Lesson 9: Solving Radical Equations

- Let’s practice solving radical equations.

9.1: Math Talk: Radical Equations

Solve these equations mentally:

$\sqrt[3]{x} = 1$

$\sqrt{7} = \sqrt{x - 1}$

$\sqrt{100} = 2x$

$\sqrt{x + 1} = -5$

9.2: Getting to the Root of the Problem

Find the solution(s) to each of these equations, or explain why there is no solution.

1. $\sqrt{a-5}=5$

2. $\sqrt[3]{a-5}=5$

3. $\sqrt[3]{b}=-2$

4. $\sqrt{c}+2=0$

5. $\sqrt[3]{3-d}+4=0$

6. $\sqrt{7}=\sqrt{x-1}$

7. $\sqrt{36}=3y$

8. $22z=\sqrt[3]{11}$

9.3: Write Your Own Equation

1. Write an equation that includes a radical symbol with:
    a. one solution

    b. no solutions

    c. two solutions

2. Switch with a partner and solve their equations.

Are you ready for more?

Find all solutions to the equation $\sqrt{x} = \sqrt[3]{x}$. Explain how you know those are all of the solutions.

Lesson 9 Summary

Whenever we have an equation with a radical symbol that contains a variable, we can solve it by isolating the radical and then raising each side of the equation to a power in order to get a new equation without radicals. Here is an example:

$$\begin{aligned} \text{-}4 &= \sqrt[3]{5p+1} \\ (\text{-}4)^3 &= \left(\sqrt[3]{5p+1}\right)^3 \\ \text{-}64 &= 5p+1 \\ \text{-}65 &= 5p \\ \text{-}13 &= p \end{aligned}$$

Sometimes this results in an equation with solutions that do not make the original equation true. If we use this strategy, it is good to check the solutions to the new equation we got after raising each side to a power, to be sure they make the original equation true. In this example, we did find a solution to the original equation because $\sqrt[3]{5(\text{-}13)+1} = \text{-}4$.

Another way to solve these equations is to reason about what the answer is, instead of raising each side to a power. For example, if we are solving $\sqrt{1-x} + 5 = 11$, we can rearrange it to get $\sqrt{1-x} = 6$ and then think, “If the positive square root of $1-x$ is 6, then $1-x$ must be 36, since the positive square root of 36 is 6. So x must be $\text{-}35$, since $1-(\text{-}35) = 36$.” If we check this result, we see that $\text{-}35$ is a solution to the original equation because $\sqrt{1-(\text{-}35)} + 5 = 11$.