## Lesson 3: Congruent Triangles, Part 1

* Let’s use transformations to be sure that two triangles are congruent.

### 3.1: True or . . . Sometimes True?: Triangles

If triangle  is congruent to triangle . . .

1. What must be true?
2. What could possibly be true?
3. What definitely can’t be true?

### 3.2: Invisible Triangles

Player 1: You are the transformer. Take the transformer card.

Player 2: Select a triangle card. Do not show it to anyone. Study the diagram to figure out which sides and which angles correspond. Tell Player 1 what you have figured out.

Player 1: Take notes about what they tell you so that you know which parts of their triangles correspond. Think of a sequence of rigid motions you could tell your partner to get them to take one of their triangles onto the other. Be specific in your language. The notes on your card can help with this.

Player 2: Listen to the instructions from the transformer. Use tracing paper to follow their instructions. Draw the image after each step. Let them know when they have lined up 1, 2, or all 3 vertices on your triangles.

#### Are you ready for more?

Replay invisible triangles, but with a twist. This time, the transformer can only use reflections—the last 2 sentence frames on the transformer card. You may wish to include an additional sentence frame: Reflect \_\_\_\_\_ across the angle bisector of angle \_\_\_\_\_.

### 3.3: Why Do They Coincide?

Noah and Priya were playing Invisible Triangles. For card 3, Priya told Noah that in triangles and :



Here are the steps Noah had to tell Priya to do before all 3 vertices coincided:

* Translate triangle by the directed line segment from to .
* Rotate the image, triangle , using  as the center, so that rays and line up.
* Reflect the image, triangle , across line .

After those steps, the triangles were lined up perfectly. Now Noah and Priya are working on explaining why their steps worked, and they need some help. Answer their questions.

First, we translate triangle by the directed line segment from to . Point will coincide with because we defined our transformation that way. Then, rotate the image, triangle , by the angle , so that rays and line up.

1. We know that rays and line up because we said they had to, but why do points and have to be in the exact same place?
2. Finally, reflect the image, triangle across .
   1. How do we know that now, the image of ray  and ray  will line up?
   2. How do we know that the image of point  and point  will line up exactly?

### Lesson 3 Summary

If all corresponding parts of 2 triangles are congruent, then one triangle can be taken exactly onto the other triangle using a sequence of translations, rotations, and reflections. The congruence of corresponding parts justifies that the vertices of the triangles will line up exactly.

One of the most common ways to line up the vertices is through a translation to get one pair of vertices to line up, followed by a rotation to get a second pair of vertices to line up, and if needed, a reflection to get the third pair of vertices lined up. There are multiple ways to justify why the vertices must line up if the triangles are congruent, but here is one way to do it:

First, translate triangle by the directed line segment from to . Points and coincide after translating because we defined our translation that way! Then, rotate the image of triangle  using as the center, so that rays and line up.



We know that rays and line up because that’s how we defined the rotation. The distance is the same as the distance , because translations and rotations don’t change distances. Since points and are the same distance along the same ray from , they have to be in the same place.



If necessary, reflect triangle across so that the image of is on the same side of as . We know angle is congruent to angle because translation, rotation, and reflection don’t change angles.



must be on ray since both and are on the same side of and make the same angle with it at . We know the distance is the same as the distance , so that means is the same distance from as is from (because translations and rotations preserve distance). Since and are the same distance along the same ray from , they have to be in the same place.



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