## Lesson 21: Odd and Even Numbers

* Let’s explore even and odd numbers.

### 21.1: Math Talk: Evens and Odds

Evaluate mentally.

$64+88$

$65+89$

$14⋅5$

$14⋅4$

### 21.2: Always Even, Never Odd

Here are some statements about the sums and products of numbers. For each statement:

* decide whether it is *always* true, true for *some* numbers but not others, or *never* true
* use examples to explain your reasoning
1. Sums:
	1. The sum of 2 even numbers is even.
	2. The sum of an even number and an odd number is odd.
	3. The sum of 2 odd numbers is odd.
2. Products:
	1. The product of 2 even numbers is even.
	2. The product of an even number and an odd number is odd.
	3. The product of 2 odd numbers is odd.

### 21.3: Even + Odd = Odd

How do we know that the sum of an even number and an odd number *must* be odd? Examine this proof and answer the questions throughout.

Let $a$ represent an even number, $b$ represent an odd number, and $s$ represent the sum $a+b$.

1. What does it mean for a number to be even? Odd?
* Assume that $s$ is even, then we will look for a reason the original statement cannot be true. Since $a$ and $s$ are even, we can write them as 2 times an integer. Let $a=2k$ and $s=2m$ for some integers $k$ and $m$.
1. Can this always be done? To convince yourself, write 4 different even numbers. What is the value for $k$ for each of your numbers when you set them equal to $2k$?
* Then we know that $a+b=s$ and $2k+b=2m$.
* Divide both sides by 2 to get that $k+\frac{b}{2}=m$.
* Rewrite the equation to get $\frac{b}{2}=m−k$.
* Since $m$ and $k$ are integers, then $\frac{b}{2}$ must be an integer as well.
1. Is the difference of 2 integers always an integer? Select 4 pairs of integers and subtract them to convince yourself that their difference is always an integer.
2. What does the equation $\frac{b}{2}=m−k$ tell us about $\frac{b}{2}$? What does that mean about $b$?
3. Look back at the original description of $b$. What is wrong with what we have discovered?

The logic for everything in the proof works, so the only thing that could’ve gone wrong was our assumption that $s$ is even. Therefore, $s$ must be odd.



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