## Lesson 10: Interpreting and Writing Logarithmic Equations

### 10.1: Reading Logs

The expression $log\_{10}1,​000=3$ can be read as: “The log, base 10, of 1,000 is 3.”

It can be interpreted as: “The exponent to which we raise a base 10 to get 1,000 is 3.”

Take turns with a partner reading each equation out loud. Then, interpret what they mean.

* $log\_{10}100,​000,​000=8$
* $log\_{10}1=0$
* $log\_{2}16=4$
* $log\_{5}25=2$

### 10.2: Base 2 Logarithms

|  |  |
| --- | --- |
| $x$ | $log\_{2}(x)$ |
| 1 | 0 |
| 2 | 1 |
| 3 | 1.5850 |
| 4 | 2 |
| 5 | 2.3219 |
| 6 | 2.5850 |
| 7 | 2.8074 |
| 8 | 3 |
| 9 | 3.1699 |
| 10 | 3.3219 |

|  |  |
| --- | --- |
| $x$ | $log\_{2}(x)$ |
| 11 | 3.4594 |
| 12 | 3.5845 |
| 13 | 3.7004 |
| 14 | 3.8074 |
| 15 | 3.9069 |
| 16 | 4 |
| 17 | 4.0875 |
| 18 | 4.1699 |
| 19 | 4.2479 |
| 20 | 4.3219 |

|  |  |
| --- | --- |
| $x$ | $log\_{2}(x)$ |
| 21 | 4.3923 |
| 22 | 4.4594 |
| 23 | 4.5236 |
| 24 | 4.5850 |
| 25 | 4.6439 |
| 26 | 4.7004 |
| 27 | 4.7549 |
| 28 | 4.8074 |
| 29 | 4.8580 |
| 30 | 4.9069 |

|  |  |
| --- | --- |
| $x$ | $log\_{2}(x)$ |
| 31 | 4.9542 |
| 32 | 5 |
| 33 | 5.0444 |
| 34 | 5.0875 |
| 35 | 5.1293 |
| 36 | 5.1699 |
| 37 | 5.2095 |
| 38 | 5.2479 |
| 39 | 5.2854 |
| 40 | 5.3219 |

1. Use the table to find the exact or approximate value of each expression. Then, explain to a partner what each expression and its approximated value means.
	1. $log\_{2}2$
	2. $log\_{2}32$
	3. $log\_{2}15$
	4. $log\_{2}40$
2. Solve each equation. Write the solution as a logarithmic expression.
	1. $2^{y}=5$
	2. $2^{y}=70$
	3. $2^{y}=999$

### 10.3: Exponential and Logarithmic Forms

These equations express the same relationship between 2, 16, and 4:

$log\_{2}16=4$

$2^{4}=16$

1. Each row shows two equations that express the same relationship. Complete the table.

|  |  |  |
| --- | --- | --- |
| *
 | * exponential form
 | * logarithmic form
 |
| * a.
 | * $2^{1}=2$
 | *
 |
| * b.
 | * $10^{0}=1$
 | *
 |
| * c.
 | *
 | * $log\_{3}81=4$
 |
| * d.
 | *
 | * $log\_{5}1=0$
 |
| * e.
 | * $10^{-1}=\frac{1}{10}$
 | *
 |
| * f.
 | * $9^{\frac{1}{2}}=3$
 | *
 |
| * g.
 | *
 | * $log\_{2}\frac{1}{8}=-3$
 |
| * h.
 | * $2^{y}=15$
 | *
 |
| * i.
 | *
 | * $log\_{5}40=y$
 |
| * j.
 | * $b^{y}=x$
 | *
 |

1. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use “?” for the unknown value.
	1. “To what exponent do we raise the number 4 to get 64?”
	2. “What is the log, base 2, of 128?”

#### Are you ready for more?

1. Is $log\_{2}(10)$ greater than 3 or less than 3? Is $log\_{10}(2)$ greater than or less than 1? Explain your reasoning.
2. How are these two quantities related?

### Lesson 10 Summary

Many relationships that can be expressed with an exponent can also be expressed with a logarithm. Let’s look at this equation: $2^{7}=128$ The base is 2 and the exponent is 7, so it can be expressed as a logarithm with base 2:

$log\_{2}128=7$

In general, an exponential equation and a logarithmic equation are related as shown here:



Exponents can be negative, so a logarithm can have negative values. For example $3^{-4}=\frac{1}{81}$, which means that $log\_{3}\frac{1}{81}=-4$.

An exponential equation cannot always be solved by observation. For example, $2^{x}=19$ does not have an obvious solution. The logarithm gives us a way to represent the solution to this equation: $x=log\_{2}19$. The expression $log\_{2}19$ is approximately 4.25, but $log\_{2}19$ is an exact solution.



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