## Lesson 24: Polynomial Identities (Part 2)

### 24.1: Revisiting an Old Theorem

Instructions to make a right triangle:

* Choose two integers.
* Make one side length equal to the sum of the squares of the two integers.
* Make one side length equal to the difference of the squares of the two integers.
* Make one side length equal to twice the product of the two integers.

Follow these instructions to make a few different triangles. Do you think the instructions always produce a right triangle? Be prepared to explain your reasoning.

### 24.2: Theorems and Identities

Here are the instructions to make a right triangle from earlier:

* Choose two integers.
* Make one side length equal to the sum of the squares of the two integers.
* Make one side length equal to the difference of the squares of the two integers.
* Make one side length equal to twice the product of the two integers.
1. Using $a$ and $b$ for the two integers, write expressions for the three side lengths.
2. Why do these instructions make a right triangle?

### 24.3: Identifying Identities

Here is a list of equations. Circle all the equations that are identities. Be prepared to explain your reasoning.

1. $a=-a$
2. $a^{2}+2ab+b^{2}=(a+b)^{2}$
3. $a^{2}−2ab+b^{2}=(a−b)^{2}$
4. $a^{2}−b^{2}=(a−b)(a−b)$
5. $(a+b)(a^{2}−ab+b^{2})=a^{3}−b^{3}$
6. $(a−b)^{3}=a^{3}−b^{3}−3ab(a+b)$
7. $a^{2}(a−b)^{4}−b^{2}(a−b)^{4}=(a−b)^{5}(a+b)$

### 24.4: Egyptian Fractions



In Ancient Egypt, all non-unit fractions were represented as a sum of distinct unit fractions. For example, $\frac{4}{9}$ would have been written as $\frac{1}{3}+\frac{1}{9}$ (and not as $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ or any other form with the same unit fraction used more than once). Let’s look at some different ways we can rewrite $\frac{2}{15}$ as the sum of distinct unit fractions.

1. Use the formula $\frac{2}{d}=\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{6d}$ to rewrite the fraction $\frac{2}{15}$, then show that this formula is an identity.
2. Another way to rewrite fractions of the form $\frac{2}{d}$ is given by the identity $\frac{2}{d}=\frac{1}{d}+\frac{1}{d+1}+\frac{1}{d(d+1)}$. Use it to re-write the fraction $\frac{2}{15}$, then show that it is an identity.

#### Are you ready for more?

For fractions of the form $\frac{2}{pq}$, that is, fractions with a denominator that is the product of two positive integers, the following formula can also be used: $\frac{2}{pq}=\frac{1}{pr}+\frac{1}{qr}$, where $r=\frac{p+q}{2}$. Use it to re-write the fraction $\frac{2}{45}$, then show that it is an identity.

### Lesson 24 Summary

Sometimes we can think something is an identity when it actually isn’t. Consider the following equations that are sometimes mistaken as identities:

$(a+b)^{2}=a^{2}+b^{2}$

$(a−b)^{2}=a^{2}−b^{2}$

Both of these are true for some very specific values of $a$ and $b$, for example when either $a$ or $b$ is 0, but they are not true for most values of a and b, for example $a=2$ and $b=1$ (try it!). The actual identities associated with the expressions on the left side are $(a+b)^{2}=a^{2}+2ab+b^{2}$ and $(a−b)^{2}=a^{2}−2ab+b^{2}$.

Are polynomials the only types of expressions you can find in identities? Not at all! Here is an identity that shows a relationship between rational expressions:

$\frac{1}{x}=\frac{1}{x+1}+\frac{1}{x(x+1)}$

We can show that this identity is true by adding the terms in the expression on the right using a common denominator:

$\begin{matrix}\frac{1}{x+1}+\frac{1}{x(x+1)}&=\frac{1}{x+1}⋅\frac{x}{x}+\frac{1}{x(x+1)}\\&=\frac{x}{x(x+1)}+\frac{1}{x(x+1)}\\&=\frac{x+1}{x(x+1)}\\&=\frac{1}{x}\end{matrix}$

An important difference from polynomial identities is that identities involving rational expressions could have a few exceptional values of $x$ where they are not true because the rational expressions on one side or the other are not defined. For example, the identity above is true for all values of $x$ except $x=0$ and $x=-1$.



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