Lesson 14: Distances on a Coordinate Plane

Goals

- Compare and contrast (orally and in writing) the coordinates for points in different locations on the coordinate plane.
- Determine the vertical or horizontal distance between two points on the coordinate plane that share the same x- or y-coordinate.
- Generalize (orally) about the coordinates of points that are reflected across the x- or y-axis.

Learning Targets

- I can find horizontal and vertical distances between points on the coordinate plane.

Lesson Narrative

In this lesson, students explore ways to find vertical and horizontal distances in the coordinate plane. In the first activity, students use repeated reasoning to explore the relationship between points with opposite coordinates (MP8). In the second activity, students develop strategies for finding the distance between two points where the coordinates might not be integers. Students can use previous strategies, such as considering the distance of a point from zero, or counting squares. Students will use these skills in Grade 7 to find distances on maps. In Grade 8, they will use these skills to draw slope triangles in the coordinate plane and find the lengths of their sides when considering graphs of proportional and non-proportional relationships.

Alignments

Addressing

- 6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.C.6.b: Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share

Required Preparation

It may be useful, but not required, to provide access to tracing paper or rulers to help students think through the misconception that the length of a diagonal is equal to the length of a related horizontal or vertical distance in the coordinate plane.

Student Learning Goals

Let's explore distance on the coordinate plane.

14.1 Coordinate Patterns

Warm Up: 10 minutes (there is a digital version of this activity)
The purpose of this warm-up is for students to review plotting and labeling points that include negative coordinates and use repeated reasoning to generalize patterns in the coordinates of points in each quadrant (MP8).

Addressing

- 6.NS.C.6

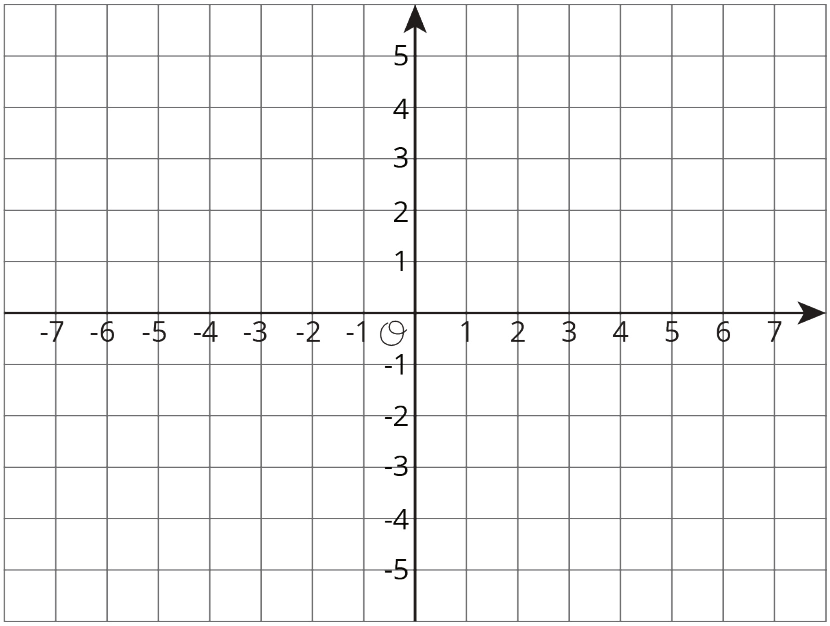
Launch

Arrange students in groups of 4. Assign each person in a group a different quadrant. Tell students you will give them 2 minutes to plot and label at least three points in their assigned quadrant, and up to six if they have time. Give students 2 minute of quiet work time. Give students 4 minutes to share their points and their coordinates with their small group and look for patterns in the coordinates of points in each quadrant. Follow with whole-class discussion.

Though the paper and pencil version may be preferred, a digital applet is available for this activity.

Student Task Statement

Plot points in your assigned quadrant and label them with their coordinates.



Student Response

Responses vary. Sample response: The points $(-5, 5)$, $(-3, 6)$, and $(-2, 4)$ are all in quadrant II. If the first coordinate is negative and the second coordinate is positive, then the point is in quadrant II.

Activity Synthesis

The focus of the discussion is for students to explain why the following patterns emerge:

- In quadrants I and IV, the x-coordinate of a point (or the first number in an ordered pair) is positive.
- In quadrants II and III, the x-coordinate of a point is negative.
- In quadrants I and II, the y-coordinate of a point (or the second number in an ordered pair) is positive.
- In quadrants III and IV, the y-coordinate of a point in is negative.

Ask the students to share any patterns they noticed among the coordinates of the points in each quadrant. After each student shares, ask the rest of the class if they noticed the same pattern within their small group. Record and display these patterns for all to see. If possible, plot and label a few example points in each quadrant based on students' observations.

14.2 Signs of Numbers in Coordinates

15 minutes (there is a digital version of this activity)

The purpose of this task is for students to connect opposite signs in coordinates with reflections across one or both axes. Students investigate relationships between several pairs of points in order to make this connection more generally (MP8). The square grid, spaced in units, means that students can use counting squares as a strategy for finding distances.

The use of the word "reflection" is used informally to describe the effect of opposite signs in coordinates. In grade 8, students learn a more precise, technical definition of the word "reflection" as it pertains to rigid transformations of the plane.

Addressing

- 6.NS.C.6.b
- 6.NS.C.8

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 3-4 minutes of quiet work time and 1-2 minutes to check results with their partner for questions 1 and 2. Tell students to pause after question 2 for whole-class discussion. At that time, briefly check that students have the correct coordinates for points A, B, C, D, and E before moving on to the rest of the questions. Give students 4 minutes to answer the rest of the questions with their partner, followed by whole-class discussion.

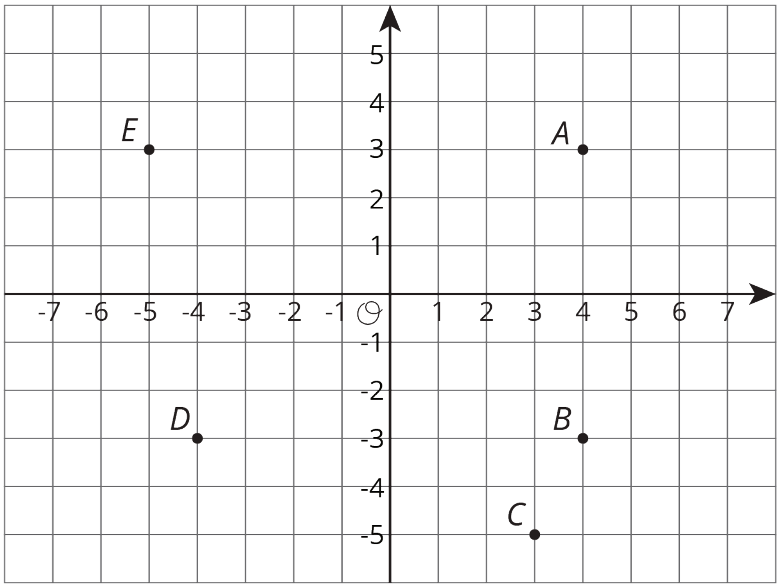
Though the paper and pencil version may be preferred, a digital applet is available for this activity.

Access for English Language Learners

Speaking: MLR1 Stronger and Clearer Each Time. Use this routine with successive pair shares to give students a structured opportunity to revise and refine their response to “How far away are the points from the x-axis and y-axis?” Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help teams strengthen their ideas and clarify their language (e.g., “How did you use the horizontal/vertical distance to help?”) Students can borrow ideas and language from each partner to strengthen their final response.
Design Principle(s): Cultivate conversation

Student Task Statement

1. Write the coordinates of each point.



$A =$

$B =$

$C =$

$D =$

$E =$

2. Answer these questions for each pair of points.

    - How are the coordinates the same? How are they different?
    - How far away are they from the y-axis? To the left or to the right of it?
    - How far away are they from the x-axis? Above or below it?

    a. A and B

    b. B and D

    c. A and D

    Pause here for a class discussion.

3. Point F has the same coordinates as point C, except its y-coordinate has the opposite sign.

    a. Plot point F on the coordinate plane and label it with its coordinates.

    b. How far away are F and C from the x-axis?

    c. What is the distance between F and C?

4. Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.

    a. Plot point G on the coordinate plane and label it with its coordinates.

    b. How far away are G and E from the y-axis?

    c. What is the distance between G and E?

5. Point H has the same coordinates as point B, except its *both* coordinates have the opposite sign. In which quadrant is point H?

Student Response

1. $A = (4, 3)$, $B = (4, \text{-}3)$, $C = (3, \text{-}5)$, $D = (\text{-}4, \text{-}3)$, $E = (\text{-}5, 3)$
2. Answers vary. Sample response:
    a. A and B have the same x-coordinate (4) but opposite y-coordinates (3 and -3). Both points are 4 units to the right of the y-axis. Both are 3 units from the x-axis, but A is above the x-axis and B is below it.

    b. B and D have the opposite x-coordinates (4 and -4) but the same y-coordinate of -3. Both points are 4 units away from the y-axis but one is to the left of it and the other to the right of it. Both are 3 units below the x-axis.

    c. A and D have opposite x-coordinates (4 and -4) and y-coordinates (3 and -3). Both are 4 units away from y-axis and 3 units away from the x-axis, but in opposite directions.
3. a. $F = (3, 5)$

    b. Each point is 5 units from the x-axis.

    c. 10 units
4. a. $G = (5, 3)$

    b. Each point is 5 units from the y-axis.

    c. 10 units
5. Quadrant II

Activity Synthesis

The key takeaway is that coordinates with opposite signs correspond to reflections across the axes. Encourage students to share ideas that they discussed with their partner. Ask students first about what patterns they noticed for pairs of points whose x-coordinates had opposite signs. Push students to give specific examples of pairs of points and their coordinates, and to describe what opposite x-coordinates mean in terms of the coordinate plane. Record students' explanations for all to see. Students may use phrasing like "the point flips across the y-axis." This would be a good opportunity to use the word "reflection" and discuss the similarities between reflections across the y-axis and reflections in a mirror.

Repeat this discussion for pairs of points where the y-coordinates had opposite signs to get to the idea that they are reflections across the x-axis. Close by discussing the relationship between points H and B, where both the x- and y-coordinates have opposite signs. Ask students how they might describe the relationship between H and B visually on the coordinate plane. While students may describe the relationship in terms of 2 reflections (once across the x-axis and again across the y-axis or vice versa), it is not expected that students see their relationship in terms of rotation.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students describe what they noticed, use color and annotations to scribe their thinking on a display of the coordinate plane that is visible for all students.
Supports accessibility for: Visual-spatial processing; Conceptual processing

14.3 Finding Distances on a Coordinate Plane

15 minutes (there is a digital version of this activity)
The purpose of this activity is for students to develop strategies for finding the distance between two points in the coordinate plane when the coordinates might not be integers. These distances are restricted to horizontal and vertical distances; use of the general two-dimensional distance formula is not expected, nor are students expected to add or subtract negative numbers fluently. More general strategies for finding distance in the coordinate plane are developed in grade 8, and rational number arithmetic is developed more completely in grade 7.

Addressing

- 6.NS.C.8

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 5–6 minutes of quiet work time followed by 3–4 minutes to check results with their partner. Follow with a whole-class discussion.

Though the paper and pencil version may be preferred, a digital applet is available for this activity.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students within the first 2–3 minutes of work time to ensure they understood the directions. If students are unsure how to begin, suggest that they use previous strategies, such as considering the distance of a point from zero.
Supports accessibility for: Organization; Attention

Access for English Language Learners

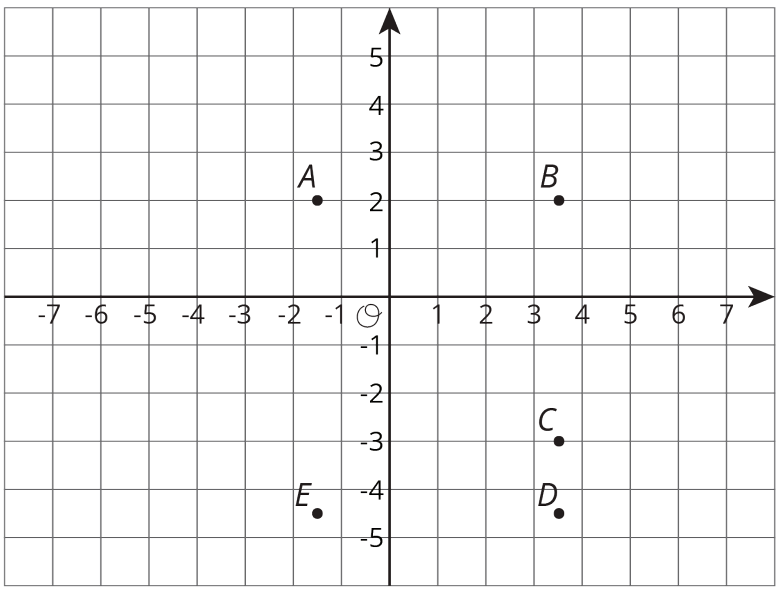
Conversing: MLR5 Co-Craft Questions. Display only the first line of the first question with the coordinate grid and ask pairs of students to write possible mathematical questions about the coordinate grid. Then, invite pairs to share their questions with the class. This helps students begin the conversation of horizontal and vertical distance and even possibly talking about diagonal distances between two points.
Design Principle(s): Cultivate conversation

Anticipated Misconceptions

Some students may assume that a diagonal line across a number of squares has the same length. Ask students to use a ruler or tracing paper to compare the length of the diagonal distance in question against the horizontal or vertical distance the student claims is equal.

Student Task Statement

1. Label each point with its coordinates.



2. Find the distance between each of the following pairs of points.

    a. Point B and C

    b. Point D and B

    c. Point D and E

3. Which of the points are 5 units from $(-1.5, -3)$?

4. Which of the points are 2 units from $(0.5, -4.5)$?

5. Plot a point that is both 2.5 units from A and 9 units from E. Label that point M and write down its coordinates.

Student Response

1. $A = (\text{-}1.5, 2)$, $B = (3.5, 2)$, $C = (3.5, \text{-}3)$, $D = (3.5, \text{-}4.5)$, $E = (\text{-}1.5, \text{-}4.5)$
2. a. 5 units

   b. 6.5 units

   c. 5 units
3. Points A and C
4. Point E
5. $M = (\text{-}1.5, 4.5)$

Are You Ready for More?

Priya says, “There are exactly four points that are 3 units away from $(\text{-}5, 0)$.” Lin says, “I think there are a whole bunch of points that are 3 units away from $(\text{-}5, 0)$.”

Do you agree with either of them? Explain your reasoning.

Student Response

Answers vary. Possible response: I agree with Lin, because I can measure a length of 3 units from the point $(\text{-}5, 0)$ in any direction. I don't have to just go up, down, left, or right.

(Students will learn in grade 7 that a circle is the (infinite) set of points that are equidistant from a center point. For grade 6 we focus on vertical and horizontal distances where naming the coordinates of points at a given distance is clear.)

Activity Synthesis

The important idea students should come away with is that they can continue to use strategies they have developed for finding horizontal and vertical distances even without a context. Here are some questions for discussion:

- How did finding lengths in this activity compare to the previous activity? How were they the same? How were they different?
- Were any of the points reflections across the axes? How could you tell by looking at the coordinate plane? How could you tell by looking at the coordinates?
- Were there any points of disagreement with your partner? How did you come to agreement?

Lesson Synthesis

The activities in this lesson asked students to analyze the effect of replacing coordinates with their opposites and to find horizontal and vertical distances in the coordinate plane. Here are some questions for discussion:

- "Without graphing, what can you say about the points $(5, \text{-}3)$ and $(5, 3)$ on the coordinate plane?" (Sample responses: The first point is in quadrant IV and the second point is in quadrant I. They are both 3 units from the x-axis. They are reflections across the x-axis. They are 6 units apart. They are both on the same vertical line.)
- "Without graphing, what can you say about $(\text{-}6, 4)$ and $(6, 4)$ on the coordinate plane?" (Sample responses: The first point is in quadrant II and the second point is in quadrant I. They are both 6 units from the y-axis. They are reflections across the y-axis. They are 12 units apart. They are both on the same horizontal line)
- "Without graphing, what can you say about $(2.5, 1)$ and $(6, 1)$ on the coordinate plane?" (Sample responses: they are both in quadrant I. They are both on the same horizontal line because they have the same y-value. The second point is 3.5 units to the right of the first point.)

If time allows, challenge students to draw, for example, a rectangle with given side lengths, and identify its vertices. This will lead nicely into the next lesson, where students will explore shapes in the coordinate plane. Select student responses to display for all to see.

14.4 Points and Distances

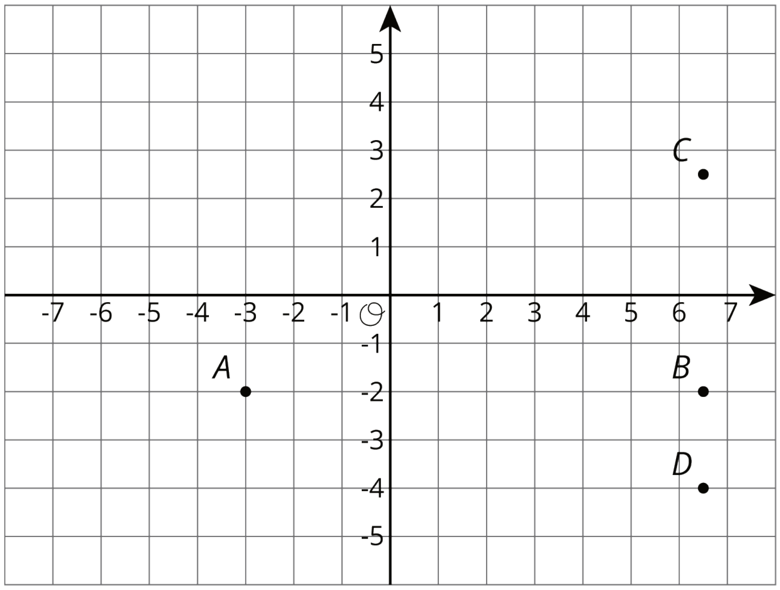
Cool Down: 5 minutes

Addressing

- 6.NS.C.8

Student Task Statement

Here are four points on a coordinate plane.



1. What is the distance between points A and B?

2. What is the distance between points C and D?

3. Plot the point $(-3, 2)$. Label it E.

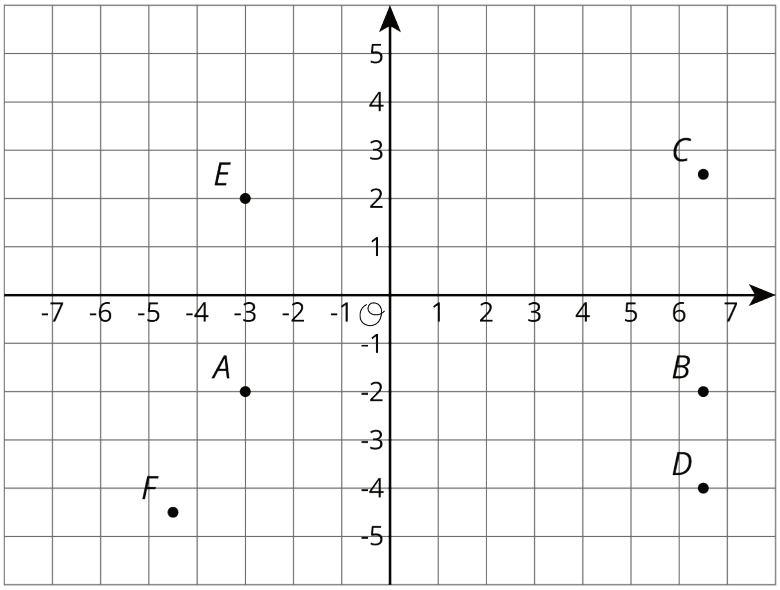
4. Plot the point $(-4.5, -4.5)$. Label it F.

Student Response

1. About 9.5 units

2. About 6.5 units

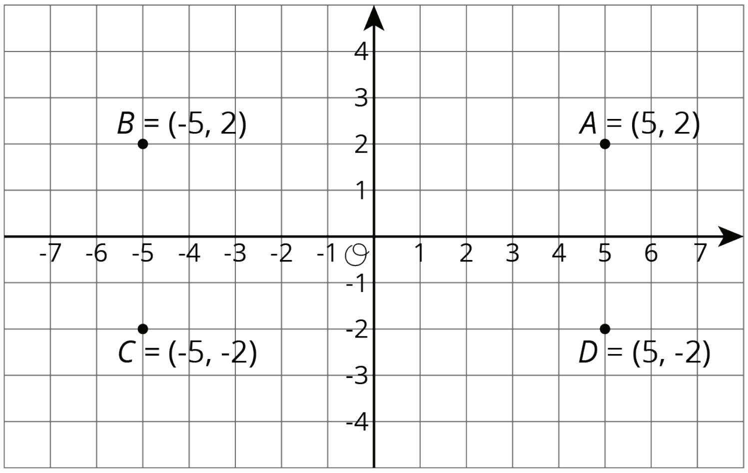
3.



4. See solution for question 3.

Student Lesson Summary

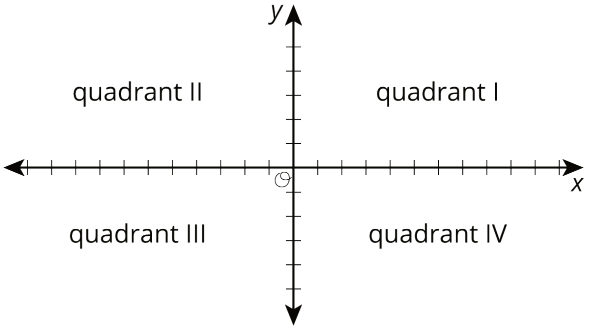
The points $A = (5,2)$, $B = (-5,2)$, $C = (-5,-2)$, and $D = (5,-2)$ are shown in the plane. Notice that they all have almost the same coordinates, except the signs are different. They are all the same distance from each axis but are in different quadrants.



Notice that the vertical distance between points A and D is 4 units, because point A is 2 units above the horizontal axis and point D is 2 units below the horizontal axis. The horizontal distance between points A and B is 10 units, because point B is 5 units to the left of the vertical axis and point A is 5 units to the right of the vertical axis.

We can always tell which quadrant a point is located in by the signs of its coordinates.

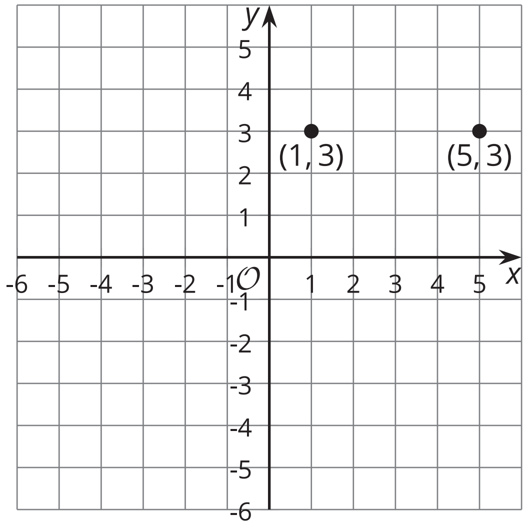
x	y	**quadrant**
positive	positive	I
negative	positive	II
negative	negative	III
positive	negative	IV



In general:

- If two points have x-coordinates that are opposites (like 5 and -5), they are the same distance away from the vertical axis, but one is to the left and the other to the right.
- If two points have y-coordinates that are opposites (like 2 and -2), they are the same distance away from the horizontal axis, but one is above and the other below.

When two points have the same value for the first or second coordinate, we can find the distance between them by subtracting the coordinates that are different. For example, consider $(1, 3)$ and $(5, 3)$:



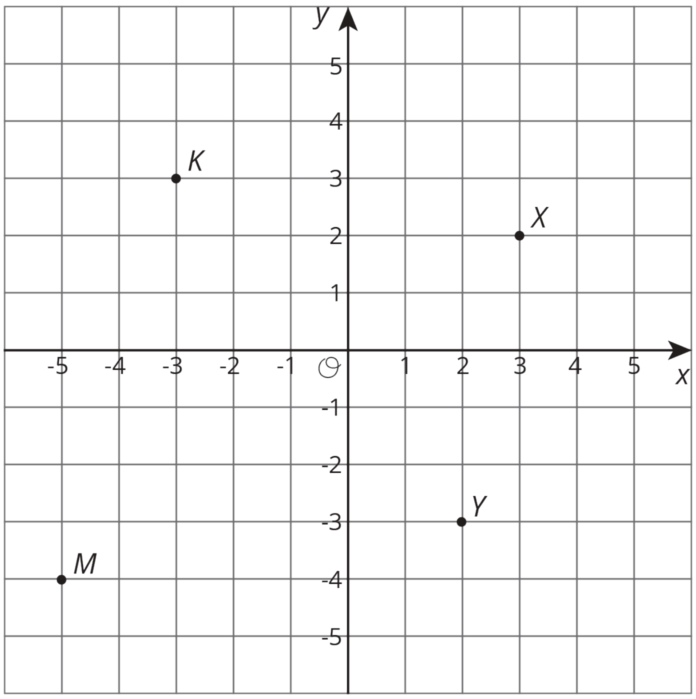
They have the same y-coordinate. If we subtract the x-coordinates, we get $5 - 1 = 4$. These points are 4 units apart.

Lesson 14 Practice Problems

Problem 1

Statement

Here are 4 points on a coordinate plane.



a. Label each point with its coordinates.

b. Plot a point that is 3 units from point K. Label it P.

c. Plot a point that is 2 units from point M. Label it W.

Solution

a. X = $(3, 2)$, Y = $(2, -2)$, K = $(-3, 3)$, M = $(-5, -4)$

b. Answers vary. Sample responses: $(0, 3)$ or $(-3, 6)$

c. Answers vary. Sample responses: $(-3, -4)$ or $(-5, -2)$

Problem 2

Statement

Each set of points are connected to form a line segment. What is the length of each?

a. A = $(3, 5)$ and B = $(3, 6)$

b. C = $(-2, -3)$ and D = $(-2, -6)$

c. E = $(-3, 1)$ and F = $(-3, -1)$

Solution

a. 1 unit

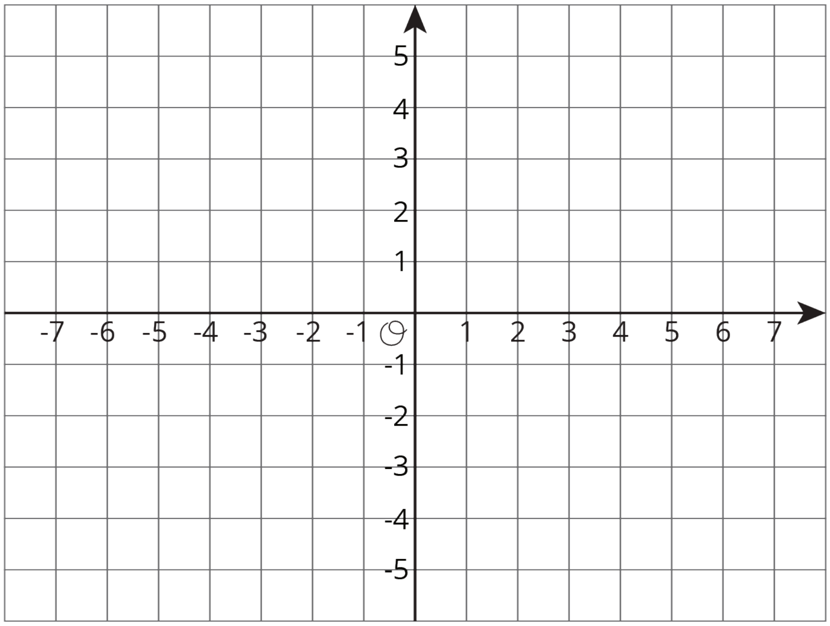
b. 3 units

c. 2 units

Problem 3

Statement

On the coordinate plane, plot four points that are each 3 units away from point $P = (-2, -1)$. Write the coordinates of each point.



Solution

Answers vary. Sample response: $A = (-5, -1)$, $B = (1, -1)$, $C = (-2, 2)$, $D = (-2, -4)$. (Students are unlikely to come up with other possible solutions at this stage.)

Problem 4

Statement

Noah's recipe for sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

a. Noah prepares large batches of sparkling orange juice for school parties. He usually knows the total number of liters, t, that he needs to prepare. Write an equation that shows how Noah can find s, the number of liters of soda water, if he knows t.

b. Sometimes the school purchases a certain number, j, of liters of orange juice and Noah needs to figure out how much sparkling orange juice he can make. Write an equation that Noah can use to find t if he knows j.

Solution

a. $s = \frac{5}{9}t$

b. $t = \frac{9}{4}j$

(From Unit 6, Lesson 16.)

Problem 5

Statement

For a suitcase to be checked on a flight (instead of carried by hand), it can weigh at most 50 pounds. Andre's suitcase weighs 23 kilograms. Can Andre check his suitcase? Explain or show your reasoning. (Note: 10 kilograms ≈ 22 pounds)

Solution

No, Andre will not be able to check his suitcase if they are strict about following the rule. Possible explanation:

1 kg weighs 2.2 pounds, so 23 kg weighs $(2.2) \cdot 23 = 50.6$ pounds.

weight (kilograms)	weight (pounds)
10	22
20	44
1	2.2
3	6.6
23	50.6

(From Unit 3, Lesson 4.)