Lesson 12: Tangent

- Let’s learn more about tangent.

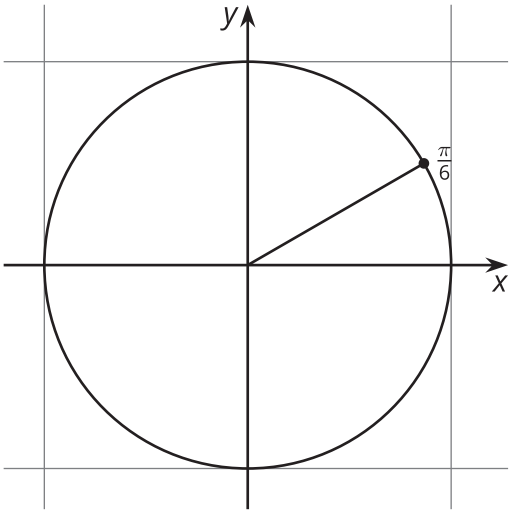
12.1: Notice and Wonder: An Unusual Function

What do you notice? What do you wonder?

θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$
$-\frac{\pi}{2}$	0	-1	
$-\frac{\pi}{3}$	0.5	-0.87	
$-\frac{\pi}{6}$	0.87	-0.5	
0	1	0	
$\frac{\pi}{6}$	0.87	0.5	
$\frac{\pi}{3}$	0.5	0.87	
$\frac{\pi}{2}$	0	1	

12.2: A Tangent Ratio

1. Complete the table. For each positive angle in the table, add the corresponding point and the segment between it and the origin to the unit circle.



θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$
$-\frac{\pi}{2}$	0	-1	
$-\frac{\pi}{3}$	0.5	-0.87	
$-\frac{\pi}{6}$	0.87	-0.5	
0	1	0	
$\frac{\pi}{6}$	0.87	0.5	
$\frac{\pi}{3}$	0.5	0.87	
$\frac{\pi}{2}$	0	1	
$\frac{2\pi}{3}$			
$\frac{5\pi}{6}$			
π			
$\frac{7\pi}{6}$			
$\frac{4\pi}{3}$			
$\frac{3\pi}{2}$			
$\frac{5\pi}{3}$			
$\frac{11\pi}{6}$			
2π			

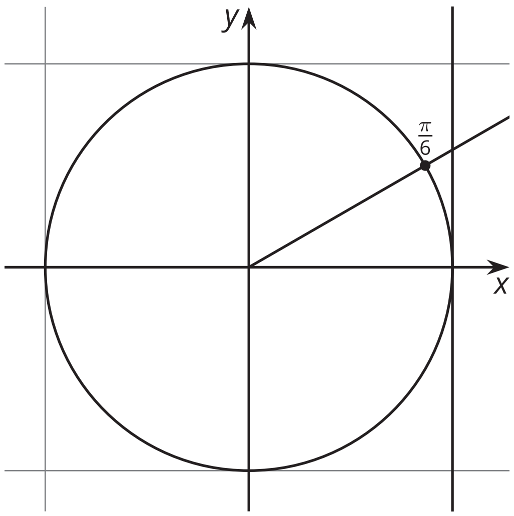
2. How are the values of $\tan(\theta)$ like the values of $\cos(\theta)$ and $\sin(\theta)$? How are they different?

Are you ready for more?

1. Where does the line $x = 1$ intersect the line that passes through the origin and the point corresponding to the angle $\frac{\pi}{6}$?

2. Where does the line $x = 1$ intersect the line that passes through the origin and the point corresponding to the angle θ?

3. Where do you think the name "tangent" of an angle comes from?



12.3: The Tangent Function

Before we graph $y = \tan(\theta)$, let's figure out some things that must be true.

1. Explain why the graph of $\tan(\theta)$ has a vertical asymptote at $x = \frac{\pi}{2}$.

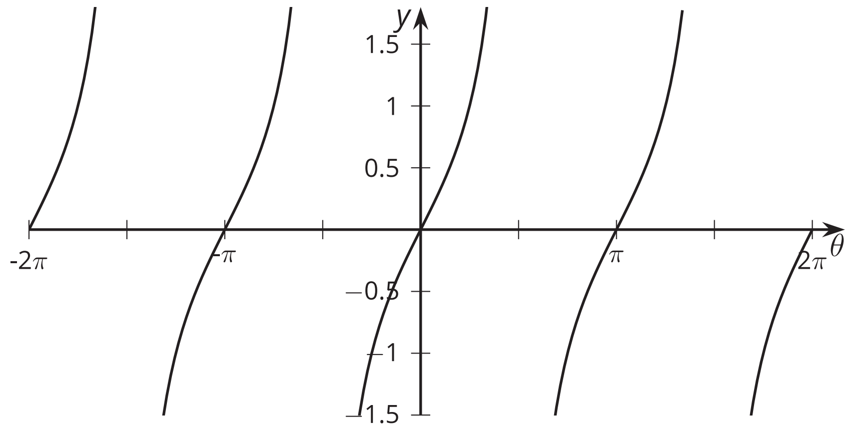
2. Does the graph of $\tan(\theta)$ have other vertical asymptotes? Explain how you know.

3. For which values of θ is $\tan(\theta)$ zero? For which values of θ is $\tan(\theta)$ one? Explain how you know.

4. Is the graph of $\tan(\theta)$ periodic? Explain how you know.

Lesson 12 Summary

The tangent of an angle θ, $\tan(\theta)$, is the quotient of the sine and cosine: $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. Here is a graph of $y = \tan(\theta)$.



We can see from the graph that $\tan(\theta) = 0$ when θ is $\text{-}2\pi, \text{-}\pi, 0, \pi, \text{or } 2\pi$. This makes sense because the sine is 0 for these values of θ. Since sine and cosine are never 0 at the same θ, we can say that tangent has a value of 0 whenever sine has a value of 0.

We can also see the asymptotes of tangent $\text{-}\frac{3\pi}{2}, \text{-}\frac{\pi}{2}, \frac{\pi}{2}, \text{and } \frac{3\pi}{2}$. Let's look more closely at what happens when $\theta = \frac{\pi}{2}$. We have $\sin\frac{\pi}{2} = 1$ and $\cos\frac{\pi}{2} = 0$. This means $\tan\left(\frac{\pi}{2}\right) = \frac{1}{0}$, which is not defined. Whenever $\cos(\theta) = 0$, the tangent is not defined and has a vertical asymptote.

Like the sine and cosine functions, the tangent function is periodic. This makes sense because it is defined using sine and cosine. The period of tangent is only π while the period of sine and cosine is 2π.