Lesson 6: Picking Representatives

Goals

- Compare and contrast different ways to distribute representatives, and recognize that changing the way the votes are grouped can affect the outcome.
- Critique (orally and in writing) whether a method for distributing representatives is fair.
- Suggest a method for distributing representatives and justify (orally) why is it fair.

Lesson Narrative

This lesson is optional. The five activities in this third lesson on the mathematics on voting return to the situation of an election with two choices. However, rather than directly choosing the result, voters elect representatives, each of whom then casts a single vote for all the people they represent. The activities explore ways to “share” the representatives fairly between groups of people. In the first activity, numbers have been designed so that representatives (or computers) can be shared exactly proportionally between several groups. In later activities, it’s impossible to share representatives fairly; students may use division with decimal quotients or with remainders to try to find the least unfair way. The final activity asks students to *gerrymander* several districts: to divide it into sections in two ways to influence the final voting result in opposite ways. The mathematics here involves geometric properties of shapes on maps: area and connectedness, as well as some proportional reasoning.

Most of the activities use students’ skills from earlier units to reason about ratios and proportional relationships (MP2) in the context of real-world problems (MP4). While some of the activities do not involve much computation, they all require serious thinking and decision making (MP3).

Most importantly, this lesson addresses topics that are important for citizens in a democracy to understand. Teachers may wish to collaborate with a civics or government teacher to learn how the fictional middle-school situations in this lesson relate to real-world elections.

Alignments

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- 6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. $Expectations for unit rates in this grade are limited to non-complex fractions.
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct

Required Materials

Four-function calculators

> ### Student Learning Goals
> Let's think about fair representation.

6.1 Computers for Kids

10 minutes

This activity introduces the question for the last five activities in the voting unit: How can we fairly share a small number of representatives between several groups of people?

The first question of the activity asks students to distribute computers to families with children. In this question, computers can be shared so that the same number of children share a computer in each family. In the second question, fair sharing is not possible, so students need to decide and explain which alternative is the fairest, or the least unfair (MP3). Monitor for students who chose different ways to distribute the computers.

Addressing

- 6.NS.B.3
- 6.RP.A.3

Launch

Arrange students in groups of 2. Give students 5 minutes quiet think time and then ask them to compare their work with a partner.

Anticipated Misconceptions

Division has two roles in this activity, as discussed in the unit on dividing fractions. Dividing 16 children by 8 computers is like putting 16 pounds of almonds into 8 bags, giving 2 pounds of almonds per bag. Bagging 6 pounds of almonds at 2 pounds of almonds per bag fills 3 bags.

> ### Student Task Statement
> A program gives computers to families with school-aged children. They have a certain number of computers to distribute fairly between several families. How many computers should each family get?

1. One month the program has 8 computers. The families have these numbers of school-aged children: 4, 2, 6, 2, 2.

    a. How many children are there in all?

    b. Counting all the children in all the families, how many children would use each computer? This is the number of children per computer. Call this number A.

    c. Fill in the third column of the table. Decide how many computers to give to each family if we use A as the basis for distributing the computers.

family	number of children	number of computers, using A
Baum	4	
Chu	2	
Davila	6	
Eno	2	
Farouz	2	

    d. Check that 8 computers have been given out in all.

2. The next month they again have 8 computers. There are different families with these numbers of children: 3, 1, 2, 5, 1, 8.

    a. How many children are there in all?

    b. Counting all the children in all the families, how many children would use each computer? This is the number of children per computer. Call this number B.

    c. Does it make sense that B is not a whole number? Why?

    d. Fill in the third column of the table. Decide how many computers to give to each family if we use B as the basis for distributing the computers.

family	number of children	number of computers, using B	number of computers, your way	children per computer, your way
Gray	3			
Hernandez	1			
Ito	2			
Jones	5			
Krantz	1			
Lo	8			

e. Check that 8 computers have been given out in all.

f. Does it make sense that the number of computers for one family is not a whole number? Explain your reasoning.

g. Find and describe a way to distribute computers to the families so that each family gets a whole number of computers. Fill in the fourth column of the table.

h. Compute the number of children per computer in each family and fill in the last column of the table.

i. Do you think your way of distributing the computers is fair? Explain your reasoning.

Student Response

1. The computers can be fairly shared by two children in each family. There are 16 children and 8 computers. Details are shown in this table.

    a. 16 children

    b. $A = 2$ children per computer

c.

family	number of children	number of computers, using A
Baum	4	2
Chu	2	1
Davila	6	3
Eno	2	1
Farouz	2	1

d. No response required.

2. It isn't possible to fairly distribute the computers, since you can't split computers between families. There are 20 children and 8 computers, so $B = 2.5$ children per computer would be fair. In practice, this means two computers for every 5 children. But this isn't possible unless there is a multiple of 5 children in each family. Other less fair solutions are possible.

    a. 20 children

    b. $B = 2.5$ children per computer. Written as a fraction, $B = \frac{5}{2}$.

    c. $B = 2.5$ makes sense. It's an average, not an actual amount for any children or families.

    d. Divide number of children by 2.5 or $\frac{5}{2}$ children per computer for each family to get number of computers.

family	number of chidren	number of computers, using B	number of computers, your way	children per computer, your way
Gray	3	1.2 or $\frac{6}{5}$	1	3
Hernandez	1	0.4 or $\frac{2}{5}$	1	1
Ito	2	0.8 or $\frac{4}{5}$	1	2
Jones	5	2	2	2.5
Krantz	1	0.4 or $\frac{2}{5}$	1	1
Lo	8	3.2 or $\frac{16}{5}$	2	4

e. The sum of entries in column 3 is 8.

f. It doesn't make sense for a family to get a fractional or decimal amount of a computer; they only work when they are whole. A half of a computer is a broken computer.

g. Responses vary. One solution is given in column 4 of the table. This distribution gives one computer to each family, which uses 6 computers. The two last computers are given to the two largest families. A student could argue that this distribution is somewhat fair because all children at least have access to a computer.

h. See table.

i. It's not completely fair because the Hernandez and Krantz children get the computer all to themselves, while the Lo children need to share with 3 others.

Activity Synthesis

Invite students to share their answers.

For question 1, there should be an agreement that every two children share a computer.

For question 2, find students who chose different ways to distribute the computers.

Ask students which distribution they think is more fair and explain why. There may be no answer that everyone agrees on.

6.2 School Mascot (Part 1)

15 minutes

This lesson explores mathematical difficulties that arise in a representative democracy, where people do not vote individually, but vote for representatives who vote for all their constituents. This is in part a sharing problem. If all the people in a town are to be represented by a few people, the representatives should be shared as equally as possible. However, sometimes the groups to be represented are predetermined, such as classrooms or states. It's not always possible to have the same numbers of constituents per representative. This part of the activity is mathematically the same as sharing computers among families, as in the previous activity.

A further difficulty is that different people in the same group will usually have different opinions. The representative has only one vote, so it's impossible to fairly represent the opinions of all the constituents.

The mathematical issues involve unit rates and division, usually resulting in decimals.

This activity uses a voting situation with one vote per class. Similarly to the families with computers, three classrooms need to share three votes. The vote for the class is whichever choice wins a majority in the class election. Students discover that this system is unfair, since a class voting heavily for one choice counts for the same as a class barely voting for the choice ("yessiness"). They use division to try to devise a more fair system. Monitor for different ways to assign votes.

Note: a banana slug is a bright yellow snail without a shell that lives in redwood forests. Students at the University of California Santa Cruz voted for the banana slug as their mascot; the administration thought sea lions were more dignified.

Addressing

- 6.NS.B.3
- 6.RP.A.3

Launch

Arrange students in groups of 2. Students start with 5 minutes quiet think time, followed by comparing work with a partner. Provide access to four-function calculators.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about calculations involving fractions and percentages. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may be confused about the two-step process: each class votes, then the representative votes for the winner of the class vote. There are only three people voting at the second (representative) level. If necessary, act out the vote by dividing the class into three groups, not necessarily equal, and appointing a representative for each.

Students may object that this system is not fair. They are right: there is the potential for a minority choice to win the election.

Student Task Statement

A school is deciding on a school mascot. They have narrowed the choices down to the Banana Slugs or the Sea Lions.

The principal decided that each class gets one vote. Each class held an election, and the winning choice was the one vote for the whole class. The table shows how three classes voted.

	banana slugs	sea lions	class vote
class A	9	3	banana slug
class B	14	10	
class C	6	30	

1. Which mascot won, according to the principal's plan? What percentage of the votes did the winner get under this plan?
2. Which mascot received the most student votes in all? What percentage of the votes did this mascot receive?
3. The students thought this plan was not very fair. They suggested that bigger classes should have more votes to send to the principal. Make up a proposal for the principal where there are as few votes as possible, but the votes proportionally represent the number of students in each class.
4. Decide how to assign the votes for the results in the class. (Do they all go to the winner? Or should the loser still get some votes?)
5. In your system, which mascot is the winner?
6. In your system, how many representative votes are there? How many students does each vote represent?

Student Response

1. The banana slugs win with $\frac{2}{3}$, or 67% of the representatives' vote (rounded), since Classes A and B voted for banana slugs. Banana slugs got a total of 29 student votes and sea lions got 43.
2. So sea lions should have won with about 60% of the vote because they got 43 votes out of a total of 72 student votes.
3. The smallest number of representatives to give proportional representation has Class A with 1 vote, Class B with 2 votes, and Class C with 3 votes.
   Answers vary. Possible response: Use fractions. Class A has 12 students, or $\frac{12}{72} = \frac{1}{6}$ of all the

students, Class B has 24 students for $\frac{24}{72} = \frac{2}{6}$ of the students, and Class C has 36 students for $\frac{36}{72} = \frac{3}{6}$ of the students. So 6 representatives can be shared fairly among the three classes.
Another method: The greatest common divisor of 12, 24, and 36 is 12. A proportional system of votes would give one vote to every 12 students, so Class A would get 1 vote, Class B 2 votes, and Class C 3 votes.
Other choices are possible, but they will need more than 6 representatives. For representation to be exactly proportional, there must be a multiple of 6 representatives.

4. If the choice winning a majority in a class determines all the votes for the class, then A gives its one vote to banana slugs, B gives 2 votes for banana slugs, and all 3 of C's votes go to sea lions. It's a tie! If you try to assign votes proportionally within classes, then Class B should probably give one vote for banana slugs and one for sea lions, since the numbers are fairly close. This would give 2 total class votes for banana slugs and 4 for sea lions. Now sea lions win.
   There should be a criterion for how to split the class votes; is 14 to 10 closer to 1 to 1, or 2 to 0?

5. Sea lions win if B's votes are split between banana slugs and sea lions. It's a tie if the winner of the class election gets all the votes for that class.

6. There are 6 representatives who vote. Each represents 12 students.

Activity Synthesis

Invite students to share their ways to assign votes and the reasons for their system. Select students who have different methods.

Students should recognize, after discussion, that the system the principal proposed is unfair. A majority in a small class voting for banana slugs can overwhelm a larger number voting for sea lions in a larger class.

A more fair system should take the sizes of the classes into account.

6.3 Advising the School Board

30 minutes
This activity includes two problems of assigning representatives proportionally, with schools sending students to advise the school board. In the first problem, school sizes have been carefully planned so that each school has the same number of students per representative as the district as a whole. In the second, this is not possible, in part because of a very large and a very small school.

The mathematics includes finding rates by division, quotients and divisors that are decimals, and rounding, as well as quantitative reasoning (MP2).

The mathematics is the same as the previous activity distributing computers to families. It is also the same as the problem of assigning congressional representatives to states. Some states have very large populations, and others have very small populations.

As students work, monitor for different ways that they assign advisors.

Addressing

- 6.NS.B.3
- 6.RP.A.2

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2–4. Provide access to four-function calculators if desired.

Explain the situation: “The school board (the elected people who make major decisions about all the schools) wants students from the schools to help them decide, and to give the board advice about what the students at each school think. They would like 10 students to be chosen to come to school board meetings. These students will be called advisors. Big schools should send more advisors than small schools, but even the tiniest school should send at least one advisor. If possible, the number of advisors should be proportional to the number of students at the school.”

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. After students have solved the first 2–3 problems, check in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as inviting them to ask any questions they have before continuing.
Supports accessibility for: Organization; Attention

Access for English Language Learners

Speaking, Writing, Representing: MLR2 Collect and Display. Use this routine to record language that students use to explain their strategies for discovering fair ratios of advisors. While students work through the problems, circulate and record language they use in explaining their methods for achieving fairness in the ratios. There will be multiple ways to calculate or compare. Encourage students to compare numbers, explain how they decided what is fair, and to describe the ideal situation. Pay attention to what they find problematic about working with partial numbers (fractions or decimals) concerning advisor count. Expect students to use specific language like “proportional,” “rates,” “ _____ per_____.”
Design Principle(s): Support sense-making; Maximize meta-awareness

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Student Task Statement

1. In a very small school district, there are four schools, D, E, F, and G. The district wants a total of 10 advisors for the students. Each school should have at least one advisor.

school	number of students	number of advisors, using A
D	48	
E	12	
F	24	
G	36	

   a. How many students are in this district in all?

   b. If the advisors could represent students at different schools, how many students per advisor should there be? Call this number A.

   c. Using A students per advisor, how many advisors should each school have? Complete the table with this information for schools D, E, F, and G.

2. Another district has four schools; some are large, others are small. The district wants 10 advisors in all. Each school should have at least one advisor.

school	number of students	number of advisors, using B	number of advisors, your way	students per advisor, your way
Dr. King School	500			
O'Connor School	200			
Science Magnet School	140			
Trombone Academy	10			

   a. How many students are in this district in all?

   b. If the advisors didn't have to represent students at the same school, how many students per advisor should there be? Call this number B.

c. Using B students per advisor, how many advisors should each school have? Give your quotients to the tenths place. Fill in the first "number of advisors" column of the table. Does it make sense to have a tenth of an advisor?

d. Decide on a consistent way to assign advisors to schools so that there are only whole numbers of advisors for each school, and there is a total of 10 advisors among the schools. Fill in the "your way" column of the table.

e. How many students per advisor are there at each school? Fill in the last row of the table.

f. Do you think this is a fair way to assign advisors? Explain your reasoning.

Student Response

1.

school	number of students	number of advisors, using A
D	48	4
E	12	1
F	24	2
G	36	3

a. 120 students in the district

b. A is 12 students per advisor

c. See table.

2.

school	number of students	number of advisors, using B	number of advisors, your way	students per advisor, your way
Dr. King School	500	5.9	6	83.3
O'Connor School	200	2.4	2	100
Science Magnet School	140	1.6	1	140
Trombone Academy	10	0.1	1	10

a. 850 students in the district

b. B is 85 students per advisor

c. See table. It doesn't make sense to have a tenth of an advisor; you can't have a fraction of a person.

d. Responses vary. The lesson plan shows a strategy: round to the nearest whole number, then adjust. The first attempt gives the Trombone Academy no advisors. The second attempt gives them one of the two from the Science Magnet.

e. Responses vary. See table.

f. Responses vary. The example is unfair because the Trombone Academy is very small but still gets one advisor, and there are 10 students per advisor, compared to the Science Magnet, with 140 students per advisor. Dr. King School gets almost the ideal number of students per advisor.

Activity Synthesis

Invite several students to present their work, especially their attempts to assign advisors fairly. Choose students who chose different ways to assign advisors.

The ideal number of students per representative is 85, an average. King School is very close to this ideal. The other schools have much higher or lower numbers.

The big idea here is that it's impossible to be completely fair. The Trombone Academy will have more clout than the bigger schools because their advisor is representing only 10 students. On the other hand, if the Trombone Academy gets no advisors, then their views aren't represented at all, so this isn't fair either.

6.4 School Mascot (Part 2)

10 minutes

In the previous activities, representatives (“advisors”) were assigned to groups that couldn’t be changed: schools. Sometimes the groups or districts for representatives can be changed, as in districts for the U.S. House of Representatives, and for state legislatures, wards in cities, and so on. Often, the people in an area have similar opinions, so it’s possible to design districts where you can reliably predict the outcomes.

In this lesson, students use geometric reasoning about areas and connectedness to experiment with drawing districts in a way that predict the outcome of elections. This is often called gerrymandering.

Addressing

- 6.RP.A.3
- 6.RP.A.3.c

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2–4.

Access for Students with Disabilities

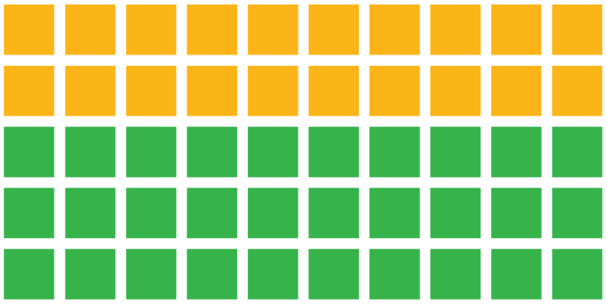
Representation: Develop Language and Symbols. Activate or supply background knowledge about calculating area. Share examples of expressions for area in a few different forms to illustrate how area can be expressed to represent district blocks' votes for seal lions and banana slugs mascots. Allow continued access to concrete manipulatives such as snap cubes for students to view or manipulate.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Student Task Statement

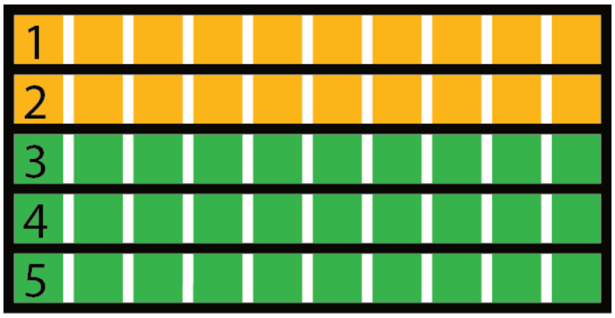
The whole town gets interested in choosing a mascot. The mayor of the town decides to choose representatives to vote.

There are 50 blocks in the town, and the people on each block tend to have the same opinion about which mascot is best. Green blocks like sea lions, and gold blocks like banana slugs. The mayor decides to have 5 representatives, each representing a district of 10 blocks.

Here is a map of the town, with preferences shown.



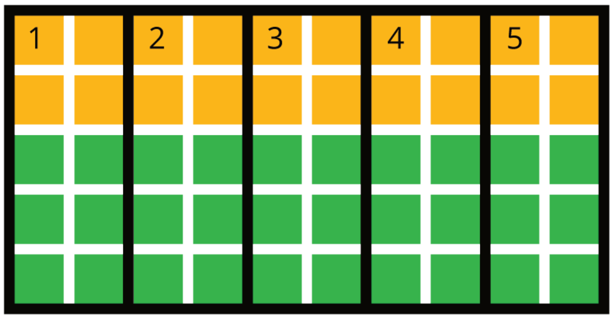
1. Suppose there were an election with each block getting one vote. How many votes would be for banana slugs? For sea lions? What percentage of the vote would be for banana slugs?
2. Suppose the districts are shown in the next map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1	10	0		banana slugs
2				
3				
4				
5				

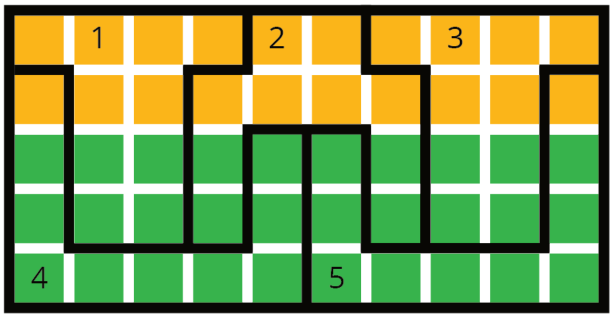
3. Suppose, instead, that the districts are shown in the new map below. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

4. Suppose the districts are designed in yet another way, as shown in the next map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

5. Write a headline for the local newspaper for each of the ways of splitting the town into districts.

6. Which systems on the three maps of districts do you think are more fair? Are any totally unfair?

Student Response

1. 20 votes for banana slugs, 30 votes for sea lions, so sea lions win with 60% of the vote.

2. Sea lions win with 3 of 5 representatives.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative's vote
1	10	0	100%	banana slugs
2	10	0	100%	banana slugs
3	0	10	0%	sea lions
4	0	10	0%	sea lions
5	0	10	0%	sea lions

3. Sea lions win with all 5 representatives.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative’s vote
1	4	6	40%	sea lions
2	4	6	40%	sea lions
3	4	6	40%	sea lions
4	4	6	40%	sea lions
5	4	6	40%	sea lions

4. Banana slugs win with 3 of 5 representatives.

district	number of blocks choosing banana slugs	number of blocks choosing sea lions	percentage of blocks choosing banana slugs	representative’s vote
1	6	4	60%	banana slugs
2	6	4	60%	banana slugs
3	6	4	60%	banana slugs
4	1	9	10%	sea lions
5	1	9	10%	sea lions

5. Responses vary. Examples:

   First map: 60% of Districts and 60% of People Vote for Sea Lions
   Second map: All Districts, but only 60% of People Vote for Sea Lions
   Third map: Banana Slugs Win with 60% of Districts, but Only 40% of People

6. Responses vary. The first map seems fairest since the percentages of the people and the representatives match. The second map has the same winner as the vote of the people but different percentages. The third map seems totally unfair: the percentages are reversed. More than half the people voted for sea lions, but banana slugs won.

Activity Synthesis

Ask students to share some of their headlines. Discuss the fairness of the different district arrangements.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Use this routine to help students identify an error in calculating a percent and to justify their own calculations and "fairness" reasoning. Display the following statement before discussing the final problem: "Both map 1 and 2 are equally fair since 60% of people prefer sea lions. That means they show the same results." Give students 1–2 minutes to improve on the statement in writing. Look for students to notice that the error is in the total number of districts that voted, even though out of that number, 60% did choose sea lions. Give students 2–3 minutes to discuss the original statement and their improvement with a partner. Ask, "Is there an error in the reasoning behind this statement? If so, what is it?" For extra support, encourage students to discuss the "total number of districts." Encourage students to explain what was not "equal" and how the 60% interpretation did not address "equal total values." Select 1–2 students to share their rewritten statements with the class.
Design Principle(s): Optimize output (for explanation); Cultivate conversation; Maximize meta-awareness

6.5 Fair and Unfair Districts

30 minutes
Students design districts in three towns to "gerrymander" the results of elections. In two of the towns, the election results can be skewed to either color. In the third, it isn't possible to skew the results.

Addressing

- 6.RP.A.3
- 6.RP.A.3.c

Launch

Arrange students in groups of 2–4.

Explain the history of gerrymandering: Sometimes people in charge of designing districts make them in strange shapes to give the election results they want. One of the first was Elbridge Gerry (governor of Massachusetts in 1812), whose party designed a district that many people thought looked like a salamander. They called a Gerrymander, and the name stuck. It means a very strangely shaped, spread-out district designed to produce a certain result.

Usually districts are required to be connected: a person traveling to all parts of the district should be able to stay inside the district. There should be no "islands" that are separated by parts of other districts.

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to four out of six task statements.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

The white streets on the maps do not disconnect the districts.

However, probably squares that are only connected at their corners should not be considered connected to each other. (But you or the students can make their own rule, since they are in charge of making districts.)

Student Task Statement

1. Smallville's map is shown, with opinions shown by block in green and gold. Decompose the map to create three connected, equal-area districts in two ways:

    a. Design three districts where *green* will win at least two of the three districts. Record results in Table 1.

    Table 1:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

    b. Design three districts where *gold* will win at least two of the three districts. Record results in Table 2.

    Table 2:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

2. Squaretown's map is shown, with opinions by block shown in green and gold. Decompose the map to create five connected, equal-area districts in two ways:

    a. Design five districts where *green* will win at least three of the five districts. Record the results in Table 3.

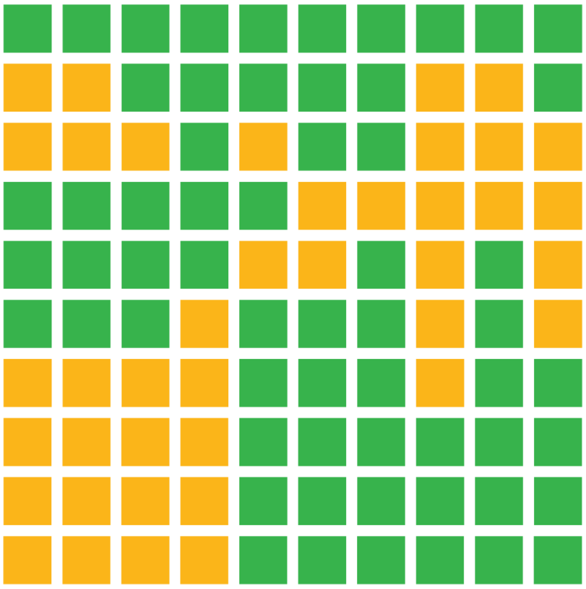


Table 3:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

b. Design five districts where *gold* will win at least three of the five districts. Record the results in Table 4.

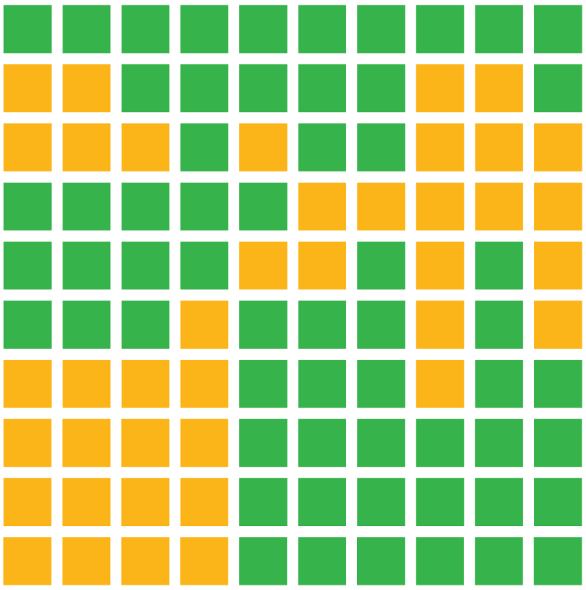


Table 4:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

3. Mountain Valley's map is shown, with opinions by block shown in green and gold. (This is a town in a narrow valley in the mountains.) Can you decompose the map to create three connected, equal-area districts in the two ways described here?

   a. Design three districts where *green* will win at least 2 of the 3 districts. Record the results in Table 5.

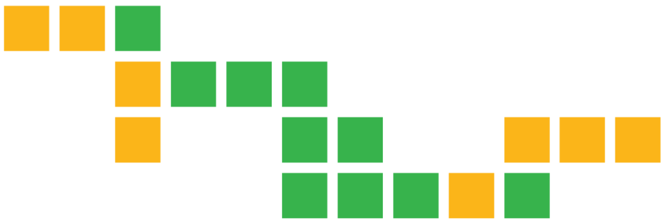
   

   Table 5:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

   b. Design three districts where *gold* will win at least 2 of the 3 districts. Record the results in Table 6.

   Table 6:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

Student Response

1. Answers vary. Districts can be designed so that either color wins the election. Two examples are shown.

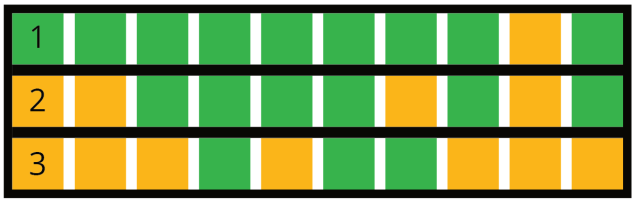


Table 1 shows that green wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	9	1	90%	green
2	6	4	60%	green
3	3	7	30%	gold

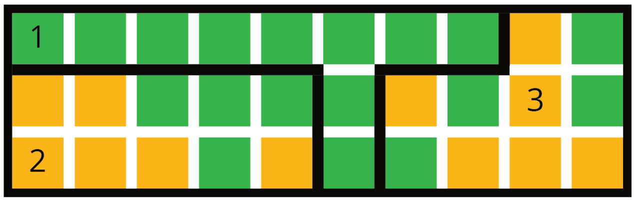


Table 2 shows that gold wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	10	0	100%	green
2	4	6	40%	gold
3	4	6	40%	gold

2. Answers vary. Districts can be designed so that either color wins the election. Two examples are shown.

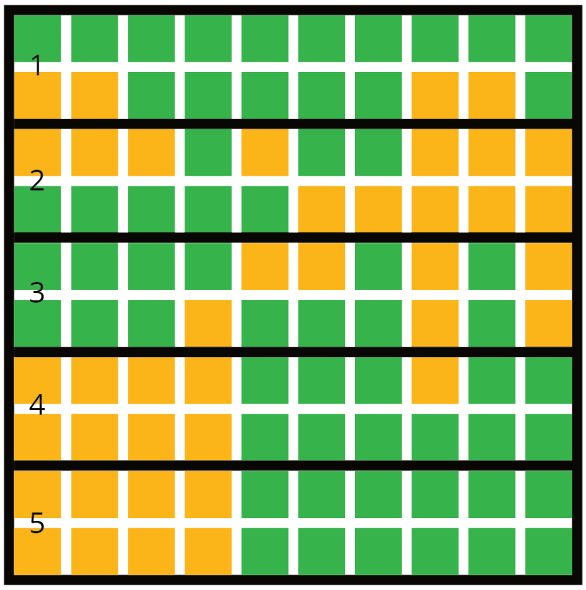


Table 3 shows that green wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative’s vote
1	16	4	80%	green
2	8	12	40%	gold
3	13	7	65%	green
4	11	9	55%	green
5	12	8	60%	green

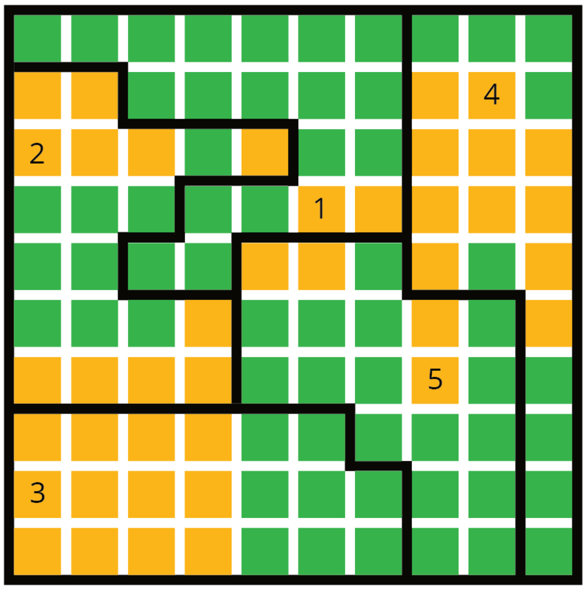


Table 4 shows that gold wins.

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	18	2	90%	green
2	9	11	45%	gold
3	8	12	40%	gold
4	9	11	45%	gold
5	16	4	80%	green

3. Gold wins the election. There is only one way to draw connected districts because the town is so narrow.

   It is impossible to complete table 5.

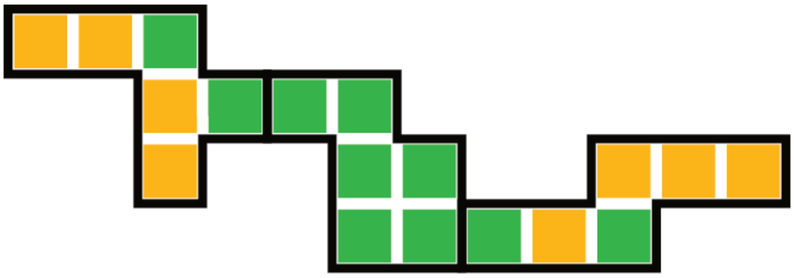


Table 6:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1	2	4	33%	gold
2	6	6	100%	green
3	2	4	33%	gold

Activity Synthesis

This set of questions is similar to the previous activity, except that the patterns are irregular, so more planning and computation is needed to design districts.

Ask several students with different maps to show and explain their work.

The important result for this problem is that it’s possible to design both fair districts (where the result of the vote is similar to the vote if all individual votes were counted) and unfair districts. According to the rules (equal area and connected), these sets of districts are legal. Encourage students to try to devise some rules for drawing districts that would be more fair.