________ than this height.

2. Write two inequalities—one to show your estimate for the *minimum* height of the basketball hoop, and another for the *maximum* height. Use an inequality symbol and the variable h to represent the unknown height.

3. Plot each estimate for minimum or maximum value on a number line.

   - Minimum:

   - Maximum:

4. Suppose a classmate estimated the value of h to be 19 feet. Does this estimate agree with your inequality for the maximum height? Does it agree with your inequality for the minimum height? Explain or show how you know.

5. Ask a partner for an estimate of h. Record the estimate and check if it agrees with your inequalities for maximum and minimum heights.

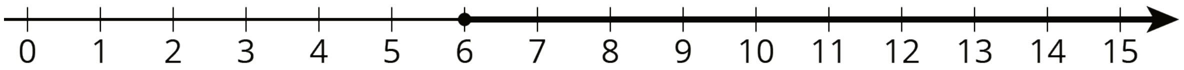
Student Response

1. a. Estimates vary. Sample response: 6 ft, shorter, because the word "minimum" means the lowest possible value, so the hoop cannot be shorter than 6 ft.

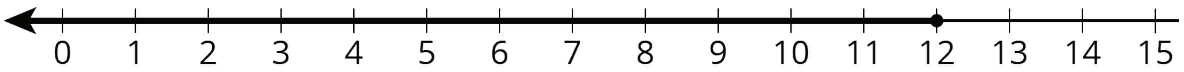
   b. Estimates vary. Sample response: 12 ft, taller, because the word "maximum" means the highest possible value, so the hoop cannot be taller than 12 ft.

2. Answers vary based on the estimates in previous question. Sample responses:
   - Minimum: $h > 6$ and $h = 6$
   - Maximum: $h < 12$ and $h = 12$

3. Answers vary. Sample response:
   Minimum:



   Maximum:

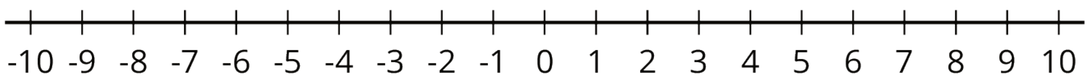


4. Answers vary depending on previous question. Sample response: Since 19 ft is greater than 12 ft, this estimate does not agree with my inequality for the maximum height of the hoop. Using the inequality, $h < 12$, when $h = 19$, the inequality would be $19 < 12$, which is false. Since 19 ft is greater than 6 ft, this answer does agree with my inequality for the minimum height of the hoop. Using the inequality, $h > 6$, when $h = 19$, the inequality would be $19 > 6$, which is true. Students may also show that 19 falls within or outside of the shaded regions on their number lines.

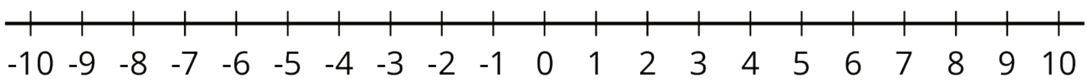
5. Answers vary. Students should use similar reasoning to what they did in the previous question.

Are You Ready for More?

1. Find 3 different numbers that a could be if $|a| < 5$. Plot these points on the number line. Then plot as many other possibilities for a as you can.

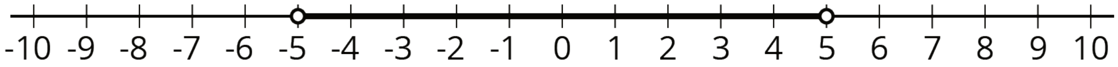


2. Find 3 different numbers that b could be if $|b| > 3$. Plot these points on the number line. Then plot as many other possibilities for b as you can.

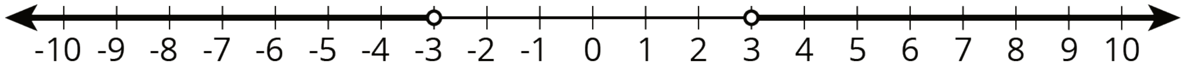


Student Response

1. Answers vary. Sample answers: $a = 1$, $a = 0$, $a = -3$



2. Answers vary. Sample answers: $b = 4$, $b = -4$, $b = -7$



Activity Synthesis

The discussion should focus on the concepts of minimum and maximum. While students are not yet using the formal definition of “solutions to an inequality,” they are reasoning about whether a value makes an inequality statement true. A student’s estimated minimum is the least value that makes their inequality true. Similarly, their maximum is the greatest value that makes their inequality true.

Ask students to share their responses to question 5. Select one group to share an estimate that makes a particular inequality true and an estimate that makes the same inequality false. Ask students to describe the values that make both inequalities true at the same time. Record their responses and display for all to see. Include both inequality symbols and number line representations.

Lesson Synthesis

Ask students to think about situations where a quantity might have a maximum or minimum value (for example, the number of students in a class, the pounds of fruit purchased by the school cafeteria, the budget for a trip). Ask students to create a variable and represent the situation with an inequality statement using their variable. Ask them whether the maximum or minimum is included in the range of possible values of the variable (for example, does the number of students in a class have to be less than 30, or can it also be equal to 30?). Then ask them to graph the possible values on a number line. Invite selected students to share their situations, inequalities, and graphs.

8.4 A Box of Paper Clips

Cool Down: 5 minutes

Addressing

- 6.EE.B.6

Student Task Statement

Andre looks at a box of paper clips. He says: “I think the number of paper clips in the box is less than 1,000.”

Lin also looks at the box. She says: “I think the number of paper clips in the box is more than 500.”

1. Write an inequality to show Andre's statement, using p for the number of paper clips.

2. Write another inequality to show Lin's statement, also using p for the number of paper clips.
3. Do you think both Lin and Andre would agree that there could be 487 paperclips in the box? Explain your reasoning.
4. Do you think both Lin and Andre would agree that there could be 742 paperclips in the box? Explain your reasoning.

Student Response

1. $p < 1,000$
2. $p > 500$
3. No. Andre would agree because the inequality, $487 < 1,000$ is a true statement. However, Lin would not agree because the inequality $487 > 500$ is a false statement.
4. Yes. Both inequalities are true for 742 paper clips: $742 < 1{,}000$ and $742 > 500$. This means that according to Lin and Andre, there could be 742 paperclips in the box.

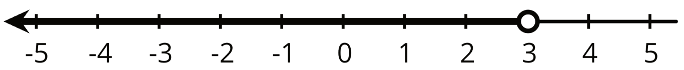
Student Lesson Summary

An inequality tells us that one value is *less than* or *greater than* another value.

Suppose we knew the temperature is *less than* 3°F, but we don't know exactly what it is. To represent what we know about the temperature t in $^\circ\text{F}$ we can write the inequality:

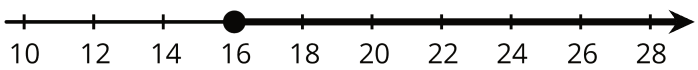
$$t < 3$$

The temperature can also be graphed on a number line. Any point to the left of 3 is a possible value for t. The open circle at 3 means that t cannot be *equal* to 3, because the temperature is *less than* 3.



Here is another example. Suppose a young traveler has to be at least 16 years old to fly on an airplane without an accompanying adult.

If a represents the age of the traveler, any number greater than 16 is a possible value for a, and 16 itself is also a possible value of a. We can show this on a number line by drawing a closed circle at 16 to show that it meets the requirement (a 16-year-old person can travel alone). From there, we draw a line that points to the right.



We can also write an inequality and equation to show possible values for a:

$$a > 16$$
$$a = 16$$

Lesson 8 Practice Problems

Problem 1

Statement

At the book sale, all books cost less than \$5.

a. What is the most expensive a book could be?

b. Write an inequality to represent costs of books at the sale.

c. Draw a number line to represent the inequality.

Solution

a. \$4.99

b. Answer varies. Sample response: If p is the price of a book, then $p < 5$.

c. The number line has an open circle and an arrow drawn to the left starting with 5.

Problem 2

Statement

Kiran started his homework *before* 7:00 p.m. and finished his homework *after* 8:00 p.m. Let h represent the number of hours Kiran worked on his homework.

Decide if each statement it is definitely true, definitely not true, or possibly true. Explain your reasoning.

a. $h > 1$

b. $h > 2$

c. $h < 1$

d. $h < 2$

Solution

a. Definitely true. Kiran worked from 7:00 until 8:00 and some additional time.

b. Possibly true. It is true if Kiran started his homework at 6:15 and stopped at 8:30. It could also be false if Kiran started his work at 6:45 and finished at 8:15.

c. Definitely false. $h > 1$ is true.

d. Possibly true. $h > 2$ might be true, but it might also be false.

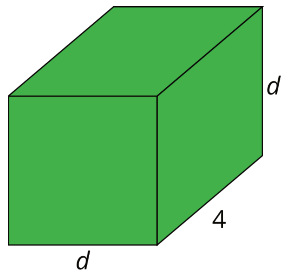
Problem 3

Statement

Consider a rectangular prism with length 4 and width and height d.

a. Find an expression for the volume of the prism in terms of d.

b. Compute the volume of the prism when $d = 1$, when $d = 2$, and when $d = \frac{1}{2}$.



Solution

a. $4d^2$

b. When $d = 1$, the volume is 4. When $d = 2$, the volume is 16. When $d = \frac{1}{2}$, the volume is 1.

(From Unit 6, Lesson 14.)

Problem 4

Statement

Match the statements written in English with the mathematical statements. All of these statements are true.

A. The number -15 is further away from 0 than the number -12 on the number line.

B. The number -12 is a distance of 12 units away from 0 on the number line.

C. The distance between -12 and 0 on the number line is greater than -15.

D. The numbers 12 and -12 are the same distance away from 0 on the number line.

E. The number -15 is less than the number -12.

F. The number 12 is greater than the number -12.

1. $|-12| > -15$
2. $-15 < -12$
3. $|-15| > |-12|$
4. $|-12| = 12$
5. $12 > -12$
6. $|12| = |-12|$

Solution

- A: 3
- B: 4
- C: 1
- D: 6
- E: 2
- F: 5

(From Unit 7, Lesson 7.)

Problem 5

Statement

Here are five sums. Use the distributive property to write each sum as a product with two factors.

a. $2a + 7a$

b. $5z - 10$

c. $c - 2cd$

d. $r + r + r + r$

e. $2x - \frac{1}{2}$

Solution

Answers vary. Sample responses:

a. $(2 + 7)a$ or $9a$

b. $5(z - 2)$

c. $c(1 - 2d)$

d. $(1 + 1 + 1 + 1)r$ or $4r$

e. $2(x - \frac{1}{4})$ or $\frac{1}{2}(4x - 1)$

(From Unit 6, Lesson 11.)