## Lesson 3: Types of Transformations

* Let’s analyze transformations that produce congruent and similar figures.

### 3.1: Why is it a Dilation?

Point $B$ was transformed using the coordinate rule $(x,y)\rightarrow (3x,3y)$.



1. Add these auxiliary points and lines to create 2 right triangles: Label the origin $P$. Plot points $M=(2,0)$ and $N=(6,0)$. Draw segments $PB^{′},MB,$ and $NB^{′}$.
2. How do triangles $PMB$ and $PNB^{′}$ compare? How do you know?
3. What must be true about the ratio $PB:PB^{′}$?

### 3.2: Congruent, Similar, Neither?

Match each image to its rule. Then, for each rule, decide whether it takes the original figure to a congruent figure, a similar figure, or neither. Explain or show your reasoning.

A



B

C



D



1. $(x,y)\rightarrow \left(\frac{x}{2},\frac{y}{2}\right)$
2. $(x,y)\rightarrow (y,-x)$
3. $(x,y)\rightarrow (-2x,y)$
4. $(x,y)\rightarrow (x−4,y−3)$

#### Are you ready for more?

Here is triangle $A$.



1. Reflect triangle $A$ across the line $x=2$.
2. Write a single rule that reflects triangle $A$ across the line $x=2$.

### 3.3: You Write the Rules



1. Write a rule that will transform triangle $ABC$ to triangle $A^{′}B^{′}C^{′}$.
2. Are $ABC$ and $A^{′}B^{′}C^{′}$ congruent? Similar? Neither? Explain how you know.
3. Write a rule that will transform triangle $DEF$ to triangle $D^{′}E^{′}F^{′}$.
4. Are $DEF$ and $D^{′}E^{′}F^{′}$ congruent? Similar? Neither? Explain how you know.

### Lesson 3 Summary

Triangle $ABC$ has been transformed in two different ways:

* $(x,y)\rightarrow (-y,x)$, resulting in triangle $DEF$
* $(x,y)\rightarrow (x,3y)$, resulting in triangle $XYC$



Let’s analyze the effects of the first transformation. If we calculate the lengths of all the sides, we find that segments $AB$ and $DE$ each measure $\sqrt{5}$ units, $BC$ and $EF$ each measure 5 units, and $AC$ and $DF$ each measure $\sqrt{20}$ units. The triangles are congruent by the Side-Side-Side Triangle Congruence Theorem. That is, this transformation leaves the lengths and angles in the triangle the same—it is a rigid transformation.

Not all transformations keep lengths or angles the same. Compare triangles $ABC$ and $XYC$. Angle $X$ is larger than angle $A$. All of the side lengths of $XYC$ are larger than their corresponding sides. The transformation $(x,y)\rightarrow (x,3y)$ stretches the points on the triangle 3 times farther away from the $x$-axis. This is not a rigid transformation. It is also not a dilation since the corresponding angles are not congruent.



© CC BY 2019 by Illustrative Mathematics®