Lesson 10: Interpreting Inequalities

Goals

- Critique (orally and in writing) possible values given for a situation with more than one constraint, including whether fractional or negative values are reasonable.
- Interpret unbalanced hanger diagrams (orally and in writing) and write inequality statements to represent relationships between the weights on an unbalanced hanger diagram.
- Write and interpret inequality statements that include more than one variable.

Learning Targets

- I can explain what the solution to an inequality means in a situation.
- I can write inequalities that involves more than one variable.

Lesson Narrative

In this final lesson on inequalities, students explore situations in which some of the solutions to inequalities do not make sense in the situation’s context. Students learn to think carefully about a situation’s constraints when coming up with reasonable solutions to an inequality. Students also see that inequalities can represent a comparison of two or more unknown quantities.

Alignments

Addressing

- 6.EE.A.2.b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8+7)$ as a product of two factors; view $(8+7)$ as both a single entity and a sum of two terms.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.8: Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share
- True or False

Student Learning Goals

Let’s examine what inequalities can tell us.

10.1 True or False: Fractions and Decimals

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to reason about properties of operations in equivalent expressions. While students may evaluate each side of the equation to determine if it is true or false, encourage students to think about the properties of arithmetic operations in their reasoning (MP7).

Addressing

- 6.EE.A.2.b

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 1 minute of quiet think time and follow with a whole-class discussion.

Student Task Statement

Is each equation true or false? Be prepared to explain your reasoning.

1. $3(12+5)=(3\cdot 12)\cdot(3\cdot 5)$
2. $\frac{1}{3}\cdot\frac{3}{4}=\frac{3}{4}\cdot\frac{2}{6}$
3. $2\cdot(1.5)\cdot 12=4\cdot(0.75)\cdot 6$

Student Response

1. False. Possible responses: 51 does not equal 540; When we use the distributive property, the 2 terms on the right side of the equation should be added not multiplied.
2. True. Possible responses: Both sides equal $\frac{1}{4}$; Since $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, we can see that the left side and right side of this equation show the same numbers being multiplied just in a

different order. By the commutative property of multiplication, both sides of this equation are equal.

3. False. Possible responses: 36 does not equal 18; If we change the order and the groupings of the numbers on both sides using the commutative and associative properties of multiplication, we can group the whole numbers together like this: $(2 \cdot 12) \cdot 1.5 = (4 \cdot 6) \cdot 0.75$, so that the equation becomes: $24 \cdot 1.5 = 24 \cdot 0.75$. Since 1.5 does not equal 0.75, we see that the two sides of this equation are not equal.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. After each true equation, ask students if they could rely on that same reasoning to think about or solve other problems that are similar in type. After each false equation, ask students how we could make the equation true.

To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to _____’s reasoning?”

10.2 Basketball Game

15 minutes
Students interpret inequalities that represent constraints or conditions in a real-world problem. They find solutions to an inequality and reason about the context’s limitations on solutions (MP2).

Addressing

- 6.EE.B.5
- 6.EE.B.8

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Allow students 10 minutes quiet work time to complete all questions followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Provide students with access to blank number lines. Encourage students to attempt more than one strategy for at least one of the problems.
Supports accessibility for: Visual-spatial processing; Organization

Anticipated Misconceptions

Students might have trouble interpreting $15 < n$ because of the placement of the variable on the right side of the inequality. Encourage students to reason about the possible values of n that would make this inequality true.

Student Task Statement

Noah scored n points in a basketball game.

1. What does $15 < n$ mean in the context of the basketball game?
2. What does $n < 25$ mean in the context of the basketball game?
3. Draw two number lines to represent the solutions to the two inequalities.
4. Name a possible value for n that is a solution to both inequalities.
5. Name a possible value for n that is a solution to $15 < n$, but not a solution to $n < 25$.
6. Can -8 be a solution to n in this context? Explain your reasoning.

Student Response

1. Noah scored more than 15 points.
2. Noah scored less than 25 points.
3. $n > 15$: open circle at 15, shaded line and arrow to the right, $n < 25$: open circle at 25, shaded line and arrow to the left.
4. Answers vary. Sample response: $n = 17$.
5. Answers vary. Sample response: $n = 30$.
6. No, -8 cannot be a solution in this context because the score of a basketball game cannot be below 0.

Activity Synthesis

Invite selected students to justify their answers. Extend the discussion of the basketball game to consider how scoring works and whether any number could represent the points scored by a player. For example, could a player have scored 1 point? $2\frac{1}{2}$ points? 0 points? -3 points? Is it reasonable for a player to score 200 points in a game?

Access for English Language Learners

Reading, Speaking, Representing: MLR3 Clarify, Critique, Correct. To support students in their ability to read inequalities and their number line representations as well as to critique the reasoning of others, present an incorrect response to the prompt “Draw two number lines that represent the two inequalities.” For example, use the first number line representing $15 < n$ to contain an error by shading the line and arrow to the left instead of the right. Then correctly display the second number line. Tell students the response is not drawn correctly even though the numbers and open circles are correct. Ask students to identify the error, explain how they know that the response is incorrect, and revise the incorrect number line. This helps prompt student reflection with an incorrect written mathematical statement, and for students to improve upon the written work by correcting errors and clarifying meaning.
Design Principle(s): Optimize output (for explanation)

10.3 Unbalanced Hangers

15 minutes
In this activity, students describe unbalanced hanger diagrams with inequalities. Students construct viable arguments and critique the reasoning of others during partner and whole-class discussions about how unknown values relate to each other (MP3).

Addressing

- 6.EE.B.6
- 6.EE.B.8

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

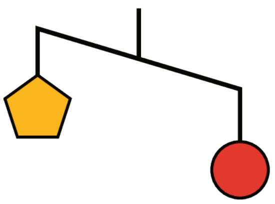
Arrange students in groups of 2. Give students 7 minutes quiet work time, followed by 3–5 minutes for partner discussion. Tell students to check in with their partners and, if there are disagreements, work to come to an agreement. Follow with whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2–3 minutes of work time. Invite students to share the strategies they used for the first unbalanced hanger.
Supports accessibility for: Organization; Attention

Student Task Statement

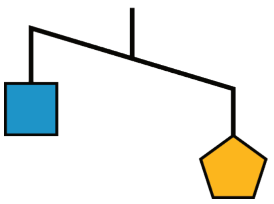
1. Here is a diagram of an unbalanced hanger.



   a. Jada says that the weight of one circle is greater than the weight of one pentagon. Write an inequality to represent her statement. Let p be the weight of one pentagon and c be the weight of one circle.

   b. A circle weighs 12 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

2. Here is another diagram of an unbalanced hanger.



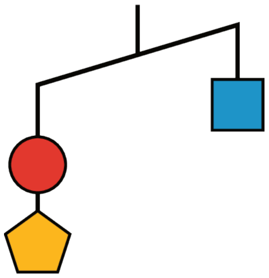
   a. Write an inequality to represent the relationship of the weights. Let p be the weight of one pentagon and s be the weight of one square.

   b. One pentagon weighs 8 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

   c. Graph the solutions to this inequality on a number line.

3. Based on your work so far, can you tell the relationship between the weight of a square and the weight of a circle? If so, write an inequality to represent that relationship. If not, explain your reasoning.

4. This is another diagram of an unbalanced hanger.



Andre writes the following inequality: $c + p < s$. Do you agree with his inequality? Explain your reasoning.

5. Jada looks at another diagram of an unbalanced hangar and writes: $s + c > 2t$, where t represents the weight of one triangle. Draw a sketch of the diagram.

Student Response

1. a. $p < c$

   b. $p < 12$. This means that the pentagon weighs less than 12 ounces.

2. a. $s < p$

   b. $s < 8$. This means that the square weighs less than 8 ounces.

   c. On the graph, there is an open circle at 8 and all values less than 8 are shaded.

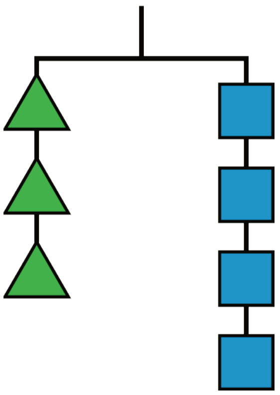
3. Yes, the square weighs less than the circle, so $s < c$. Explanations vary. Sample response: The square weighs less than the pentagon, which in turn weighs less than the circle. So the square weighs less than the circle.

4. No. Explanations vary. Sample response: The combined weight of the circle and the pentagon is greater than the weight of the square, so the inequality should be $c + p > s$.

5. The diagram should show a circle and a square on one side and two triangles on the other. The side with the circle and square outweighs the side with the triangles so it hangs lower.

Are You Ready for More?

Here is a picture of a balanced hanger. It shows that the total weight of the three triangles is the same as the total weight of the four squares.



1. What does this tell you about the weight of one square when compared to one triangle? Explain how you know.

2. Write an equation or an inequality to describe the relationship between the weight of a square and that of a triangle. Let s be the weight of a square and t be the weight of a triangle.

Student Response

1. The weight of a square is less than the weight of a triangle, or the weight of a triangle is more than the weight of a square. Sample reasoning: It takes 4 squares to weigh the same as 3 triangles, so each square must be lighter than each triangle.

2. $s < t$; $t > s$; or $3t = 4s$.

Activity Synthesis

The purpose of the discussion is to let students explain how they used inequalities to compare the weights of different shapes on the hanger diagrams. Invite groups to describe any disagreements or difficulties they had and how they resolved them. Select students to share how they reasoned about the quantities when there were two or more unknowns. Ask students if they can think of other situations comparing two or more unknown quantities (people's heights, weights of backpacks). Invite them to represent the quantities with variables and write inequality statements to compare them.

If time allows, display a circle opposite a pentagon and square for all to see. Ask students which side they think would be heavier. In this case, which side is heavier depends on how much the square weighs. Since the circle is 12 ounces and the pentagon is 8 ounces, the square would have to be less than 4 ounces for the circle to be heavier and greater than 4 ounces for the pentagon and square to be heavier.

Access for English Language Learners

Representing, Speaking: MLR8 Discussion Supports. To help students be more precise in their use of language related representations of inequalities, use the sentence frames to support their discussion. Some examples include "I know the circle is heavier because _____," "The inequality _____ represents the hanger because _____," or "If the circle weighs 12 ounces, I know _____."
Design Principle(s): Support sense-making; Maximize meta-awareness

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Lesson Synthesis

Ask students to think about situations where limits or ranges of values can be important to public health or safety (e.g., weight limitations on an elevator, safe dosage for medication, tire pressure, speed limit, temperature for growing carrots, etc.). Ask them to define variables and write inequalities to represent these situations. Select 2 or 3 students to share their responses. Record and display those responses for all to see using the appropriate symbols. Here are some questions to consider during discussion:

- "Do solutions that are not whole numbers make sense in this situation?"
- "Do solutions that are negative numbers make sense in this situation?"

- “Do the numbers on the boundary count as solutions? For example, if an elevator has a maximum capacity of 2,500 pounds, can it handle exactly 2,500 pounds?”

10.4 Lin and Andre’s Heights

Cool Down: 5 minutes

Addressing

- 6.EE.B.6
- 6.EE.B.8

Student Task Statement

1. Lin says that the inequalities $h > 150$ and $h < 160$ describe her height in centimeters. What do the inequalities tell us about her height?

2. Andre notices that he is a little taller than Lin but is shorter than their math teacher, who is 164 centimeters tall. Write two inequalities to describe Andre's height. Let a be Andre's height in centimeters.

3. Select all heights that could be Andre's height in centimeters. If you get stuck, consider drawing a number line to help you.

    a. 150

    b. 154.5

    c. 160

    d. 162.5

    e. 164

Student Response

1. These inequalities tell us that Lin is between 150 and 160 cm tall.
2. $a < 164$ and $a > h$ (or $h < a$).
3. B, C, D

Student Lesson Summary

When we find the solutions to an inequality, we should think about its context carefully. A number may be a solution to an inequality outside of a context, but may not make sense when considered in context.

- Suppose a basketball player scored more than 11 points in a game, and we represent the number of points she scored, s, with the inequality $s > 11$. By looking only at $s > 11$, we can say that numbers such as 12, $14\frac{1}{2}$, and 130.25 are all solutions to the inequality because they each make the inequality true.

$$12 > 11 \qquad 14\frac{1}{2} > 11 \qquad 130.25 > 11$$

In a basketball game, however, it is only possible to score a whole number of points, so fractional and decimal scores are not possible. It is also highly unlikely that one person would score more than 130 points in a single game.

In other words, the context of an inequality may limit its solutions.

Here is another example:

- The solutions to $r < 30$ can include numbers such as $27\frac{3}{4}$, 18.5, 0, and -7. But if r represents the number of minutes of rain yesterday (and it did rain), then our solutions are limited to positive numbers. Zero or negative number of minutes would not make sense in this context.

  To show the upper and lower boundaries, we can write two inequalities:

  $$0 < r \qquad r < 30$$

Inequalities can also represent comparison of two unknown numbers.

- Let's say we knew that a puppy weighs more than a kitten, but we did not know the weight of either animal. We can represent the weight of the puppy, in pounds, with p and the weight of the kitten, in pounds, with k, and write this inequality:

  $$p > k$$

Lesson 10 Practice Problems

Problem 1

Statement

There is a closed carton of eggs in Mai's refrigerator. The carton contains e eggs and it can hold 12 eggs.

a. What does the inequality $e < 12$ mean in this context?

b. What does the inequality $e > 0$ mean in this context?

c. What are some possible values of e that will make both $e < 12$ and $e > 0$ true?

Solution

a. There are *fewer* than 12 eggs in the carton; the carton is not full.

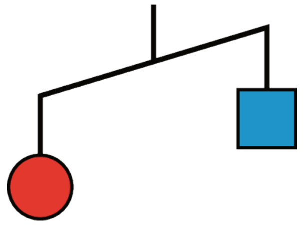
b. There are *more* than 0 eggs in the carton; the carton is not empty.

c. There could be as few as 1 egg or as many as 11 eggs in the carton: any whole number of eggs from 1 up to 11.

Problem 2

Statement

Here is a diagram of an unbalanced hanger.



a. Write an inequality to represent the relationship of the weights. Use s to represent the weight of the square in grams and c to represent the weight of the circle in grams.

b. One red circle weighs 12 grams. Write an inequality to represent the weight of one blue square.

c. Could 0 be a value of s? Explain your reasoning.

Solution

a. $s < c$

b. $s < 12$

c. No, 0 could not be a value of s because the square represents an object. It must have some weight, even if it is very small.

Problem 3

Statement

a. Jada is taller than Diego. Diego is 54 inches tall (4 feet, 6 inches). Write an inequality that compares Jada's height in inches, j, to Diego's height.

b. Jada is shorter than Elena. Elena is 5 feet tall. Write an inequality that compares Jada's height in inches, j, to Elena's height.

Solution

a. $j > 54$

b. $j < 60$

(From Unit 7, Lesson 8.)

Problem 4

Statement

Tyler has more than \$10. Elena has more money than Tyler. Mai has more money than Elena. Let t be the amount of money that Tyler has, let e be the amount of money that Elena has, and let m be the amount of money that Mai has. Select **all** statements that are true:

A. $t < j$

B. $m > 10$

C. $e > 10$

D. $t > 10$

E. $e > m$

F. $t < e$

Solution

["A", "B", "C", "F"]

Problem 5

Statement

Which is greater, $\frac{-9}{20}$ or -0.5? Explain how you know. If you get stuck, consider plotting the numbers on a number line.

Solution

$\frac{-9}{20}$ is larger. Explanations vary. Sample explanation: $\frac{-9}{20} = -0.45$, and this is to the right of -0.5 on the number line. So, $\frac{-9}{20}$ is larger.

(From Unit 7, Lesson 3.)

Problem 6

Statement

Select **all** the expressions that are equivalent to $\left(\frac{1}{2}\right)^3$.

A. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

B. $\frac{1}{2^3}$

C. $\left(\frac{1}{3}\right)^2$

D. $\frac{1}{6}$

E. $\frac{1}{8}$

Solution

["A", "B", "E"]

(From Unit 6, Lesson 13.)