Lesson 5: Negative Rational Exponents

- Let's investigate negative exponents.

5.1: Math Talk: Don't Be Negative

Evaluate mentally.

9^2 9^{-2} $9^{\frac{1}{2}}$ $9^{-\frac{1}{2}}$

5.2: Negative Fractional Powers Are Just Numbers

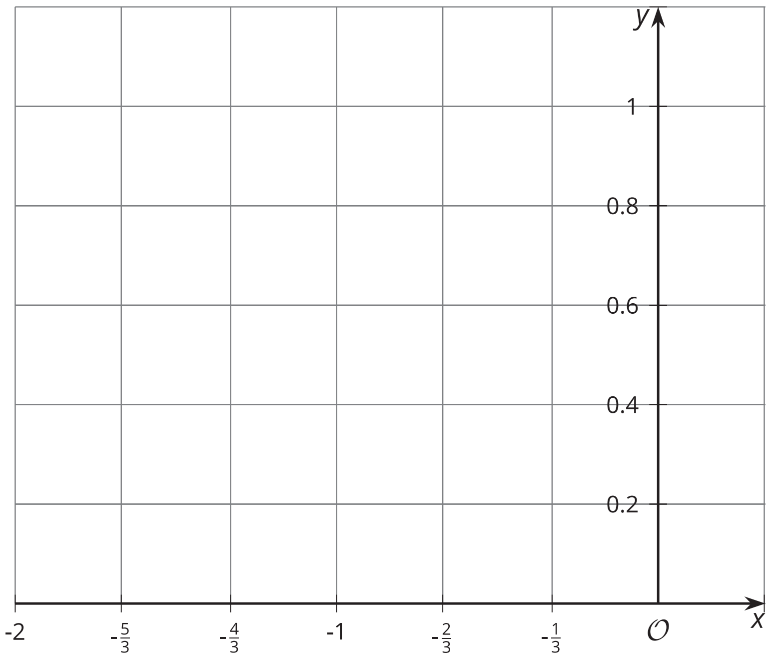
1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

x	-2	$-\frac{5}{3}$	$-\frac{4}{3}$	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0
2^x (using exponents)	2^{-2}	$2^{-\frac{5}{3}}$	$2^{-\frac{4}{3}}$	2^{-1}	$2^{-\frac{2}{3}}$	$2^{-\frac{1}{3}}$	2^0
2^x (decimal approximation)							

a. Plot these powers of 2 in the coordinate plane.

b. Connect the points as smoothly as you can.

c. Use your graph of $y = 2^x$ to estimate the value of the other powers in the table, and write your estimates in the table.



2. Let's investigate $2^{-\frac{1}{3}}$.

a. Write $2^{-\frac{1}{3}}$ using radical notation.

b. What is the value of $\left(2^{-\frac{1}{3}}\right)^3$?

c. Raise your estimate of $2^{-\frac{1}{3}}$ to the third power. What should it be? How close did you get?

3. Let's investigate $2^{-\frac{2}{3}}$.

a. Write $2^{-\frac{2}{3}}$ using radical notation.

b. What is $\left(2^{-\frac{2}{3}}\right)^3$?

c. Raise your estimate of $2^{-\frac{2}{3}}$ to the third power. What should it be? How close did you get?

5.3: Any Fraction Can Be an Exponent

1. For each set of 3 numbers, cross out the expression that is not equal to the other two expressions.
   a. $8^{\frac{4}{5}}$, $\sqrt[4]{8}^{5}$, $\sqrt[5]{8}^{4}$
   b. $8^{-\frac{4}{5}}$, $\frac{1}{\sqrt[5]{8^4}}$, $-\frac{1}{\sqrt[5]{8^4}}$
   c. $\sqrt{4^3}$, $4^{\frac{3}{2}}$, $4^{\frac{2}{3}}$
   d. $\frac{1}{\sqrt{4^3}}$, $-4^{\frac{3}{2}}$, $4^{-\frac{3}{2}}$
2. For each expression, write an equivalent expression using radicals.
   a. $17^{\frac{3}{2}}$
   b. $31^{-\frac{3}{2}}$
3. For each expression, write an equivalent expression using only exponents.
   a. $\left(\sqrt{3}\right)^4$
   b. $\frac{1}{\left(\sqrt[3]{5}\right)^6}$

Are you ready for more?

Write two different expressions that involve only roots and powers of 2 which are equivalent to $\frac{4^{\frac{2}{3}}}{8^{\frac{1}{4}}}$.

5.4: Make These Exponents Less Complicated

Match expressions into groups according to whether they are equal. Be prepared to explain your reasoning.

$\left(\sqrt{3}\right)^4$	$\sqrt{3^2}$	$\left(3^{\frac{1}{2}}\right)^4$	$(\sqrt{3})^2 \cdot (\sqrt{3})^2$
$\left(3^2\right)^{\frac{1}{2}}$	3^2	$3^{\frac{4}{2}}$	$\left(3^{\frac{1}{2}}\right)^2$

Lesson 5 Summary

When we have a number with a negative exponent, it just means we need to find the reciprocal of the number with the exponent that has the same magnitude, but is positive. Here are two examples:

$$7^{-5} = \frac{1}{7^5}$$

$$7^{-\frac{6}{5}} = \frac{1}{7^{\frac{6}{5}}}$$

The table shows a few more examples of exponents that are fractions and their radical equivalents.

x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
5^x (using exponents)	5^{-1}	$5^{-\frac{2}{3}}$	$5^{-\frac{1}{3}}$	5^0	$5^{\frac{1}{3}}$	$5^{\frac{2}{3}}$	5^1
5^x (equivalent expressions)	$\frac{1}{5}$	$\frac{1}{\sqrt[3]{5^2}}$ or $\frac{1}{\sqrt[3]{25}}$	$\frac{1}{\sqrt[3]{5}}$	1	$\sqrt[3]{5}$	$\sqrt[3]{5^2}$ or $\sqrt[3]{25}$	5